The effects of Lorentz violation on unitarity in $e^+e^- \to \mu^+\mu^-$

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Abstract

We investigated two distinct models incorporating LIV with nonminimal coupling in scattering processes of $\boldsymbol{e^+e^- \to \mu^+\mu^-}$. We verified that employing the model with dual electromagnetic tensor ($\boldsymbol{\tilde{F}_{\mu\nu}}$) led to violations of unitarity in both vector and axial scenarios. Conversely, utilizing the model with nonminimal coupling with ($\boldsymbol{F_{\mu\nu}}$) preserved unitarity in both vector and axial cases. Consequently, this could hold promising implications, considering that the nonminimal coupling model with dual electromagnetic tensor $\boldsymbol{\tilde{F}_{\mu\nu}}$ appeared potentially superior to the electromagnetic tensor $\boldsymbol{F_{\mu\nu}}$. Hence, we anticipate that these findings might offer a valuable roadmap for further exploration into the investigation of CPT and Lorentz breaking phenomena, with phenomenological implications that are certainly not trivial.

Keywords: Lorentz invariance; nonminimal coupling; muon-pair production; unitarity violation.

1 Introduction

In the wake of Carroll-Field-Jackiw (CFJ) seminal work [1] and the subsequent evolution delineated in the Standard Model Extension (SME) by Colladay and Kostelecky [2, 3], the scrutiny of Lorentz covariance breakdown within the realm of Quantum Field Theory (QFT) has undergone rigorous examination. This exploration spans across a plethora of domains, encompassing a broad spectrum of topics ranging from supersymmetry [4, 5] to non-commutative field theory [6–8], and even the realm of gravity [9]. Another significant area of inquiry intersects intriguingly with the emergence of Lorentz Invariance Violation (LIV) in string theory [10], the resurgence of the concept of preferred reference frames—a modern reinterpretation of the age-old notion of the ether [11], and recent developments in condensed matter scenarios [12, 13]. These diverse domains have been investigated in our prior research endeavors (see [14] and the references therein).

Having said that, it is imperative still to envisage that the concept of LIV has been contemplated as a potential solution to enduring enigmas across various domains of physics. A particularly noteworthy instance of this lies within the domain of cosmic ray physics [15]. While LIV may not exclusively offer resolutions to these complex issues, it often emerges as a highly plausible candidate. This suggests that LIV transcends mere peripheral significance in the scope of challenging-to-explore physics, implying substantial implications beyond our current experimental capabilities. Indeed, if alternative explanations fail to adequately address these puzzles, LIV could potentially emerge as an indispensable component of the ultimate theory, surpassing both the Standard Model (SM) of particle physics and general relativity. Ensuring the validation of theoretical predictions requires the corroboration through empirical evidence. Despite recognizing the inherent difficulties in promptly acquiring such evidence, the efforts delineated in the tables [16] offer an initial insight into potential avenues for validation.

The SME provides a comprehensive framework for investigating LIV across both high and low-energy physics domains. By augmenting the SM lagrangian with Lorentz-violating interaction terms, the SME encompasses various criteria, including power counting renormalizability (*i.e.* with mass dimension ≤ 4), gauge invariance[1] $SU(3)\times SU(2)\times U(1)$, and incorporation of fields from the SM. However, within the realm of Quantum Electrodynamics (QED), LIV studies take on particular significance.[2] Pioneered CFJ, initial investigations in this field continue to be central to ongoing inquiry. The CFJ model explores modifications to electrodynamics through the introduction of a Chern-Simons (CS) term, highlighting deviations from Lorentz symmetry. This CS term, also incorporated within the SME framework, yields measurable effects on photodynamics, thereby providing avenues for investigating LIV phenomena. Moreover, its induction by radiative corrections from other sectors of the theory underscores the intricate interplay among different facets of the theoretical framework.

Experimental tests of Lorentz Invariance (LI) stand as paramount endeavors, serving to scrutinize the fundamental symmetries underpinning the fabric of the universe.[3] Within this context, understanding LIV becomes imperative. LIV, rather than denoting a forfeiture of covariance, suggests nuanced variances within specific particle sectors, potentially stemming from phenomena such as quantum gravity at immensely high energies. Despite Lorentz symmetry enduring all experimental scrutiny within feasible energy scales, the quest for LIV necessitates meticulous precision. Detecting LIV entails remarkable accuracy in low-energy physics experiments and relies on discerning scaling effects in (ultra) high-energy domains. In low-energy realms, LIV is typified by LIV-tensors, while modified dispersion relations categorize it in high-energy contexts. The Planck energy scale $E_P = \sqrt{\frac{\hbar c^5}{G}} \approx 1.2\times 10^{19}\,\text{GeV}$ emerges as a pivotal threshold, modulating these relations, yet the substantial energies observed in astrophysical phenomena facilitate detection despite potential suppression. Consequently, experimental inquiries into LI constrain parameters of modified models or LIV functions, demanding particle-specific interpretations.

