# The properties of $\phi(2170)$ and its three-body nature 

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#### Abstract

We summarize the results we obtained for the partial decay widths of $\phi(2170)$ into two-body final states formed by a $\bar{K}$ and a Kaonic resonance, like $K(1460), K_{1}(1270)$, as well as to final states constituted by a $\phi$ and an $\eta / \eta^{\prime}$ mesons. The results obtained are compared with the values extracted from experimental data on the corresponding branching ratios, which were determined by the BESIII collaboration. A reasonable agreement is found, which together with the previous reproduction of the mass, width, and cross section for the process $e^{+} e^{-} \rightarrow \phi f_{0}$ strongly indicates the molecular nature of $\phi(2170)$ as a $\phi K \bar{K}$ system.


## 1 Introduction

Since its discovery in 2006 by the BaBar collaboration, several experimental collaborations have been trying to understand the properties of the $\phi(2170)$ meson [1-7]. Recently, the BESIII collaboration [6, 8, 9] have determined the product between the decay width of $\phi(2170) \rightarrow e^{+} e^{-}$and the branching fraction of $\phi(2170) \rightarrow \bar{K} K_{R}, \phi \eta, \phi \eta^{\prime}$, with $K_{R}$ being a Kaonic resonance, from fits to the corresponding data. The results found seem to challenge the theoretical predictions for the partial decay widths of $\phi(2170)$ to the same $\bar{K} K_{R}, \phi \eta, \phi \eta^{\prime}$ channels obtained within a $s \bar{s}$, hybrid or tetraquark picture for its inner structure [6, 8-11]. In Ref. [12], the $\phi K \bar{K}$ system was studied considering interactions in s-wave, and the solution of the Faddeev equations was obtained for such system within the approach of Refs. [1315]. As a result, the three-body $T$-matrix for the system shows the generation of a state with mass and width compatible with that of $\phi(2170)$ when the $K \bar{K}$ system is forming $f_{0}(980)$. The $e^{+} e^{-} \rightarrow \phi f_{0}(980)$ cross section determined by the BaBar collaboration was also well reproduced with the model of Ref. [12] by implementing the final state interaction in the $e^{+} e^{-} \rightarrow \phi f_{0}(980)$ cross section calculated with the approach of Ref. [16], which explained the background of the process, but not the signal observed for $\phi(2170)$.

Given the recent data obtained by the BESIII collaboration about some partial decay widths of $\phi(2170)$, it would be interesting to know the corresponding values determined with the model of Ref. [12] and check if they agree, or not, with the experimental data. Such compatibility with the data is by no means trivial, since models considering $\phi(2170)$ as a $s \bar{s}$, hybrid, tetraquark, etc., do not seem to give compatible results for all the known experimental data for $\phi(2170)$, which include the previous mentioned partial decay widths, cross sections

[^0]

Figure 1. Decay mechanism for $\phi(2170)$ to $\bar{K} K_{R}, K_{R} \equiv K(1460), \bar{K} K_{1}, K_{1} \equiv K_{1}(1270), K_{1}(1400)$, and to $\phi \eta, \phi \eta^{\prime}$.
obtained from $e^{+} e^{-}$collisions, mass, and width. For instance, the BESIII collaboration has found that the decay mode of $\phi(2170)$ to $K^{*}(892) \bar{K}^{*}(892)$ is suppressed as compared to other $\bar{K} K_{R}$ final states. This fact alone does not seem to be understood considering $\phi(2170)$ to be a $s \bar{s}$ or hybrid state.

