

Coordination Capacity for Classical-Quantum Correlations

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Abstract

Network coordination is considered in three basic settings, characterizing the generation of separable and classical-quantum correlations among multiple parties. First, we consider the simulation of a classical-quantum state between two nodes with rate-limited common randomness (CR) and communication. Furthermore, we study the preparation of a separable state between multiple nodes with rate-limited CR and no communication. At last, we consider a broadcast setting, where a sender and two receivers simulate a classical-quantum-quantum state using rate-limited CR and communication. We establish the optimal tradeoff between communication and CR rates in each setting.

Index Terms

Quantum communication, coordination, reverse Shannon theorem.

I. INTRODUCTION

Coordination is essential in communication systems, as it ensures that different components can work together in harmony to achieve a common goal. For example, in sensor networks, the sensors do not just share information in the conventional sense, but also collaborate in the transmission of data [1]. Furthermore, coordination plays a major role in cooperative computing between distributed systems [2], rate-distortion theory for secrecy systems [3], simulation of distributed quantum measurements [4], and quantum nonlocal games [5]. It is envisioned that quantum information technology will enhance future communication systems from different perspectives, such as efficiency [6, 7], security [8–10], and computing [11]. These advances motivate the study of coordination in quantum networks [12].

Cuff et al. [13] studied classical coordination in various communication networks with different topologies, and considered two coordination types, empirical coordination and strong coordination. Empirical coordination requires that the average frequency of joint actions in the network approaches a desired distribution with high certainty. On the other hand, strong coordination sets a requirement on the joint distribution of all actions. Efficient coordination codes are constructed in [14]. Source coding [15–18], state coordination [19], channel simulation [20–25], and distributed source simulation [26, 27] can be viewed as instances of network coordination. In particular, Bennet et al. [22] considered simulation of a quantum channel, under the assumption that pre-shared entanglement is available to the sender and the receiver, and derived the quantum reverse Shannon theorem [28, 29]. The optimal simulation rate turns out to be identical to the entanglement-assisted quantum capacity [22]. Recently, the authors [30] considered entanglement coordination using quantum communication links.

The general problem of quantum coordination can be formulated as follows. Consider a quantum network that consists of m nodes, where Node i performs an encoding operation $\mathcal{E}_i$ on a quantum system A_i , which is required to be in a certain desired correlation with the rest of the network. In other words, the goal is to simulate a particular joint state, $\omega_{A_1 A_2 \dots A_m}$. In general, some of the nodes are not free to choose their encoding, but rather their state is dictated by Nature, according to a certain physical process. Node i can also send a sequence of bits or qubits to Node j via a communication link of a limited rate, $R_{i,j}$. The ultimate performance is defined by the set of rates $\{R_{i,j}\}$ that are necessary and sufficient in order to simulate the quantum correlation.

Cuff et al. [13] introduced the classical version of this problem, where the encoders, decoders, and rates are all classical, and the goal is to simulate a prescribed probability distribution. In the basic two-node setting, the simulation of a joint distribution p_{XY} requires a rate $R_{1,2} \geq C(X; Y)$, where $C(X; Y)$ is Wyner's common information [31]. The quantum analog was recently established by George et al. [27], in the context of distributed source simulation. Under the assumption that Alice and Bob share unlimited common randomness (CR) a priori, simulation can be performed at a lower rate, $R_{1,2} \geq I(X; Y)$, where $I(X; Y)$ is the mutual information. The capacity region describes the optimal tradeoff between the communication and CR rates [32].

In this paper, we consider three coordination settings. Our models are motivated by quantum-enhanced Internet of Things (IoT) networks in which the communication links are classical [33–36], as opposed to our work in [30]. First, we consider the simulation of a classical-quantum state ω_{XB} between two nodes with rate-limited CR. We characterize the optimal tradeoff between the required rate of description and the amount of CR used. Our second model is a quantum no-communication network. The network comprises three nodes, where no-communication is allowed between the nodes, yet CR is available at a classical rate R_0 . Thereby, only separable states can be simulated. We show that a joint state ω_{ABC} can be simulated at a CR rate of $R_0 \geq I(ABC; U)_\sigma$, with respect to an extension σ_{UABC} of ω_{ABC} , where U is an auxiliary classical random variable that satisfies a Markov property. At last, we consider a broadcast setting, where a sender and two receivers simulate a

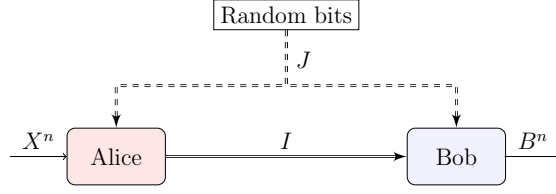


Fig. 1. Two-node network.

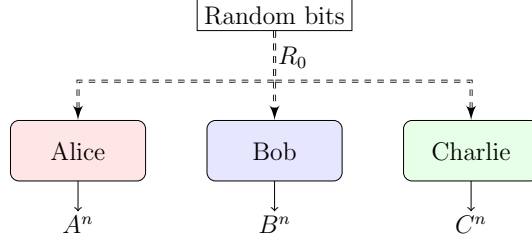


Fig. 2. No-communication network

classical-quantum-quantum (c-q-q) state using rate-limited CR and communication. We establish the optimal tradeoff between communication and CR rate. In the analysis, we use random coding and apply quantum resolvability results [37–39].