Exploring the high-energy frontier offers a unique avenue to interrogate nature's fundamental laws, motivating endeavors such as muon colliders. The potential to reach multi-TeV energy regimes in a muon collider unlocks opportunities for probing Higgs boson properties. Projects like LEMMA [20] investigate muon production from e^+e^- annihilation, aiming to exploit the process's threshold for muon pair production. Experimental data near this threshold are scarce, urging precise measurements of production cross sections and muon pair kinematics to validate theoretical predictions.

While leading-order QED calculations for $e^+e^-\to\mu^+\mu^-$ are robust, higher-order radiative effects, particularly due to Coulomb interaction, gain significance near the kinematic threshold. Experimental setups, like those employing 45 GeV positrons on Beryllium or Carbon targets, are devised to study muon pair production in detail [21]. Specifically, a system comprising segmented and instrumented absorbers, deployed at CERN North Area beam lines, facilitated the investigation of $e^+e^-\to\mu^+\mu^-$ near the production threshold. Initially designed for studying hadronic showers from LHC interactions, these absorbers were repurposed to identify and discriminate muons from other particles, contributing significantly to experimental efforts in this domain.

The structure of this paper unfolds as follows. In Section 2, we explore the analysis of the cross-sectional area for $e^+e^-\to\mu^+\mu^-$ annihilation within the framework of QED. Our investigation entails examining contributions from Feynman diagrams at the tree level. In Section 3, we examine a modified version of QED in four dimensions with LIV where the coupling between the photon and the fermions consists of two distinct terms: a minimal coupling and a nonminimal coupling. In Section 4 we present the calculations of the cross sections with the electromagnetic tensor and the dual electromagnetic tensor in the vector and axial scenarios. Finally, in the Section 5, we discuss and comment on the possible implications of the results obtained.

[1] By enforcing the customary $SU(3)\times SU(2)\times U(1)$ gauge invariance and confining scrutiny to phenomena at low-energy levels, the expansion of the standard model is adequately approximated by the conventional standard model augmented with every conceivable Lorentz-violating expression of mass dimension equal to or below four, derived from standard-model constituents.

[2] In recent years, there has been significant scholarly focus on investigating the implications of Lorentz and CPT violation within the framework of QED. This attention stems primarily from the prospect of spontaneous symmetry breaking occurring at exceedingly high energies, specifically at the Planck scale.

[3] Further details can be found in [17–19] (and in references therein).

2 Muon-pair production by electron-positron scattering

In the realm of QED, the preeminent influence on the determination of a cross-sectional area or decay rate typically emanates from the Feynman diagram boasting the scantiest array of interaction vertices, distinguished as the lowest-order diagram. For the annihilation process $e^+e^- \to \mu^+\mu^-$, a singular lowest-order QED diagram presides, elegantly depicted in Figure 1. Within this diagram, a duet of QED interaction vertices adorn its architecture, each endowing the matrix element with a factor of $ie\gamma^\mu$, where e is the charge of the electron. Consequently, irrespective of ancillary deliberations, the squared matrix element $|\mathcal{M}|^2$ exhibits a direct proportionality to e^4, or equivalently $|\mathcal{M}|^2 \propto \alpha^2$, where α stands as the adimensional fine-structure constant, $\alpha = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c}$. Broadly speaking, every QED vertex contributes a coefficient of α to the expressions dictating cross-sectional areas and decay rates.

There is one tree diagram for this process

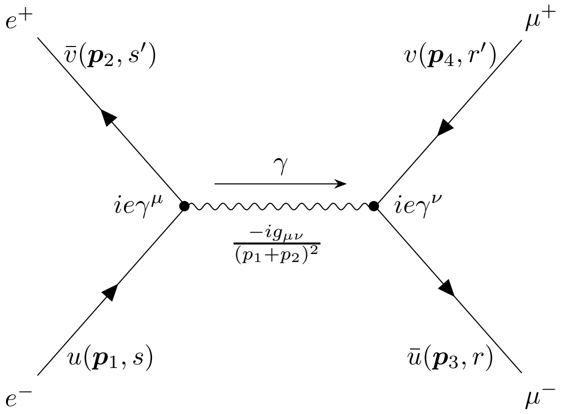

Figure: 1 Scattering $e^+ + e^- \to \mu^+ + \mu^-$.