## 2 Formalism

The partial decay widths of $\phi(2170)$ to the aforementioned channels not only depend on the nature of $\phi(2170)$, but also on that of the Kaonic resonances present in the final state, like $\mathbb{K} \equiv K(1460), K_{1}(1270), K_{1}(1400)$. Having a good description of the properties of these latter states is relevant to having reliable partial decay widths for $\phi(2170)$. In the case of $K(1460)$, we consider the model of Ref. [17] in which the state is described from the $K K \bar{K}$ interaction, with a large coupling to the $K f_{0}(980)$ configuration. In the case of $K_{1}(1270)$ and $K_{1}(1400)$ we consider three different approaches: (1) In Ref. [18], the $K \rho$ and pseudoscalarvector coupled channel dynamics were studied and generation of $K_{1}(1270)$ was found as a consequence of the superposition of two poles, one at $z_{1}=M-i \Gamma / 2=1195-i 123 \mathrm{MeV}$ and other at $z_{2}=1284-i 73 \mathrm{MeV}$. In this case, no signal for $K_{1}(1400)$ was obtained. We call this model $A$; (2) In Ref. [19], a tensor formalism for the vector mesons was used and $K_{1}(1270)$ and $K_{1}(1400)$ were described as states obtained from the mixing of the $K_{1 A}$ and $K_{1 B}$ states belonging to the nonet of axial resonances. Mixing angles of $29^{\circ}-62^{\circ}$ were shown to be compatible with the experimental data available for these states. We call this model $B$; (3) Instead of relying on the results found within the previous two models for the coupling constants of the $K_{1}$ states to pseudoscalar-vector meson channels, we could directly use the data on the radiative decay of $K_{1}(1270)$ and $K_{1}(1400)$ available on the particle data book to estimate such couplings. We call this model $C$.

Having in ming the coupling of $K_{1}(1270)$ and $K_{1}(1400)$ to pseudoscalar-vector channels, the molecular nature of $\phi(2170)$ as a $\phi f_{0}(980)$ state and that of $f_{0}(980)$ as a state obtained from the $K \bar{K}$ and pseudoscalar-pseudoscalar coupled channel dynamics [20,21], the decay of $\phi(2170) \rightarrow \bar{K} K_{R}, \phi \eta$ and $\phi \eta^{\prime}$ proceeds as depicted in Fig. 1.

Following Refs. [12, 17, 18, 20], the states $\phi(2170), K(1460), K_{1}(1270)$, and $f_{0}(980)$ are generated from the s-wave interactions of three or two hadron systems. Thus, the contribution of the vertices $\phi(2170) \rightarrow \phi f_{0}(980), f_{0} K^{+} \rightarrow K^{+}(1460), \phi \rightarrow K_{1}^{+} K^{-}$present in the decay mechanisms of Fig. 1 can be written as,

$$
\begin{align*}
& t_{\phi_{R}}=g_{\phi_{R} \rightarrow \phi f_{0}} \epsilon_{\phi R} \cdot \epsilon_{\phi}, \quad t_{K_{R}}=g_{K_{R}^{+} \rightarrow K^{+} f_{0}}, \\
& t_{f_{0} \rightarrow \mathcal{P} \mathcal{P}^{\prime}}=g_{f_{0} \rightarrow \mathcal{P} \mathcal{P}^{\prime}}, \quad t_{K_{1}^{+} \rightarrow \phi K^{+}}=g_{K_{1}^{+} \rightarrow \phi K} \epsilon_{K_{1}^{+}} \cdot \epsilon_{\phi}, \tag{1}
\end{align*}
$$

where $g_{i \rightarrow j}$ is the coupling for the process $i \rightarrow j, \mathcal{P}$ and $\mathcal{P}^{\prime}$ represent pseudocalar particles and $\epsilon_{k}$ is the corresponding polarization vector for particle $k$. To determine the amplitude for
the $\phi \rightarrow \mathcal{P}_{1} \mathcal{P}_{2}$ vertex, we consider the Lagrangian [22]

$$
\begin{equation*}
\mathcal{L}_{V P P}=-i g\left\langle V^{\mu}\left[\mathbb{P}, \partial_{\mu} \mathbb{P}\right]\right\rangle, \tag{2}
\end{equation*}
$$