II. PROBLEM DEFINITIONS

We consider three coordination settings as described below. We use standard notation in quantum information theory, as in [40], $X, Y, Z, \dots$ are discrete random variables on finite alphabets $\mathcal{X}, \mathcal{Y}, \mathcal{Z}, \dots$, respectively, $x^n = (x_i)_{i \in [n]}$ denotes a sequence in $\mathcal{X}^n$. A quantum state is described by a density operator, ρ_A , on the Hilbert space $\mathcal{H}_A$. Denote the set of all such operators by $\mathfrak{S}(\mathcal{H}_A)$. A c-q channel is a map $\mathcal{N}_{X \rightarrow B} : \mathcal{X} \rightarrow \mathfrak{S}(\mathcal{H}_B)$. The quantum mutual information is defined as $I(A; B)_\rho = H(\rho_A) + H(\rho_B) - H(\rho_{AB})$, where $H(\rho) \equiv -\text{Tr}[\rho \log(\rho)]$, the conditional quantum entropy as $H(A|B)_\rho = H(\rho_{AB}) - H(\rho_B)$, and $I(A; B|C)_\rho$, accordingly.

A. Two-Node Network

Consider the two-node network in Figure 1. Alice and Bob wish to simulate a c-q state $\omega_{XB}^{\otimes n}$, using the following scheme. Node 1 (Alice) receives a classical source sequence x^n , drawn by Nature according to a given PMF p_X . The source sequence is encoded into an index i at a rate R_1 . Node 2 (Bob) is quantum. Both nodes have access to a CR element j at a given rate R_0 , i.e., j is uniformly distributed over $[2^{nR_0}]$, and it is independent of X^n .

Formally, a $(2^{nR_0}, 2^{nR_1}, n)$ coordination code for the simulation of a c-q state ω_{XB} consists of a classical encoding channel, $F : \mathcal{X}^n \times [2^{nR_0}] \rightarrow [2^{nR_1}]$, and a c-q decoding channel $\mathcal{D}_{IJ \rightarrow B^n}$. The protocol works as follows. A classical sequence $x^n \sim p_X^n$ is generated by Nature. Given the sequence x^n and the CR element j , Alice selects a random index,

$$i \sim F(\cdot | x^n, j) \quad (1)$$

and sends it through a noiseless link. As Bob receives the message i and the CR element j , he prepares the state

$$\rho_{B^n}^{(i,j)} = \mathcal{D}_{IJ \rightarrow B^n}(i, j). \quad (2)$$

Hence, the resulting joint state is

$$\hat{\rho}_{X^n B^n} = \frac{1}{2^{nR_0}} \sum_{j \in [2^{nR_0}]} \sum_{x^n \in \mathcal{X}^n} \left(p_X^n(x^n) |x^n\rangle\langle x^n|_{X^n} \otimes \sum_{i \in [2^{nR_1}]} F(i | x^n, j) \rho_{B^n}^{(i,j)} \right). \quad (3)$$

Definition 1. A coordination rate pair (R_0, R_1) is achievable for the simulation of ω_{XB} , if for every $\varepsilon > 0$ and sufficiently large n , there exists a $(2^{nR_0}, 2^{nR_1}, n)$ code that achieves

$$\|\hat{\rho}_{X^n B^n} - \omega_{XB}^{\otimes n}\|_1 \leq \varepsilon. \quad (4)$$

The coordination capacity region of the two-node network, $\mathcal{R}_{2\text{-node}}(\omega)$, with respect to the c-q state ω_{XB} , is the closure of the set of all achievable rate pairs.

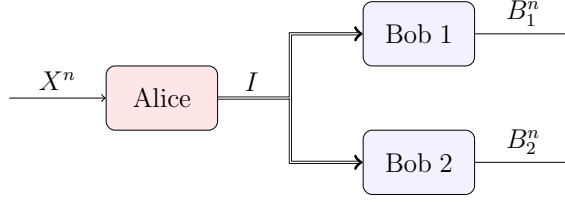


Fig. 3. Broadcast Network. The CR element is omitted for simplicity.

The coordination capacity, $C_{2\text{-node}}^{(0)}(\omega)$, without CR, is the supremum of rates R_1 such that $(0, R_1) \in \mathcal{R}_{2\text{-node}}(\omega)$. The CR-assisted coordination capacity, $C_{2\text{-node}}^{(\infty)}(\omega)$, i.e., with unlimited CR, is the supremum of rates R_1 such that $(R_0, R_1) \in \mathcal{R}_{2\text{-node}}(\omega)$ for some $R_1 \geq 0$.

B. No-Communication Network

Consider a network that consists of three users: Alice, Bob and Charlie, holding quantum systems A , B , and C , respectively. The users cannot communicate, but they share a CR element j at a rate R_0 , as illustrated in Figure 2. Given j , each user prepares a quantum state separately.