In the Figure 1 $u(p)$ represents the electron spinor, $\bar{v}(p) = v^\dagger(p)\gamma^0$ the adjoint spinor positron, $v(p)$ the anti-muon spinor and $\bar{u}(p) = v^\dagger(p)\gamma^0$ the adjoint spinor muon. The indices r, r', s, s' represent the spins of particles and antiparticles. In the center-of-mass frame, the momentum vectors are chosen such that $p_{1,2} = (E_e, 0, 0, \pm p)$, $p_{3,4} = (E_\mu, \pm E_\mu \sin\theta, 0, \pm E_\mu \cos\theta)$[4], $E = \sqrt{(pc)^2 + (m_0c^2)^2}$ represents the total energy, and m_0 denotes the particle rest mass (~ 0.5 MeV/c^2 for the electron and 106 MeV/c^2 for the muon). Using the Feynman rules of QED, we can now write the amplitude for this process.

$$
\begin{aligned}
i\mathcal{M} &= \bar{v}^{s'}(p_2)(-ie\gamma^\mu)u^s(p_1) \times \\
&\left(\frac{-ig_{\mu\nu}}{s}\right)\bar{u}^r(p_3)(-ie\gamma^\nu)v^{r'}(p_4), \quad (1)
\end{aligned}
$$

where $-ie\gamma^\mu$ is vertex factor usual and $\sqrt{s} = p_1 + p_2 = p_3 + p_4$[5] is the photon momentum. The amplitude of the Eq. (1) depends on the spins of all four particles involved. But in this work we shall focus on the unpolarized cross-section. Therefore, the unpolarized cross section in the centre-of-mass frame can be written in terms of the scattering angle this way[6]

$$
\begin{aligned}
\left(\frac{d\sigma}{d\Omega}\right)_{\text{QED}} &= \frac{\alpha^2}{4s}\sqrt{1 - \frac{m_\mu^2}{E^2}} \times \\
&\left[1 + \frac{m_\mu^2}{E^2} + \left(1 - \frac{m_\mu^2}{E^2}\right)\cos^2\theta\right], \quad (2)
\end{aligned}
$$

where $\alpha \approx 1/137$ is fine-structure constant. In the high-energy assume that the beam energy E[7] is much greater than either the muon mass m_μ, so we found

$$
\left(\frac{d\sigma}{d\Omega}\right)_{QED} = \frac{\alpha^2}{4s}\left(1 + \cos^2\theta\right). \quad (3)
$$

The total cross section is obted integrating over $d\Omega$, so

$$
\left(\frac{d\sigma}{d\Omega}\right)_{total} = \frac{4\pi\alpha^2}{3s}. \quad (4)
$$

3 Models with nonminimal coupling

The Lorentz violation terms are generated as vacuum expectation values of tensors defined in a high energy scale. We propose the investigation of nonminimal coupling terms in the calcule of cross section in the scatering $e^+ + e^- \to \mu^+ + \mu^-$. In this section four models will be presented that induce the violation of Lorentz and CPT symmetry from the presence of a background vector field that couples the fermion field to the electromagnetic field. This background four-vector indicates a preferred direction in spacetime, thus violating Lorentz symmetry.

[4] θ is the scattering angle. The azimuth angle ϕ cancels out in scattering calculations.
[5] $s = E_{cm}^2$, E_{cm} is the total energy in the center-of-mass frame.
[6] In this result it was used using the experimental fact that the muon is much heavier than the electron, $m_\mu \approx 207 m_e$.
[7] $E = |\boldsymbol{p_1}| = |\boldsymbol{p_2}| = |\boldsymbol{p_3}| = |\boldsymbol{p_4}| = E_{cm}/2$.

3.1 Nonminimal coupling with electromagnetic tensor

3.1.1 Vectorial nonminimal coupling

The vectorial nonminimal coupling model is built from a modification in the covariant derivative of the QED:

$$D_\mu^{vect} = \partial_\mu + ieA_\mu + ig(k_{AF})^\nu F_{\mu\nu}, \tag{5}$$

where g is a effective coupling constant (real) with mass dimension -2, $(k_{AF})^\mu = (b_0, \vec{b}) = b^\mu$ is the Carroll-Field-Jackiw 4-vector responsible for that breaks the Lorentz symmetry and CPT its has dimensions $[b^\mu] = -1$ and assuming that it couples to the electromagnetic field strength $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ corresponding to the gauge field A_μ and whose components are $F_{0i} = -F_{i0} = -E_i$ and $F_{ij} = -F_{ji} = \epsilon_{ijk}B^k$. This CPT-odd modification in the covariant derivative above affects all electron-photon interactions already at tree level. This LIV scenario with the dimension operator 5 has been proposed in Ref. [22] in the context of topological phases. The gauge invariant modified Dirac equation is written in the form