with $V^{\mu}$ and $\mathbb{P}$ being matrices having as elements the vector and pseudoscalar meson octet fields, respectively, $g=M_{V} /\left(2 f_{\pi}\right), M_{V} \simeq M_{\rho}, f_{\pi} \simeq 93 \mathrm{MeV}$, and 〈 > indicating the $\mathrm{SU}(3)$ trace. The coupling constant $g_{f_{0} \rightarrow \mathcal{P} \nu}$ is obtained from the residue of the two-body $t$-matrix describing the interaction between two pseudoscalars. This $t$-matrix is obtained by solving the Bethe-Salpeter equation with a kernel $V$ which is determined from the lowest-order chiral Lagrangian $\mathcal{L}_{\mathbb{P} P}$, implementing the $\eta-\eta^{\prime}$ mixing [23-25],

$$
\begin{equation*}
\mathcal{L}_{\mathbb{P} \mathbb{P}}=\frac{1}{12 f^{2}}\left\langle\left(\partial_{\mu} \mathbb{P P}-\mathbb{P} \partial_{\mu} \mathbb{P}\right)^{2}+M \mathbb{P}^{4}\right\rangle \tag{3}
\end{equation*}
$$

with

$$
\mathbb{P}=\left(\begin{array}{ccc}
A(\beta) \eta+B(\beta) \eta^{\prime}+\frac{\pi^{0}}{\sqrt{2}} & \pi^{+} & K^{+}  \tag{4}\\
\pi^{-} & A(\beta) \eta+B(\beta) \eta^{\prime}-\frac{\pi^{0}}{\sqrt{2}} & K^{0} \\
K^{-} & \bar{K}^{0} & C(\beta) \eta+D(\beta) \eta^{\prime}
\end{array}\right)
$$

where

$$
\begin{align*}
& A(\beta)=-\frac{\sin \beta}{\sqrt{3}}+\frac{\cos \beta}{\sqrt{6}}, \quad B(\beta)=\frac{\sin \beta}{\sqrt{6}}+\frac{\cos \beta}{\sqrt{3}} \\
& C(\beta)=-\frac{\sin \beta}{\sqrt{3}}-\sqrt{\frac{2}{3}} \cos \beta, \quad D(\beta)=-\sqrt{\frac{2}{3}} \sin \beta+\frac{\cos \beta}{\sqrt{3}} \tag{5}
\end{align*}
$$

with the mixing angle $\beta$ being between $-15^{\circ}$ to $-22^{\circ}$, instead of simply considering ideal mixing (i.e., $\sin \beta=-1 / 3$, thus $\beta \simeq-19.47^{\circ}$ ) [26], and $M$ is a matrix having as elements

$$
M=\left(\begin{array}{ccc}
m_{\pi}^{2} & 0 & 0  \tag{6}\\
0 & m_{\pi}^{2} & 0 \\
0 & 0 & 2 m_{K}^{2}-m_{\pi}^{2}
\end{array}\right)
$$

where $m_{\pi}, m_{K}$ represent the masses of the pion and the kaon, respectively. When calculating the coupling of $f_{0}(980)$ to $\mathcal{P} \overline{\mathcal{P}}^{\prime}$, two models were considered: (I) We use in Eq. (3) different weak decay constants for the pseudoscalars; (II) We consider a common value $f=f_{\pi}=93$ MeV .

We refer the reader to Refs. [10, 11] for the values of the coupling constants involved in the vertices depicted in Fig. 1. Using the amplitudes of Eq. (1), we can determine the contribution of the processes depicted in Fig. 1, which depend on different tensor integrals. As a consequence of the four-momenta dependence of the vertices, these tensor integrals can be written as $d^{4} q$ integrals of a numerator that depends on $q_{\mu}, q_{\nu} q_{\mu}$, etc., and a denominator which is a function of $q, P$ and $k$, with the latter dependence being related to the propagators of the particles in the triangular loops of Fig. 1. Using Lorentz covariance, we can write these tensor integrals in terms of a linear combination of $k_{\mu}, P_{\mu}$ or products of $k_{\mu}$ and $P_{\mu}$, depending on the order of the tensor. Such a linear combination introduces several unknown coefficients, which need to be determined.