A $(2^{nR_0}, n)$ coordination code for the no-communication network consists of a CR set $[2^{nR_0}]$, and three c-q encoding channels, $\mathcal{T}_{J \rightarrow A^n}^{(1)}$, $\mathcal{T}_{J \rightarrow B^n}^{(2)}$, and $\mathcal{T}_{J \rightarrow C^n}^{(3)}$. As Alice, Bob, and Charlie receive a realization j of the CR element, each uses their encoding map to prepare their respective state. Alice prepares a quantum state, $\rho_{A^n}^j = \mathcal{T}_{J \rightarrow A^n}^{(1)}(j)$, $\rho_{B^n}^j = \mathcal{T}_{J \rightarrow B^n}^{(2)}(j)$, and $\rho_{C^n}^j = \mathcal{T}_{J \rightarrow C^n}^{(3)}(j)$, respectively. Hence,

$$\hat{\rho}_{A^n B^n C^n} = \frac{1}{2^{nR_0}} \sum_{j \in [2^{nR_0}]} \mathcal{T}^{(1)}(j) \otimes \mathcal{T}^{(2)}(j) \otimes \mathcal{T}^{(3)}(j).$$

Definition 2. A CR rate R_0 is achievable for the simulation of ω_{ABC} , if for every $\varepsilon > 0$ and sufficiently large n , there exists a $(2^{nR_0}, n)$ coordination code that achieves

$$\|\hat{\rho}_{A^n B^n C^n} - \omega_{ABC}^{\otimes n}\|_1 \leq \varepsilon. \quad (5)$$

The coordination capacity $C_{\text{NC}}(\omega)$, for the no-communication network, is the infimum of achievable rates R_0 . If there are no achievable rates, we set $C_{\text{NC}}(\omega) = +\infty$.

C. Broadcast Network

Consider the broadcast network in Figure 3. A sender, Alice, and two receivers, Bob 1 and Bob 2, wish to simulate a c-q-q state $\omega_{XB_1B_2}$, using the following scheme. Alice receives a classical source sequence $x^n \in \mathcal{X}^n$ drawn by Nature, i.i.d. according to a given PMF p_X . Alice encodes the source sequence into an index i at a rate R_1 . The other two nodes, of Bob 1 and Bob 2, are quantum. The three nodes have access to a CR element j at a rate R_0 . Similarly, a $(2^{nR_0}, 2^{nR_1}, n)$ coordination code consists of a classical encoding channel, $F : \mathcal{X}^n \times [2^{nR_0}] \rightarrow [2^{nR_1}]$, and two c-q decoding channels, $\mathcal{D}_{IJ \rightarrow B_\ell^n}^{(\ell)}$, for $\ell \in \{1, 2\}$. Given x^n and the CR element j , Alice generates $i \sim F(\cdot | x^n, j)$, and sends it to both Bob 1 and Bob 2, who then apply their decoding map.

The coordination capacity region of the broadcast network, $\mathcal{R}_{\text{BC}}(\omega)$, with respect to the c-q state $\omega_{XB_1B_2}$, is defined in a similar manner as in Definition 1.

III. RESULTS

A. Two-Nodes Network

Consider a given c-q state ω_{XB} that we wish to simulate. We now state our main result. Define the following set of c-q states. Let $\mathcal{S}_{2\text{-node}}(\omega)$ be the set of all c-q states

$$\sigma_{XUB} = \sum_{\substack{(x,u) \in \\ \mathcal{X} \times \mathcal{U}}} p_{X,U}(x, u) |x\rangle\langle x|_X \otimes |u\rangle\langle u|_U \otimes \theta_B^u \quad (6a)$$

such that

$$\sigma_{XB} = \omega_{XB} \quad (6b)$$

for $|\mathcal{U}| \leq |\mathcal{X}|^2 [\dim(\mathcal{H}_B)]^2 + 1$. Notice that given a classical value $U = u$, there is no correlation between X and B .

Theorem 1. The coordination capacity region for the two-node system described in Figure 1 is the set

$$\mathcal{R}_{2\text{-node}}(\omega) = \bigcup_{\mathcal{S}_{2\text{-node}}(\omega)} \left\{ \begin{array}{l} (R_0, R_1) \in \mathbb{R}^2 : \\ R_0 \geq I(X; U)_\sigma, \\ R_0 + R_1 \geq I(XB; U)_\sigma \end{array} \right\}. \quad (7)$$

The proof for Theorem 1 is given in Subsection IV. The following corollaries immediately follow.

Corollary 2 (Quantum Common Information [27]). The coordination capacity without CR is

$$R_{2\text{-node}}^{(0)}(\omega) = \min_{\sigma_{XUB} \in \mathcal{S}_{2\text{-node}}(\omega)} I(XB; U)_\sigma. \quad (8)$$

Corollary 3. The CR-assisted coordination capacity, i.e., with unlimited common randomness, is given by

$$R_{2\text{-node}}^{(\infty)}(\omega) \triangleq \min_{\sigma_{XUB} \in \mathcal{S}_{2\text{-node}}(\omega)} I(X; U)_\sigma \quad (9)$$

We note that in order to achieve the CR-assisted capacity, a CR rate of $R_0 = I(U; B|X)_\sigma$ is sufficient. If $B \equiv Y$ is classical, then we may substitute $U = Y$, which yields the capacity $R_{2\text{-node}}^{(\infty)}(\omega) = I(X; Y)$, and it can be achieved with CR at rate $R_0 = H(Y|X)$ [32].

B. No-Communication Network

Consider a given quantum state ω_{ABC} that we wish to simulate. We now state our main result. Define the following set of c-q states. Let $\mathcal{S}_{NC}(\omega)$ be the set of all c-q states

$$\sigma_{UABC} = \sum_{u \in \mathcal{U}} p_U(u) |u\rangle\langle u|_U \otimes \theta_A^u \otimes \theta_B^u \otimes \theta_C^u \quad (10a)$$

such that

$$\sigma_{ABC} = \omega_{ABC} \quad (10b)$$

Given $U = u$, there is no correlation between A, B and C .

Theorem 4. The coordination capacity for the no-communication network described in Figure 2 is

$$C_{NC}(\omega) = \inf_{\sigma_{UABC} \in \mathcal{S}_{NC}(\omega)} I(U; ABC)_\sigma \quad (11)$$

with the convention that an infimum over an empty set is $+\infty$.