$$(i\gamma^\mu D_\mu^{vect} - m_e)\psi = 0, \tag{6}$$

where m_e is electron mass, γ^μ is the Dirac matrix and ψ is a Dirac electron spinor. In the Eq. (5) the term ieA_μ is called minimum coupling while the term $igb^\nu F_{\mu\nu}$ of nonminimal coupling both terms are gauge invariant, however the nonminimal coupling is not renormalizable. Per the analysis in [23], the background vector b^μ can be written as

$$b^\mu = -\frac{1}{3}\, a_F{}^{(5)\alpha\mu}{}_\alpha. \tag{7}$$

In the Ref. [16] the estimated value of b^μ is about $10^{-32}GeV$. The Lagrangian density for this model is given by

$$\begin{aligned}\mathcal{L}^{vect} &= \bar{\psi}\left(i\partial\!\!\!/ - eA\!\!\!/ - m_e - gb^\mu\gamma^\nu F_{\mu\nu}\right)\psi \\ &\quad -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}.\end{aligned} \tag{8}$$

where $\bar{\psi} \equiv \psi^\dagger\gamma^0$ is the conjugate spinor. The last term corresponds to the Lagrangian of Maxwell's theory. The presence of nonminimal coupling in lagrangian modifies the vertex involving electron-photon of QED in the following way [24]

$$ie\gamma^\mu \longrightarrow ie\gamma^\mu + q\!\!\!/ b^\mu - (b\cdot q)\gamma^\mu, \tag{9}$$

where q represents the four-momentum of the photon. In section 4, the calculations of the cross sections were obtained by replacing this vertex in the scattering amplitude of Eq. (1), as well as the modified vertices that will appear in the next models that will be presented below.

3.1.2 Axial nonminimal coupling

Another possible is the axial non-minimal coupling. In this model the modified Dirac equation is written as [25]

$$D_\mu^{axial} = \partial_\mu + ieA_\mu + ig_a\gamma^5 b^\nu F_{\mu\nu}, \tag{10}$$

with the chiral Dirac matrix $\gamma_5 = \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$. The Lagrangian density of this model is given by

$$\begin{aligned}\mathcal{L}^{axial} &= \bar{\psi}\left(i\partial\!\!\!/ - eA\!\!\!/ - m_e - g_a b^\mu\gamma^\nu\gamma^5 F_{\mu\nu}\right)\psi \\ &\quad -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}.\end{aligned} \tag{11}$$

The five-dimensional non-renormalizable axial term $g_a b^\mu\gamma^\nu\gamma^5 F_{\mu\nu}$ violate C parity and preserve PT, therefore violates CPT. Differently the vectorial term in Eq. (8) preserve C parity and violate PT, so it too violates CPT.

3.2 Nonminimal coupling with dual electromagnetic tensor

3.2.1 Vectorial nonminimal coupling

In this model the covariant derivative with vectorial nonminimal coupling is chosen to be [25, 26]

$$\tilde{D}_\mu^{vect} = \partial_\mu + ieA_\mu + i\tilde{g}\tilde{b}^\nu \tilde{F}_{\mu\nu}, \tag{12}$$

where the term CPT-odd $i\tilde{g}\tilde{b}^\nu\tilde{F}_{\mu\nu}$ is gauge invariant and $\tilde{g}$ is the coupling constant with mass dimension -2, $\tilde{b}^\mu$ has similar characteristics to b^μ and $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ is the usual dual field-strength electromagnetic tensor with $\epsilon^{0123} = 1$ (Levi-Civita symbol). VSL parameters associated with other nonminimal derivative couplings involving the dual electromagnetic tensor have been limited in Ref. [27] to values $\leq 10^{-3}GeV^{-1}$. A direct consequence of the non-minimal coupling introduced in D_μ is that scalar particles display a non-trivial magnetic moment. This terms have been used for the perturbative induction of the CFJ see [28, 29]. Per the analysis in [23], the background vector b^μ can be written as

$$\tilde{b}^\mu = \frac{1}{6}\epsilon^\mu{}_{\nu\alpha\beta}\, a_F^{(5)\nu\alpha\beta}. \tag{13}$$