For example, the amplitude for the process $\phi(2170) \rightarrow \phi \mathcal{P}$ depicted in Fig. 1, where $\mathcal{P}$ represents, in this case, an $\eta$ or $\eta^{\prime}$ meson, can be written as [11]

$$
\begin{equation*}
i t_{\phi_{R} \rightarrow \phi \mathcal{P}}=\sum_{\mathcal{P}^{\prime}} 2 g_{\phi_{R} \rightarrow \phi f_{0}} g_{f_{0} \rightarrow \mathscr{P} \overline{\mathcal{P}}^{\prime}} g_{\phi \rightarrow \phi^{\prime}} \epsilon^{\mu v \alpha \beta} \epsilon_{\phi_{R} v}(P) k_{\alpha} \epsilon_{\phi \beta}(k) I_{\mu}, \tag{7}
\end{equation*}
$$

where $I_{\mu}$ is the following tensor integral:

$$
\begin{equation*}
I_{\mu}=\int_{-\infty}^{\infty} \frac{d^{4} q}{(2 \pi)^{4}} \frac{q_{\mu}}{\left[(P-k-q)^{2}-m_{f_{0}}^{2}+i \epsilon\right]} \frac{1}{\left[(k+q)^{2}-m_{\phi}^{2}+i \epsilon\right]\left[q^{2}-m_{P^{\prime}}^{2}+i \epsilon\right]} . \tag{8}
\end{equation*}
$$

As a consequence of the Lorentz covariance, we can write $I_{\mu}$ in terms of $k_{\mu}$ and $P_{\mu}$ as

$$
\begin{equation*}
I_{\mu}=a_{\mathcal{P}^{\prime}} k_{\mu}+b_{\mathcal{P}^{\prime}} P_{\mu} \tag{9}
\end{equation*}
$$

where $a_{\mathcal{P}}$, and $b_{\mathcal{P}}$ are the mentioned unknown coefficients. To calculate them, we proceed as follows: Multiplying Eq. (9) by $k^{\mu}$ and $P^{\mu}$, respectively, we get two coupled equations which permit to write $a_{\mathcal{P}^{\prime}}$ and $b_{\mathcal{P}}$, as

$$
\begin{equation*}
a_{\mathcal{P}^{\prime}}=\frac{P^{2}(k \cdot I)-(k \cdot P)(P \cdot I)}{k^{2} P^{2}-(k \cdot P)^{2}}, \quad b_{\mathcal{P}^{\prime}}=-\frac{(k \cdot P)(k \cdot I)-k^{2}(P \cdot I)}{k^{2} P^{2}-(k \cdot P)^{2}}, \tag{10}
\end{equation*}
$$

where we have introduced

$$
\begin{align*}
& k \cdot I=\int_{-\infty}^{\infty} \frac{d^{4} q}{(2 \pi)^{4}} \frac{k \cdot q}{\left[(P-k-q)^{2}-m_{f_{0}}^{2}+i \epsilon\right]} \frac{1}{\left[(k+q)^{2}-m_{\phi}^{2}+i \epsilon\right]\left[q^{2}-m_{\mathcal{P}^{\prime}}^{2}+i \epsilon\right]}, \\
& P \cdot I=\int_{-\infty}^{\infty} \frac{d^{4} q}{(2 \pi)^{4}} \frac{P \cdot q}{\left[(P-k-q)^{2}-m_{f_{0}}^{2}+i \epsilon\right]} \frac{1}{\left[(k+q)^{2}-m_{\phi}^{2}+i \epsilon\right]\left[q^{2}-m_{\mathcal{P}^{\prime}}^{2}+i \epsilon\right]} . \tag{11}
\end{align*}
$$

By working in the rest frame of the decaying particle, i.e., $P^{\mu}=\left(P^{0}, \overrightarrow{0}\right)$, with $P^{0}=m_{\phi_{R}}$, we can express the previous integrals as