Remark 1. Since the CR is classical, it cannot be used in order to create entanglement. Therefore, as Alice, Bob, and Charlie do not cooperate with one another, it is impossible to simulate entanglement. That is, we can only simulate separable states.

C. Broadcast Network

Consider a given c-q-q state $\omega_{XB_1B_2}$ that we wish to simulate. Define the following set of c-q-q states. Let $\mathcal{S}_{2\text{-BC}}(\omega)$ be the set of all c-q-q states

$$\sigma_{XUB_1B_2} = \sum_{\substack{(x,u) \in \\ \mathcal{X} \times \mathcal{U}}} p_{XU}(x, u) |x\rangle\langle x|_X \otimes |u\rangle\langle u|_U \otimes \theta_{B_1}^u \otimes \eta_{B_2}^u$$

such that

$$\sigma_{XB_1B_2} = \omega_{XB_1B_2}.$$

Note that given X, B_1 , and B_2 are uncorrelated given $U = u$.

Theorem 5. The coordination capacity region of the broadcast channel in Figure 3 network is the set

$$\mathcal{R}_{BC}(\omega) = \bigcup_{\mathcal{S}_{BC}(\omega)} \left\{ \begin{array}{l} (R_0, R_1) \in \mathbb{R}^2 : \\ R_0 \geq I(X; U)_\sigma \\ R_0 + R_1 \geq I(XB_1B_2; U)_\sigma \end{array} \right\}. \quad (12)$$

Remark 2. Since Alice's encoding is classical, she cannot distribute entanglement. Therefore, as Bob 1 and Bob 2 do not cooperate with one another, it is impossible to simulate entanglement between Bob 1 and Bob 2. That is, we can only simulate states such that $\omega_{B_1B_2}$ is separable, as in the no-communication model (see Remark 1).

IV. TWO NODE ANALYSIS

Consider the two node network in Figure 1. Our proof for Theorem 1 is based on quantum resolvability [37–39].

Theorem 6 (see [37–39]). Consider an ensemble, $\{p_X, \rho_A^x\}_{x \in \mathcal{X}}$, and a random codebook that consists of 2^{nR} independent sequence, $X^n(m)$, $m \in [2^{nR}]$, each is i.i.d. $\sim p_X$. If $R > I(X; A)_\rho$, then for every $\delta > 0$ and sufficiently large n ,

$$\mathbb{E} \left[\left\| \rho_A^{\otimes n} - \frac{1}{2^{nR}} \sum_{m=1}^{2^{nR}} \rho_{A^n}^{X^n(m)} \right\|_1 \right] \leq \delta, \quad (13)$$

where $\rho_{A^n}^{x^n} \equiv \bigotimes_{k=1}^n \rho_{A^k}^{x_k}$, and the expectation is over all realizations of the random codebook.

A. Achievability proof

Assume (R_0, R_1) is in the interior of $\mathcal{R}_{2\text{-node}}(\omega)$. We need to construct a code that consists of an encoding channel $F(i|x^n, j)$ and a c-q decoding channel $\mathcal{D}_{IJ \rightarrow B^n}$, such that the error requirement in (4) holds.

By the definition of $\mathcal{S}_{2\text{-node}}(\omega)$, there exists a c-q state

$$\sigma_{UXB} = \sum_{u \in \mathcal{U}} p_U(u) |u\rangle\langle u|_U \otimes \sigma_{XB}^u \quad (14)$$

such that

$$\sigma_{XB}^u = \sum_{x \in \mathcal{X}} p_{X|U}(x|u) |x\rangle\langle x|_X \otimes \theta_B^u, \quad u \in \mathcal{U} \quad (15)$$

$$\sigma_{XB} = \omega_{XB}, \quad (16)$$

$$R_1 \geq I(X; U)_\sigma, \quad R_0 + R_1 \geq I(XB; U)_\sigma. \quad (17)$$

Classical codebook generation: Select a random codebook $\mathcal{C} = \{u^n(i, j)\}$ by drawing $2^{n(R_0+R_1)}$ i.i.d. sequences according to the distribution $p_U^n(u^n) = \prod_{k=1}^n p_U(u_k)$. Reveal the codebook to Alice and Bob.

Let (i, j) be a pair of random indices, uniformly distributed over $[2^{nR_1}] \times [2^{nR_0}]$. Define the following PMF

$$\tilde{P}_{X^n IJ}(x^n, i, j) \equiv \frac{1}{2^{n(R_0+R_1)}} p_{X|U}^n(x^n | u^n(i, j)). \quad (18)$$

Encoder: We define the encoding channel F as the conditional distribution above, i.e., $F = \tilde{P}_{I|X^n J}$.

Decoder: As Bob receives i from Alice, and the random element j , he prepares the output state $\mathcal{D}_{IJ \rightarrow B^n}(i, j) = \theta_{B^n}^{u^n(i, j)}$.