The Lorentz-violating Lagrange density for this model is given by

$$\tilde{\mathcal{L}}^{vect} = \bar{\psi}\left(i\partial\!\!\!/ - eA\!\!\!/ - m_e - \tilde{g}\tilde{b}^{\mu}\gamma^{\nu}\tilde{F}_{\mu\nu}\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{14}$$

The vectorial nonminimal coupling can be written

$$\tilde{g}\tilde{b}_{\nu}\tilde{F}^{\mu\nu} = \tilde{g}\tilde{b}_0\vec{\gamma}\cdot\vec{B} + \tilde{g}\vec{\tilde{b}}\cdot\vec{B}\gamma_0 - \tilde{g}\vec{\gamma}\cdot(\vec{\tilde{b}}\times\vec{E}), \tag{15}$$

where $\tilde{b}_0$ and $\vec{\tilde{b}}$ are the time-like and space-like components of the coefficient $\tilde{b}_{\mu}$ respectively. We also have that γ^0 is hermitian, $\vec{\gamma}$ is anti-hermitian, and are related to the $\hat{\beta}$ and $\vec{\alpha}$ matrices through: $\gamma^0 = \hat{\beta}$, $\vec{\gamma} = \hat{\beta}\vec{\alpha}$. We choose the Dirac matrices in the form,

$$\hat{\beta} = \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}. \tag{16}$$

The presence of nonminimal coupling in Eq. (14) generates a modification in the vertices as follows (see Ref. [24]),

$$ie\gamma^{\mu} \longrightarrow ie\gamma^{\mu} - \epsilon^{\mu}{}_{\alpha\nu\beta}\gamma^{\alpha}\tilde{b}^{\nu}q^{\beta}, \tag{17}$$

where q^{μ} is the photon momentum pointing into the vertex.

3.2.2 Axial nonminimal coupling

A axial nonminimal coupling model is obtained by writing the covariant derivative in the form

$$\tilde{D}_{\mu}^{axial} = \partial_{\mu} + ieA_{\mu} + i\tilde{g}_a\gamma^5\tilde{b}^{\nu}\tilde{F}_{\alpha\beta}, \tag{18}$$

this model is called torsion-like nonminimal coupling and the Lorentz-violating Lagrange density is is written as

$$\tilde{\mathcal{L}}^{axial} = \bar{\psi}\left(i\partial\!\!\!/ - eA\!\!\!/ - m_e - \tilde{g}_a\gamma^5\tilde{b}^{\nu}\gamma^{\mu}\tilde{F}_{\alpha\beta}\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{19}$$

Similar to the previous model, the axial term $\tilde{g}_a\gamma^5\tilde{b}^{\nu}\gamma^{\mu}\tilde{F}_{\alpha\beta}$ violate C parity and preserve PT, therefore violates CPT. Differently the vectorial term in Eq. (14) preserve C parity and violate PT, so it too violates CPT.

4 Results

In this section, we proceed with the calculations by employing Feynman's rules, where we introduce a vertex modified by the nonminimal CPT-odd coupling term. Specifically, we investigate the unpolarized scattering process $e^+e^- \to \mu^+\mu^-$. Furthermore, we examine the behavior of the scattering process in the limit of high energy, where $p_{1,2}^2 = p_{3,4}^2 = m^2 = 0$. In all our results in this work we consider the scalar products involving the background field up to second order.

4.1 Vectorial nonminimal coupling with $F_{\mu\nu}$

As we said previously, we calculated the scattering amplitude by modifying the vertex of the QED in Eq. (1) by the vertex presented in Eq. (9), the result we obtained for the cross section obtained was

$$\left(\frac{d\sigma}{d\Omega}\right)^{vect} = \left(\frac{d\sigma}{d\Omega}\right)_{QED} + \frac{\alpha^2 g^2}{2s}\left(1+\cos^2\theta\right) \times \left(b\cdot p_1 + b\cdot p_2\right)^2. \tag{20}$$

In the case of a purely time-like background, i.e, $b_{\mu} = (b_0, \mathbf{0})$, the Eq. (20) will be

$$\left(\frac{d\sigma}{d\Omega}\right)_t^{vect} = \left(\frac{d\sigma}{d\Omega}\right)_{QED} + \frac{\alpha^2 g^2}{2s}\left(1+\cos^2\theta\right) \times \left(b_0\cdot p_1 + b_0\cdot p_2\right)^2. \tag{21}$$