$$
\begin{align*}
k \cdot I & =\int_{-\infty}^{\infty} \frac{d^{3} q}{(2 \pi)^{3}} \int_{-\infty}^{\infty} \frac{d q^{0}}{(2 \pi)} \frac{k^{0} q^{0}-\vec{k} \cdot \vec{q}}{\left[(P-k-q)^{2}-m_{f_{0}}^{2}+i \epsilon\right]} \frac{1}{\left[(k+q)^{2}-m_{\phi}^{2}+i \epsilon\right]\left[q^{2}-m_{\mathcal{P}^{\prime}}^{2}+i \epsilon\right]} \\
& \equiv \int_{-\infty}^{\infty} \frac{d^{3} q}{(2 \pi)^{3}}\left[k^{0} I_{1}\left(m_{f_{0}}, m_{\phi}, m_{\mathcal{P}^{\prime}}\right)-\vec{k} \cdot \vec{q} I_{0}\left(m_{f_{0}}, m_{\phi}, m_{\mathcal{P}^{\prime}}\right)\right], \\
P \cdot I & =P^{0} \int_{-\infty}^{\infty} \frac{d^{3} q}{(2 \pi)^{3}} \int_{-\infty}^{\infty} \frac{d q^{0}}{(2 \pi)} \frac{q^{0}}{\left[(P-k-q)^{2}-m_{f_{0}}^{2}+i \epsilon\right]} \frac{1}{\left[(k+q)^{2}-m_{\phi}^{2}+i \epsilon\right]\left[q^{2}-m_{\mathcal{P}^{\prime}}^{2}+i \epsilon\right]} \\
& \equiv P^{0} \int_{-\infty}^{\infty} \frac{d^{3} q}{(2 \pi)^{3}} I_{1}\left(m_{f_{0}}, m_{\phi}, m_{\mathcal{P}^{\prime}}\right), \tag{12}
\end{align*}
$$

where we have introduced

$$
\begin{equation*}
I_{n}\left(m_{1}, m_{2}, m_{3}\right) \equiv \int_{-\infty}^{\infty} \frac{d q^{0}}{(2 \pi)} \frac{\left(q^{0}\right)^{n}}{\left[(P-k-q)^{2}-m_{1}^{2}+i \epsilon\right]} \frac{1}{\left[(k+q)^{2}-m_{2}^{2}+i \epsilon\right]\left[q^{2}-m_{3}^{2},+i \epsilon\right]} \tag{13}
\end{equation*}
$$

with $n=0,1$. The integral in Eq. (13) can be calculated analytically by using Cauchy's theorem, finding

$$
\begin{equation*}
I_{n}\left(m_{1}, m_{2}, m_{3}\right)=-i \frac{N_{n}\left(m_{1}, m_{2}, m_{3}\right)}{D\left(m_{1}, m_{2}, m_{3}\right)} \tag{14}
\end{equation*}
$$

The $N_{n}$ and $D$ in Eq. (14) depend on the energy of the particles involved in the loop and we refer the reader to Refs. [10, 11] for more details. The integral in $d^{3} q$ in Eq. (12) can be obtained as

$$
\begin{equation*}
\int_{-\infty}^{\infty} \frac{d^{3} q}{(2 \pi)^{3}}(\cdots) \rightarrow \int_{0}^{\infty} \frac{d|\vec{q}||\vec{q}|^{2}}{(2 \pi)^{2}} \int_{-1}^{1} \operatorname{dcos} \theta F(\Lambda,|\vec{k}+\vec{q}|) F\left(\bar{\Lambda},\left|\vec{q}^{\mathrm{CM}}\right|\right)(\cdots), \tag{15}
\end{equation*}
$$

where we consider $\vec{k}=|\vec{k}| \hat{z}$, and $\vec{q}=|\vec{q}| \sin \theta(\cos \phi \hat{i}+\sin \phi \hat{j})+|\vec{q}| \cos \theta \hat{k}$, such that $\vec{k} \cdot \vec{q}=|\vec{k}| \vec{q} \mid \cos \theta$ and, thus, the integral in $d \phi$ is trivial. In Eq. (15), $F$ represents a form factor introduced for the different vertices to take into account the finite size of $\phi(2170), f_{0}(980)$, etc., and $\Lambda, \bar{\Lambda}$ are cutoffs of around 1000 MeV for the center-of-mass momentum of the particles forming these states. Typical expressions for the form factors in Eq. (15) are Lorentz [27],

$$
\begin{equation*}
F(\Lambda,|\vec{Q}|)=\frac{\Lambda^{2}}{\Lambda^{2}+|\vec{Q}|^{2}}, \tag{16}
\end{equation*}
$$

or Gaussian functions,

$$
\begin{equation*}
F(\Lambda,|\vec{Q}|)=e^{-\frac{-\left.\vec{Q}\right|^{2}}{2 \Lambda^{2}}} . \tag{17}
\end{equation*}
$$