Error analysis: Let $\delta > 0$. The encoder sends $i \sim F(\cdot|x^n, j)$. Given $J = j$, by the classical resolvability theorem, Cuff [32] has shown that $R_0 \geq I(X; U)_\sigma$ guarantees

$$\mathbb{E} \left\| \tilde{P}_{JX^n} - p_J \times p_X^n \right\|_1 \leq \delta \quad (19)$$

for sufficiently large n , where $\tilde{P}_{JX^n}$ is as in (18). Recall that $\tilde{P}_{JX^n}$ is random, since the codebook $\mathcal{C}$ is random. Hence, the expectation is over all realizations of $\mathcal{C}$. The resulting state is

$$\hat{\rho}_{X^n B^n} = \frac{1}{2^{nR_0}} \sum_{j, x^n} \left(p_X^n(x^n) |x^n\rangle\langle x^n|_{X^n} \otimes \sum_{i \in [2^{nR_1}]} \tilde{P}_{I|X^n J}(i|x^n, j) \theta_{B^n}^{u^n(i, j)} \right) \quad (20)$$

According to (19), the probability distributions $\tilde{P}_{I|X^n J}$ and $p_J \times p_X^n$ are close on average. Then, let

$$\hat{\tau}_{X^n B^n} \equiv \sum_{j, x^n} \tilde{P}_{JX^n}(j|x^n) |x^n\rangle\langle x^n|_{X^n} \otimes \sum_{i \in [2^{nR_1}]} \tilde{P}_{I|X^n J}(i|x^n, j) \theta_{B^n}^{u^n(i, j)}. \quad (21)$$

By (19), it follows that

$$\mathbb{E} \left\| \hat{\tau}_{X^n B^n} - \hat{\rho}_{X^n B^n} \right\|_1 \leq \delta. \quad (22)$$

Observe that

$$\begin{aligned} \hat{\tau}_{X^n B^n} &= \sum_{i, j, x^n} \tilde{P}_{IJX^n}(i, j, x^n) |x^n\rangle\langle x^n|_{X^n} \otimes \theta_{B^n}^{u^n(i, j)} \\ &= \frac{1}{2^{n(R_0+R_1)}} \sum_{i, j, x^n} p_{X|U}^n(x^n | u^n(i, j)) |x^n\rangle\langle x^n|_{X^n} \otimes \theta_{B^n}^{u^n(i, j)} \\ &= \frac{1}{2^{n(R_0+R_1)}} \sum_{i, j} \sigma_{X^n B^n}^{u^n(i, j)} \end{aligned} \quad (23)$$

where the second equality is due to the definition of $\tilde{P}$ in (18), and the last line follows from (15).

Thus, according to the quantum resolvability theorem, Theorem 6, when applied to the joint system XB , we have

$$\mathbb{E} \|\sigma_{XB}^{\otimes n} - \hat{\tau}_{X^n B^n}\|_1 \leq \delta \quad (24)$$

for sufficiently large n . Therefore, by the triangle inequality,

$$\begin{aligned} \mathbb{E} \|\omega_{XB}^{\otimes n} - \hat{\rho}_{X^n B^n}\|_1 &\leq \mathbb{E} \|\omega_{XB}^{\otimes n} - \hat{\tau}_{X^n B^n}\|_1 + \mathbb{E} \|\hat{\tau}_{X^n B^n} - \hat{\rho}_{X^n B^n}\|_1 \\ &\leq 2\delta \end{aligned} \quad (25)$$

by (16), (22) and (24). $\square$

B. Converse proof

Let (R_0, R_1) be an achievable rate pair. Then, there exists a sequence $(2^{nR_0}, 2^{nR_1}, n)$ coordination codes such that the joint quantum state $\hat{\rho}_{X^n B^n}$ satisfies

$$\|\omega_{XB}^{\otimes n} - \hat{\rho}_{X^n B^n}\|_1 \leq \varepsilon_n \quad (26)$$

where ε_n tends to zero as $n \rightarrow \infty$.

Fix an index $k \in \{1, \dots, n\}$. By trace monotonicity [40], taking the partial trace over $X_j, B_j, j \neq k$, maintains the inequality. Thus,

$$\|\omega_{XB} - \hat{\rho}_{X_k B_k}\|_1 \leq \varepsilon_n. \quad (27)$$

Then, by the AFW inequality [41],

$$\left| H(X^n B^n)_{\hat{\rho}} - nH(XB)_{\omega} \right| \leq n\beta_n, \quad (28)$$

and

$$\left| H(X_k B_k)_{\hat{\rho}} - H(XB)_{\omega} \right| \leq \beta_n, \quad (29)$$

for $k \in [n]$, where β_n tends to zero as $n \rightarrow \infty$. Therefore,

$$\begin{aligned} \left| H(X^n B^n)_{\hat{\rho}} - \sum_{k=1}^n H(X_k B_k)_{\hat{\rho}} \right| &\leq \left| H(X^n B^n)_{\hat{\rho}} - nH(XB)_{\omega} \right| + \left| nH(XB)_{\omega} - \sum_{k=1}^n H(X_k B_k)_{\hat{\rho}} \right| \\ &\leq \left| H(X^n B^n)_{\hat{\rho}} - nH(XB)_{\omega} \right| + \sum_{k=1}^n \left| H(XB)_{\omega} - H(X_k B_k)_{\hat{\rho}} \right| \\ &\leq 2n\beta_n. \end{aligned} \quad (30)$$

Now, we have

$$n(R_0 + R_1) \geq H(IJ) \quad (31)$$

$$\geq I(X^n B^n; IJ)_{\hat{\rho}} \quad (32)$$

since the conditional entropy is nonnegative for classical and c-q states, and the CR element J is statistically independent of the source X^n . Furthermore, by entropy sub-additivity [40],

$$\begin{aligned} I(X^n B^n; IJ)_{\hat{\rho}} &\geq H(X^n B^n)_{\hat{\rho}} - \sum_{k=1}^n H(X_k B_k | IJ)_{\hat{\rho}} \\ &\geq \sum_{k=1}^n I(X_k B_k; IJ)_{\hat{\rho}} - 2n\beta_n \end{aligned} \quad (33)$$

where the last inequality follows from (30). Defining a time-sharing variable $K \sim \text{Unif}[n]$, this can be written as

$$R_0 + R_1 + 2\beta_n \geq I(X_K B_K; IJ | K)_{\hat{\rho}} \quad (34)$$

with respect to the extended state:

$$\hat{\rho}_{K I J X_K B_K} = \frac{1}{n} \sum_{k=1}^n |k\rangle\langle k| \otimes \hat{\rho}_{I J X_k B_k}. \quad (35)$$