In the case of a purely space-like background, i.e, $b_{\mu} = (0, \boldsymbol{b})$, the Eq. (20) will be[8]

$$\left(\frac{d\sigma}{d\Omega}\right)_s^{vect} = \left(\frac{d\sigma}{d\Omega}\right)_{QED} + \frac{\alpha^2 g^2}{2s}\left(\boldsymbol{b}\cdot\boldsymbol{p}_1 + \boldsymbol{b}\cdot\boldsymbol{p}_2\right) \times \Big[\left(\boldsymbol{b}\cdot\boldsymbol{p}_1 + \boldsymbol{b}\cdot\boldsymbol{p}_2\right)\left(2+\cos^2\theta\right) - \left(\boldsymbol{b}\cdot\boldsymbol{p}_3 + \boldsymbol{b}\cdot\boldsymbol{p}_4\right)\Big]. \tag{22}$$

Note in cross sections of the Eqs. (20), (21) and (22) that when $s \to \infty$ the results obtained preserve the unitarity.

4.2 Axial nonminimal coupling with $F_{\mu\nu}$

Now, we present the result of the cross section from the modification of the scattering amplitude Eq. (1), we obtained

$$\left(\frac{d\sigma}{d\Omega}\right)^{axial} = \left(\frac{d\sigma}{d\Omega}\right)_{QED} + \frac{\alpha^2 g_a^2}{s}\cos\theta \times \left(b\cdot p_1 + b\cdot p_2\right)^2. \tag{23}$$

In the case of a purely time-like background, the Eq. (23) will be

[8] In this result we do not consider terms of the type $1/s^2$ and $1/s^3$.

$$\left(\frac{d\sigma}{d\Omega}\right)_t^{axial} = \left(\frac{d\sigma}{d\Omega}\right)_{QED} + \frac{\alpha^2 g_a^2}{s}\cos\theta \times (b_0 \cdot p_1 + b_0 \cdot p_2)^2 . \quad (24)$$

In the case of a purely space-like background, the Eq. (23) will be

$$\left(\frac{d\sigma}{d\Omega}\right)_s^{axial} = \left(\frac{d\sigma}{d\Omega}\right)_{QED} + \frac{\alpha^2 g_a^2}{2s^2} \times \Big\{ (\boldsymbol{b}\cdot\boldsymbol{p}_4)(\boldsymbol{b}\cdot\boldsymbol{p}_1 + \boldsymbol{b}\cdot\boldsymbol{p}_2)(p_1^2 - p_2^2) + s\,(3\boldsymbol{b}\cdot\boldsymbol{p}_1 + \boldsymbol{b}\cdot\boldsymbol{p}_2 + \boldsymbol{b}\cdot\boldsymbol{p}_3 + \boldsymbol{b}\cdot\boldsymbol{p}_4) \times (\boldsymbol{b}\cdot\boldsymbol{p}_1 + \boldsymbol{b}\cdot\boldsymbol{p}_2)\cos\theta \Big\} . \quad (25)$$

As in case of vectorial nonminimal coupling, the cross sections of the Eqs. (23), (24) and (25) that when $s \to \infty$ the results obtained preserve the unitarity.

4.3 Vectorial nonminimal coupling with $\tilde{\boldsymbol{F}}_{\boldsymbol{\mu\nu}}$

Here, we calculated the scattering amplitude by modifying the vertex of the QED in Eq. (1) by the vertex presented in Eq. (17), the result we obtained for the cross section obtained was

$$\left(\frac{d\sigma}{d\Omega}\right)^{vect} = \left(\frac{d\sigma}{d\Omega}\right)_{QED} + \frac{\alpha^2 \tilde{g}^2}{s}\left[\left(\tilde{b}\cdot p_1 + \tilde{b}\cdot p_2\right)^2 - \tilde{b}^2 s\right] \times \left(1+\cos^2\theta\right) . \quad (26)$$

In the case of a purely time-like background, the Eq. (26) will be

$$\left(\frac{d\sigma}{d\Omega}\right)_t^{vect} = \left(\frac{d\sigma}{d\Omega}\right)_{QED} - 3\alpha^2 \tilde{b}_0^2 \tilde{g}^2 \left(1+\cos^2\theta\right) . \quad (27)$$

In the case of a purely space-like background, the Eq. (26) will be

$$\left(\frac{d\sigma}{d\Omega}\right)_s^{\rm vect} = \left(\frac{d\sigma}{d\Omega}\right)_{QED} + \frac{\alpha^2 \tilde{g}^2}{s} \times \left[\left(\tilde{\boldsymbol{b}}\cdot\boldsymbol{p}_1 + \tilde{\boldsymbol{b}}\cdot\boldsymbol{p}_2\right)^2 + \tilde{\boldsymbol{b}}^2 s\right] \times \left(1+\cos^2\theta\right) . \quad (28)$$

Note in cross sections of the Eqs. (26), (27) and (28) that when $s \to \infty$ the results obtained violated the unitarity.