Once the coefficients appearing in the Lorentz expansion of the corresponding tensor integrals are determined, the partial decay width $\phi(2170) \rightarrow A B$ can be obtained by means of

$$
\begin{equation*}
\Gamma_{\phi_{R} \rightarrow A B}=\frac{\left|\vec{p}_{\mathrm{CM}}\right|}{24 \pi m_{\phi_{R}}^{2}} \sum_{\mathrm{pol}}\left|t_{\phi_{R} \rightarrow A B}\right|^{2}, \tag{18}
\end{equation*}
$$

with $\left|\vec{p}_{\mathrm{CM}}\right|$ being the modulus of the center-of-mass momentum of the particles in the final state and $\sum_{\text {pol }}$ indicating the sum over the polarizations of the initial and final states.

## 3 Results

In Tables 1, 2, and 3 we show the results obtained within our description for the branching fractions

$$
\begin{align*}
& B_{1} \equiv \frac{\Gamma_{\phi_{R} \rightarrow K^{+}(1460) K^{-}}}{\Gamma_{\phi_{R} \rightarrow K_{1}^{+}(1400) K^{-}}}=\frac{\mathcal{B} r\left[\phi_{R} \rightarrow K^{+}(1460) K^{-}\right]}{\mathcal{B} r\left[\phi_{R} \rightarrow K_{1}^{+}(1400) K^{-}\right]},  \tag{19}\\
& B_{2} \equiv \frac{\Gamma_{\phi_{R} \rightarrow K^{+}(1460) K^{-}}}{\Gamma_{\phi_{R} \rightarrow K_{1}^{+}(1270) K^{-}}}=\frac{\mathcal{B} r\left[\phi_{R} \rightarrow K^{+}(1460) K^{-}\right]}{\mathcal{B} r\left[\phi_{R} \rightarrow K_{1}^{+}(1270) K^{-}\right]},  \tag{20}\\
& B_{3} \equiv \frac{\Gamma_{\phi_{R} \rightarrow K_{1}^{+}(1270) K^{-}}}{\Gamma_{\phi_{R} \rightarrow K_{1}^{+}(1400) K^{-}}}=\frac{\mathcal{B r}\left[\phi_{R} \rightarrow K_{1}^{+}(1270) K^{-}\right]}{\mathcal{B} r\left[\phi_{R} \rightarrow K_{1}^{+}(1400) K^{-}\right]} . \tag{21}
\end{align*}
$$

The values listed in the aforementioned tables can be compared with those obtained from the experimental values: in Ref. [8], the values (in eV ) for the products $\mathcal{B} r \Gamma_{R}^{e^{+} e^{-}}$are

$$
\begin{align*}
& \mathcal{B r}\left[\phi_{R} \rightarrow K^{+}(1460) K^{-}\right] \Gamma_{R}^{e^{+} e^{-}}=3.0 \pm 3.8, \\
& \mathcal{B r}\left[\phi_{R} \rightarrow K_{1}^{+}(1400) K^{-}\right] \Gamma_{R}^{e^{+} e^{-}}=\left\{\begin{array}{c}
4.7 \pm 3.3, \text { Solution 1 } \\
98.8 \pm 7.8, \text { Solution 2 }
\end{array},\right. \\
& \mathcal{B r}\left[\phi_{R} \rightarrow K_{1}^{+}(1270) K^{-}\right] \Gamma_{R}^{e^{+} e^{-}}=\left\{\begin{array}{c}
7.6 \pm 3.7, \text { Solution 1 } \\
152.6 \pm 14.2, \text { Solution 2 }
\end{array} .\right. \tag{22}
\end{align*}
$$