Observe that by (27) and the triangle inequality,

$$\begin{aligned} \|\omega_{XB} - \hat{\rho}_{X_K B_K}\|_1 &= \left\| \omega_{XB} - \frac{1}{n} \sum_{k=1}^n \hat{\rho}_{X_k B_k} \right\|_1 \\ &\leq \varepsilon_n. \end{aligned} \quad (36)$$

Thus, by the AFW inequality,

$$\begin{aligned} I(X_K B_K; K)_{\hat{\rho}} &= H(X_K B_K)_{\hat{\rho}} - \frac{1}{n} \sum_{k=1}^n H(X_k B_k)_{\hat{\rho}} \\ &\leq \gamma_n, \end{aligned} \quad (37)$$

where γ_n tends to zero. Together with (34), it follows that

$$R_0 + R_1 + 2\beta_n + \gamma_n \geq I(X_K B_K; IJK)_{\hat{\rho}} \quad (38)$$

By similar arguments,

$$R_1 + 2\beta_n + \gamma_n \geq I(X_K; IJ) \quad (39)$$

To complete the converse proof, we identify U , X , and B with (I, J, K) , X_K , and B_K , respectively. Observe that given (i, j, k) , the joint state of X_K and B_K is $(\sum_{x_k \in \mathcal{X}} p_{X_k|IJ}(x_k|i, j) |x_k\rangle\langle x_k|_{X_K}) \otimes \rho_{B_k}^{(i, j)}$, where $p_{X^n|IJ}$ is the a posteriori probability distribution. Thus, there X and B are uncorrelated when conditioned on U , as required.

The bound on $|\mathcal{U}|$ follows by applying the Caratheodory theorem to the real-valued parametric representation of density matrices, as in [42, App. B]. $\square$

V. NO-COMMUNICATION NETWORK ANALYSIS

Consider the no-communication network in Figure 2, of a quantum state ω_{ABC} . To prove Theorem 4, we use similar tools. The achievability proof is straightforward, and it is thus omitted. Then, consider the converse part. Assume that R_0 is achievable. Therefore, there exists a sequence of $(2^{nR_0}, n)$ of coordination codes such that for sufficiently large values of n ,

$$\|\hat{\rho}_{A^n B^n C^n} - \omega_{ABC}^{\otimes n}\|_1 \leq \varepsilon_n, \quad (40)$$

where $\varepsilon_n \rightarrow 0$ as $n \rightarrow \infty$.

Applying the chain rule,

$$nR_0 \geq H(J) \quad (41)$$

$$\geq I(A^n B^n C^n; J)_{\hat{\rho}} \quad (42)$$

$$= \sum_{k=1}^n I(A_k B_k C_k; J | A^{k-1} B^{k-1} C^{k-1})_{\hat{\rho}} \quad (43)$$

For every $k \in [n]$, by trace monotonicity [40],

$$\|\omega_{ABC}^{\otimes k} - \hat{\rho}_{A^k B^k C^k}\|_1 \leq \varepsilon_n. \quad (44)$$

Then, by the AFW inequality [41] [40, Ex. 11.10.2],

$$|I(A_k B_k C_k; A^{k-1} B^{k-1} C^{k-1})_{\hat{\rho}} - I(A_k B_k C_k; A^{k-1} B^{k-1} C^{k-1})_{\omega^{\otimes k}}| \leq \beta_n, \quad (45)$$

where β_n tends to zero as $n \rightarrow \infty$. That is,

$$I(A_k B_k C_k; A^{k-1} B^{k-1} C^{k-1})_{\hat{\rho}} \leq \beta_n \quad (46)$$

since $A_k B_k C_k$ and $(A_j B_j C_j)_{j < k}$ are in a product state $\omega \otimes \omega^{\otimes (k-1)}$. Hence, by (43),

$$\begin{aligned} nR_0 &\geq \sum_{k=1}^n I(A_k B_k C_k; J | A^{k-1} B^{k-1} C^{k-1})_{\hat{\rho}} - n\beta_n \\ &\geq \sum_{k=1}^n I(A_k B_k C_k; J)_{\hat{\rho}} - n\beta_n \\ &\geq n \left(\inf_{\sigma_{UABC} \in \mathcal{S}_{NC}(\omega)} I(U; ABC)_{\sigma} - 2\beta_n \right) \end{aligned} \quad (47)$$

taking $U \equiv J$, as the encoders are uncorrelated given J . $\square$

VI. BROADCAST NETWORK ANALYSIS

Consider coordination in broadcast network, as in Figure 3 in the main text, of a classical-quantum-quantum state $\omega_{XB_1B_2}$. To prove the capacity theorem, Theorem 5, we use similar tools as in Section IV.