4.4 Axial nonminimal coupling with $\tilde{\boldsymbol{F}}_{\boldsymbol{\mu\nu}}$

Now, we present the result of the cross section from the modification of the scattering amplitude Eq. (1), we obtained

$$\left(\frac{d\sigma}{d\Omega}\right)^{axial} = \left(\frac{d\sigma}{d\Omega}\right)_{QED} + \frac{2\alpha^2 \tilde{g}_a^2}{s}\cos\theta \times \left[\left(\tilde{b}\cdot p_1 + \tilde{b}\cdot p_2\right)^2 - s\tilde{b}^2\right] . \quad (29)$$

In the case of a purely time-like background the Eq. (29) will be

$$\left(\frac{d\sigma}{d\Omega}\right)_t^{axial} = \left(\frac{d\sigma}{d\Omega}\right)_{QED} - 6\alpha^2 \tilde{b}_0^2 \tilde{g}_a^2 \cos\theta . \quad (30)$$

In the case of a purely space-like background the Eq. (29) will be

$$\left(\frac{d\sigma}{d\Omega}\right)_s^{\rm axial} = \left(\frac{d\sigma}{d\Omega}\right)_{QED} + \frac{2\alpha^2 \tilde{g}_a^2}{s}\cos\theta \times \left[\left(\tilde{\boldsymbol{b}}\cdot\boldsymbol{p}_1 + \tilde{\boldsymbol{b}}\cdot\boldsymbol{p}_2\right)^2 + s\tilde{\boldsymbol{b}}^2\right] . \quad (31)$$

As in case of vectorial nonminimal coupling, the cross sections of the Eqs. (29), (30) and (31) that when $s \to \infty$ the results violated the unitarity.

5 Final remarks

In this paper, we have investigated the breaking of Lorentz symmetry through the examination of muon-pair production via electron-positron scattering within the framework of Extended Quantum Electrodynamics. In this study, we tested two models involving nonminimal coupling in the scattering process $e^+e^- \to \mu^+\mu^-$, utilizing calculations of the cross-section. Our findings reveal that when employing the dual electromagnetic tensor, both the vector and axial scenarios exhibited unitarity violations. Conversely, utilizing the model with the electromagnetic tensor preserved unitarity in both scenarios, suggesting its potential superiority over the former model. Additionally, radiative corrections calculations [24] in massless QED have demonstrated the renormalizability of the model with radiative corrections at 1-loop, a property not observed in the model involving the dual field. This further strengthens the confidence in the model with

$F_{\mu\nu}$ coupling. In the future, we aim to explore the prospective phenomenological implications of this model, shedding light on its potential applications in various contexts. Ultimately, we anticipate that these findings may serve as a valuable roadmap for further inquiry into the investigation of CPT and Lorentz breaking.

Acknowledgments: We would like to express our gratitude to Prof. J. A. Helayël-Neto for helpful discussions and suggestions.

References

[1] S. Carroll, G. Field and R. Jackiw, *Limits on a Lorentz and Parity Violating Modification of Electrodynamics*, *Phys. Rev. D* **41**, 1231 (1990).

[2] D. Colladay and V. A. Kostelecky, *CPT violation and the standard model*, *Phys. Rev. D* **55**, 6760 (1997).

[3] D. Colladay and V. A. Kostelecky, *Lorentz-violating extension of the standard model*, *Phys. Rev. D* **58**, 116002 (1998).

[4] M. S. Berger and V. Alan Kostelecký, *Supersymmetry and Lorentz violation*, *Phys. Rev. D* **65**, 091701 (2002).

[5] Stefan Groot Nibbelink and Maxim Pospelov, *Lorentz Violation in Supersymmetric Field Theories*, *Phys. Rev. Lett.* **94**, 081601 (2005).

[6] Michael R. Douglas and Nikita A. Nekrasov, *Noncommutative field theory*, *Rev. Mod. Phys.* **73**, 977 (2001).

[7] M. M. Sheikh-Jabbari, *C, P, and T Invariance of Noncommutative Gauge Theories*, *Phys. Rev. Lett.* **84**, 5265 (2000).

[8] Sean M. Carroll, Jeffrey A. Harvey, V. Alan Kostelecký, Charles D. Lane, and Takemi Okamoto, *Noncommutative Field Theory and Lorentz Violation*, *Phys. Rev. Lett.* **87**, 141601 (2001).