where two possible solutions for $\mathcal{B r} \Gamma_{R}^{e^{+} e^{-}}$from the fits to the data were found in Ref. [8] in case of the decays $\phi(2170) \rightarrow K_{1}^{+}(1400) K^{-}, K_{1}^{+}(1270) K^{-}$. Using Eq. (22), we can obtain the experimental values for the $B_{1}, B_{2}$ and $B_{3}$ ratios of Eqs. (19)-(21), which are listed under the label "Experiment" in Tables 1-3. The theoretical values found for $B_{1}, B_{2}$, and $B_{3}$, as shown in Ref. [10] do not depend much on the form factor considered in the vertices involved in the mechanisms depicted in Fig. 1 and we provide here an average value of the results obtained with a Heaviside, a Lorentz and a Gaussian form factors.

As can be seen in Tables 1-3, we find compatible results with the values extracted from the experiment, however, there is a strong dependence of these ratios on the particular model used to describe $K_{1}(1270)$ and $K_{1}(1400)$. More precise data would be required to distinguish whether $K_{1}(1270)$ is a state generated from the pseudoscalar-vector dynamics considered in Ref. [18]. Note, however, that only if a superposition of the two poles obtained in Ref. [18] is considered in the calculation, a solution compatible with the value extracted from the experiment is obtained. Also, model B does not seem to give a good description of the ratio $B_{2}$.

Table 1. Results for the branching ratio $B_{1}$.

|  |  | $B_{1}$ |
| :---: | :---: | :---: |
| Our results | Model B | $0.62 \pm 0.20$ |
|  | Model C | $0.11 \pm 0.04$ |
| Experiment | Solution 1 | $0.64 \pm 0.92$ |
|  | Solution 2 | $0.03 \pm 0.04$ |

Table 2. Results for the ratio $B_{2}$. In the case of model $C$, due to the uncertainty in the partial decay widths for $K_{1}(1270)$ and $K_{1}(1400)$ listed in Ref. [28], three different solutions were found in Ref. [10] for the value of the coupling constant of $K_{1}$ to $\phi K$.

|  |  | $B_{2}$ |  |
| :--- | :--- | :---: | :--- |
|  |  | $1.3 \pm 0.4$ | $\left(\right.$ Poles $\left.z_{1}, z_{2}\right)$ |
| Our results | Model A | $3.6 \pm 1.2$ | $\left(\right.$ Pole $\left.z_{1}\right)$ |
|  |  | $8.8 \pm 2.8$ | $\left(\right.$ Pole $\left.z_{2}\right)$ |
|  | Model C | $16 \pm 6$ |  |
|  |  | $0.2 \pm 0.4$ | $\left(\right.$ Solution $\left.\mathbb{S}_{1}\right)$ |
|  | $0.05 \pm 0.02$ | $\left(\right.$ Solution $\left.\mathbb{S}_{2}\right)$ |  |
| $\left(\right.$ Solution $\left.\mathbb{S}_{3}\right)$ |  |  |  |
| Experiment | Solution 1 | $0.40 \pm 0.54$ |  |
|  | Solution 2 | $0.02 \pm 0.03$ |  |

Table 3. Results for the ratio $B_{3}$.

|  | $B_{3}$ |  |  |
| :--- | :--- | :--- | :--- |
| Our results | Model B | $0.04 \pm 0.01$ |  |
|  |  | $0.09 \pm 0.02$ | $\left(\right.$ Solution $\left.\mathbb{S}_{1}\right)$ |
|  |  | $0.96 \pm 0.16$ | $\left(\right.$ Solution $\left.\mathbb{S}_{2}\right)$ |
| Experiment | Solution 1 | $1.40 \pm 0.40$ | $\left(\right.$ Solution $\left.\mathbb{S}_{3}\right)$ |
|  | Solution 2 | $1.55 \pm 0.19$ |  |
|  |  |  |  |