A. Achievability proof

Assume (R_0, R_1) is in the interior of $\mathcal{R}_{\text{BC}}(\omega)$. We need to construct a code that consists of an encoding channel $F(i|x^n, j)$ and a two c-q decoding channels $\mathcal{D}_{IJ \rightarrow B_1^n}$ and $\mathcal{D}_{IJ \rightarrow B_2^n}$, such that

$$\left\| \omega_{XB}^{\otimes n} - \frac{1}{2^{nR_0}} \sum_{j \in [2^{nR_0}]} \sum_{x^n \in \mathcal{X}^n} p_X^n(x^n) |x^n\rangle\langle x^n|_{X^n} \otimes \sum_{i \in [2^{nR_1}]} F(i|x^n, j) \mathcal{D}_{IJ \rightarrow B_1^n}(i, j) \otimes \mathcal{D}_{IJ \rightarrow B_2^n}(i, j) \right\|_1 \leq \varepsilon. \quad (48)$$

According to the definition of $\mathcal{S}_{\text{BC}}(\omega)$ (see Subsection III-C), there exists a c-q state $\sigma_{XUB_1B_2}$ that can be written as

$$\sigma_{XUB_1B_2} = \sum_{(x,u) \in \mathcal{X} \times \mathcal{U}} p_{X,U}(x, u) |x\rangle\langle x|_X \otimes |u\rangle\langle u|_U \otimes \theta_{B_1}^u \otimes \eta_{B_2}^u \quad (49)$$

and satisfy

$$\sigma_{XB_1B_2} = \omega_{XB_1B_2} \quad (50)$$

We will also consider conditioning on $U = u$, and denote

$$\sigma_{XB_1B_2}^u = \sum_{x \in \mathcal{X}} p_{X|U}(x|u) |x\rangle\langle x|_X \otimes \theta_{B_1}^u \otimes \eta_{B_2}^u. \quad (51)$$

Classical codebook generation: Select a random codebook $\mathcal{C}_{\text{BC}} = \{u^n(i, j)\}$ by drawing $2^{n(R_0+R_1)}$ i.i.d. sequences according to the distribution p_U^n . Reveal the codebook.

Encoder: Define the encoding channel as $F = \tilde{P}_{I|X^nJ}$, where $\tilde{P}_{X^nIJ}$ be a joint distribution as in (18).

Decoders: As Bob 1 and Bob 2 receive i from Alice, and the random element j , they prepare the following output states,

$$\mathcal{D}_{IJ \rightarrow B_1^n}^{(1)}(i, j) = \theta_{B_1}^{u^n(i, j)}, \quad (52)$$

$$\mathcal{D}_{IJ \rightarrow B_2^n}^{(2)}(i, j) = \eta_{B_2}^{u^n(i, j)}. \quad (53)$$

Error analysis: Let $\delta > 0$. The encoder sends $i \sim F(\cdot|x^n, j)$. As in Subsection IV-A, given j , if $R_1 \geq I(X; U)$, then

$$\mathbb{E} \left\| \tilde{P}_{JX^n} - p_J \times p_X^n \right\|_1 \leq \delta \quad (54)$$

for sufficiently large n . As $\tilde{P}_{JX^n}$ depends on the random codebook $\mathcal{C}_{\text{BC}}$, the expectation is over all realizations of $\mathcal{C}_{\text{BC}}$. The resulting state is

$$\begin{aligned} \hat{\rho}_{X^n B_1^n B_2^n} &= \frac{1}{2^{nR_0}} \sum_{j \in [2^{nR_0}]} \sum_{x^n \in \mathcal{X}^n} \left(p_X^n(x^n) |x^n\rangle\langle x^n|_{X^n} \otimes \sum_{i \in [2^{nR_1}]} F(i|x^n, j) \mathcal{D}_{IJ \rightarrow B_1^n}(i, j) \otimes \mathcal{D}_{IJ \rightarrow B_2^n}(i, j) \right) \\ &= \frac{1}{2^{nR_0}} \sum_{j, x^n} \left(p_X^n(x^n) |x^n\rangle\langle x^n|_{X^n} \otimes \sum_{i \in [2^{nR_1}]} \tilde{P}_{I|X^nJ}(i|x^n, j) \theta_{B_1}^{u^n(i, j)} \otimes \eta_{B_2}^{u^n(i, j)} \right). \end{aligned} \quad (55)$$

According to (54), the probability distributions $\tilde{P}_{JX^n}$ and $p_J \times p_X^n$ are close on average. Then, let

$$\hat{\tau}_{X^n B_1^n B_2^n} \equiv \sum_{j, x^n} \tilde{P}_{JX^n}(j, x^n) |x^n\rangle\langle x^n|_{X^n} \otimes \sum_{i \in [2^{nR_1}]} \tilde{P}_{I|X^nJ}(i|x^n, j) \theta_{B_1}^{u^n(i, j)} \otimes \eta_{B_2}^{u^n(i, j)}. \quad (56)$$

Then, it follows that

$$\mathbb{E} \left\| \hat{\tau}_{X^n B_1^n B_2^n} - \hat{\rho}_{X^n B_1^n B_2^n} \right\|_1 \leq \delta, \quad (57)$$

by (54). Observe that

$$\begin{aligned} \hat{\tau}_{X^n B_1^n B_2^n} &= \sum_{i, j, x^n} \tilde{P}_{IJX^n}(i, j, x^n) |x^n\rangle\langle x^n|_{X^n} \otimes \theta_{B_1}^{u^n(i, j)} \otimes \eta_{B_2}^{u^n(i, j)} \\ &= \frac{1}{2^{n(R_0+R_1)}} \sum_{i, j, x^n} p_{X|U}^n(x^n|u^n(i, j)) |x^n\rangle\langle x^n|_{X^n} \otimes \theta_{B_1}^{u^n(i, j)} \otimes \eta_{B_2}^{u^n(i, j)} \\ &= \frac{1}{2^{n(R_0+R_1)}} \sum_{i, j} \sigma_{X^n B_1^n B_2^n}^{u^n(i, j)}, \end{aligned} \quad (58)$$

where the second equality is due to the definition of $\tilde{P}$ in (18), and the last line follows from (51).