[9] J. W. Moffat, *Lorentz violation of quantum gravity*, *Clas. Quant. Grav.* **27**, 135016 (2010).

[10] V. Alan Kostelecký and Stuart Samuel, *Spontaneous breaking of Lorentz symmetry in string theory*, *Phys. Rev. D* **39**, 683 (1989).

[11] T. Jacobson and D. Mattingly, *Einstein-aether waves*, *Phys. Rev. D* **70**, 024003 (2004).

[12] V. A. Kostelecký, R. Lehnert, N. McGinnis, M. Schreck and B. Seradjeh, *Lorentz violation in Dirac and Weyl semimetals*, *Phys. Rev. Research* **4**, 023106 (2022).

[13] Andrés Gómez, R. Martínez von Dossow, A. Martín-Ruiz, and Luis F. Urrutia, *Lorentz invariance violation and the CPT-odd electromagnetic response of a tilted anisotropic Weyl semimetal*, *Phys. Rev. D* **109**, 065005 (2024).

[14] Alessio T. B. Celeste and Adriano M. Santos, *Lorentz violation in γ-pair production ($e^+e^- \to \gamma\gamma$)*, *J. Phys. G: Nucl. Part. Phys.* **51**, 045001 (2024).

[15] Wolfgang Bietenholz, *Cosmic rays and the search for a Lorentz Invariance Violation*, *Phys. Rept.* **505**, 145-185 (2011).

[16] V. A. Kostelecky and N. Russell, *Data tables for Lorentz and CPT violation*, 2024 edition, arXiv:0801.0287v16[hep-ph].

[17] David Mattingly, *Modern Tests of Lorentz Invariance*, *Living Rev. Rel.* **8**, 5 (2005).

[18] R. Bluhm, *Overview of the Standard Model Extension: Implications and Phenomenology of Lorentz Violation*, *Lect. Notes Phys.* **702**, 191–226 (2006).

[19] T. Jacobson, S. Liberati and D. Mattingly, *Lorentz violation at high energy: Concepts, phenomena, and astrophysical constraints*, *Annals Phys.* **321**, 150-196 (2006).

[20] M. Antonelli, M. Boscolo, R. Di Nardo and P. Raimondi, *Novel proposal for a low emittance muon beam using positron beam on target*, *Nucl. Instr. Meth. Phys. Res. Sect. A.* **807**, 101-107 (2016).

[21] N. Amapane *et al.*, *Study of muon pair production from positron annihilation at threshold energy*, *JINST* **15**, P01036 (2020).

[22] K. Bakke, E. O. Silva and H. Belich, *He-McKellar-Wilkens effect and scalar Aharonov-Bohm effect for a neutral particle based on the Lorentz symmetry violation*, *J. Phys. G: Nucl. Part. Phys.* **39**, 055004 (2012).

[23] Y. M. P. Gomes and J. T. Guaitolini Junior, *Elastic light-by-light scattering in a nonminimal Lorentz violation scenario*, *Phys. Rev. D* **99**, 055006 (2019).

[24] Shan-quan Lan and Feng Wu, *Electrodynamics modified by some dimension-five Lorentz-violating interactions: radiative corrections*, *Eur. Phys. J. C* **74**, 2875 (2014).

[25] H. Belich, T. Costa-Soares, M. M. Ferreira, Jr. and J. A. Helayel-Neto, *Non-minimal coupling to a Lorentz-violating background and topological implications*, *Eur. Phys. J. C* **41**, 421-426 (2005).

[26] H. Belich, T. Costa-Soares, M. M. Ferreira, Jr., J. A. Helayel-Neto and F. M. O. Moucherek, *Lorentz-violating corrections on the hydrogen spectrum induced by a non-minimal coupling*, *Phys. Rev. D* **74**, 065009 (2006).

[27] B. Charneski, M. Gomes, R. V. Maluf and A. J. da Silva, *Lorentz violation bounds on Bhabha scattering*, *Phys. Rev. D* **86**, 045003 (2012).

[28] T. Mariz, J. R. Nascimento and A. Yu. Petrov, *On the perturbative generation of the higher-derivative Lorentz-breaking terms*, *Phys. Rev. D* **85**, 125003 (2012).

[29] G. Gazzola *et. al.*, *QED with minimal and nonminimal couplings: on the quantum generation of Lorentz-violating terms in the pure photon sector*, *J. Phys. G: Nucl. Part. Phys.* **39**, 035002 (2012).