The results for the ratio $R_{\eta / \eta^{\prime}}$ between the widths of $\phi(2170) \rightarrow \phi \eta$ and to $\phi \eta^{\prime}$ are summarized in Table 4. The results listed in this table should be compared with the ratio $R_{\eta / \eta^{\prime}}^{\exp } \equiv \mathcal{B}_{\phi \eta}^{\phi(2170)} \Gamma_{e^{+} e^{-}}^{\phi(2170)} / \mathcal{B}_{\phi \eta^{\prime}}^{\phi(2170)} \Gamma_{e^{+} e^{-}}^{\phi(2170)}$ obtained by using the values $\mathcal{B}_{\phi \mathcal{P}}^{\phi(2170)} \Gamma_{e^{+} e^{-}}^{\phi(2170)}$ found in Refs. $[6,9]$ (several solutions were found for this product by fitting the data):

$$
R_{\eta / \eta^{\prime}}^{\exp }=\left\{\begin{array}{l}
0.034_{-0.011}^{+0.018} \text { solution I, }  \tag{23}\\
1.42_{-0.48}^{+0.58} \text { solution II, }
\end{array}\right.
$$

and with the results obtained for $R_{\eta / \eta^{\prime}}^{\exp }$ by using for $\mathcal{B}_{\phi \eta}^{\phi(2170)} \Gamma_{e^{+} e^{-}}^{\phi(2170)}$ the value found in Ref. [7], which gives

$$
R_{\eta / \eta^{\prime}}^{\exp }=\left\{\begin{array}{l}
0.013 \pm 0.007 \text { solution I, }  \tag{24}\\
0.009 \pm 0.003 \text { solution II, } \\
2.4 \pm 0.4 \text { solutions III, IV }
\end{array}\right.
$$

Considering the values listed in Table 4, we find that mixing angles of $\simeq-22^{\circ}$ give rise to values for $R_{\eta / \eta^{\prime}}$ which are closer to the upper limit of the solution II of Eq. (23) and solutions III, IV of Eq. (24).

Table 4. Values for the ratio $R_{\eta / \eta^{\prime}}$ considering different $\eta-\eta^{\prime}$ mixing angles, $\beta$, and form factors. The labels $L$ and $G$ indicate the consideration of a Lorentz (L) or a Gaussian (G) form factors, while the numbers I and II refer to the model used to calculate the $\mathcal{P} \overline{\mathcal{P}}^{\prime} t$-matrix.

| $\beta$ (Degree) |  | -15 | -19.47 | -22 |
| :---: | :---: | :---: | :---: | :---: |
|  | LI | $5.12 \pm 1.57$ | $3.93 \pm 1.21$ | $3.39 \pm 1.04$ |
|  | GI | $5.47 \pm 1.68$ | $4.21 \pm 1.29$ | $3.63 \pm 1.11$ |
| $R_{\eta / \eta^{\prime}}$ | LII | $4.21 \pm 1.29$ | $3.25 \pm 1.00$ | $2.80 \pm 0.86$ |
|  | GII | $4.41 \pm 1.35$ | $3.40 \pm 1.04$ | $2.93 \pm 0.90$ |

It is worth stressing that, despite the considerable experimental uncertainty obtained for the previously determined ratios, models considering $\phi(2170)$ as a $s \bar{s}$ states, a hybrid, etc., have real challenges in finding a good reproduction of these ratios, together with the mass and width of $\phi(2170)$.

## 4 Conclusions

In this work, we have summarized our findings for the branching ratios of $\phi(2170)$ to final states involving a $\bar{K}$ and a Kaonic resonance or a $\phi$ and an $\eta / \eta^{\prime}$ mesons. The description of $\phi(2170)$ as a $\phi f_{0}(980)$ molecular state produces values compatible with the experimental findings, reinforcing the interpretation of $\phi(2170)$ as a state generated by the three-body dynamics involved in the $\phi K \bar{K}$ system in isospin 0 , with s-wave interactions and in which the $K \bar{K}$ subsystem resonates as $f_{0}(980)$. The values obtained for these ratios depend on the nature of the Kaonic resonances involved in the final state as well, and more precise data are needed to disentangle whether $K_{1}(1270)$ is a molecular state obtained from pseudoscalar-vector dynamics and the nature of $K_{1}(1400)$.

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