Thus, according to the quantum resolvability theorem 6, when applied to the joint system XB_1B_2 , we have

$$\mathbb{E} \left\| \sigma_{XB_1^n B_2^n}^{\otimes n} - \hat{\tau}_{X^n B_1^n B_2^n} \right\|_1 \leq \delta \quad (59)$$

for sufficiently large n . Therefore, by the triangle inequality,

$$\begin{aligned} \mathbb{E} \left\| \omega_{XB_1 B_2}^{\otimes n} - \hat{\rho}_{X^n B_1^n B_2^n} \right\|_1 &\leq \mathbb{E} \left\| \omega_{XB_1^n B_2^n}^{\otimes n} - \hat{\tau}_{X^n B_1^n B_2^n} \right\|_1 + \mathbb{E} \left\| \hat{\tau}_{X^n B_1^n B_2^n} - \hat{\rho}_{X^n B_1^n B_2^n} \right\|_1 \\ &\leq 2\delta \end{aligned} \quad (60)$$

by (50), (57) and (59). $\square$

B. Converse proof

Let (R_0, R_1) be an achievable coordination rate pair for the simulation of a c-q-q state $\omega_{XB_1B_2}$ in the broadcast setting. Then, there exists a sequence of $(2^{nR_0}, 2^{nR_1}, n)$ coordination codes such that the joint quantum state $\hat{\rho}_{X^n B_1^n B_2^n}$ satisfies

$$\left\| \omega_{XB_1^n B_2^n}^{\otimes n} - \hat{\rho}_{X^n B_1^n B_2^n} \right\|_1 \leq \varepsilon_n, \quad (61)$$

where ε_n tends to zero as $n \rightarrow \infty$. Fix an index $k \in \{1, \dots, n\}$. By trace monotonicity [40], taking the partial trace over X_j , B_{1j} , B_{2j} , for $j \neq k$, maintains the inequality, thus Thus,

$$\left\| \omega_{XB_1 B_2} - \hat{\rho}_{X_k B_{1k} B_{2k}} \right\|_1 \leq \varepsilon_n. \quad (62)$$

Then, by the AFW inequality [41],

$$\left| H(X^n B_1^n B_2^n)_{\hat{\rho}} - nH(XB_1B_2)_{\omega} \right| \leq n\beta_n, \quad (63)$$

and

$$\left| H(X_k B_{1k} B_{2k})_{\hat{\rho}} - H(XB_1B_2)_{\omega} \right| \leq \beta_n, \quad (64)$$

for $k \in [n]$, where β_n tends to zero as $n \rightarrow \infty$. Therefore,

$$\left| H(X^n B_1^n B_2^n)_{\hat{\rho}} - \sum_{k=1}^n H(X_k B_{1k} B_{2k})_{\hat{\rho}} \right| \leq 2n\beta_n. \quad (65)$$

Now, we also have

$$\begin{aligned} n(R_0 + R_1) &\geq H(IJ) \\ &\geq I(X^n B_1^n B_2^n; IJ)_{\hat{\rho}}, \end{aligned} \quad (66)$$

since the conditional entropy is nonnegative for classical and c-q-q states, and the CR element J is statistically independent of the source X^n . Furthermore, by entropy sub-additivity [40],

$$\begin{aligned} I(X^n B_1^n B_2^n; IJ)_{\hat{\rho}} &\geq H(X^n B_1^n B_2^n)_{\hat{\rho}} - \sum_{k=1}^n H(X_k B_{1k} B_{2k} | IJ)_{\hat{\rho}} \\ &\geq \sum_{k=1}^n I(X_k B_{1k} B_{2k}; IJ)_{\hat{\rho}} - 2n\beta_n \end{aligned} \quad (67)$$

where the last inequality follows from (65).

Defining a time-sharing variable $K \sim \text{Unif}[n]$, this can be written as

$$R_0 + R_1 + 2\beta_n \geq I(X_K B_{1K} B_{2K}; IJ | K)_{\hat{\rho}} \quad (68)$$

with respect to the extended state

$$\hat{\rho}_{K I J X_k B_{1k} B_{2k}} = \frac{1}{n} \sum_{k=1}^n |k\rangle\langle k| \otimes \hat{\rho}_{I J X_k B_{1k} B_{2k}}. \quad (69)$$

Observe that

$$\left\| \omega_{XB_1 B_2} - \hat{\rho}_{X_K B_{1K} B_{2K}} \right\|_1 = \left\| \omega_{XB_1 B_2} - \frac{1}{n} \sum_{k=1}^n \hat{\rho}_{X_k B_{1k} B_{2k}} \right\|_1 \leq \varepsilon_n \quad (70)$$

by the triangle inequality (see (62)). Thus, by the AFW inequality,

$$\begin{aligned} I(X_K B_{1K} B_{2K}; K)_{\hat{\rho}} &= H(X_K B_{1K} B_{2K})_{\hat{\rho}} - \frac{1}{n} \sum_{k=1}^n H(X_k B_{1k} B_{2k})_{\hat{\rho}} \\ &\leq \gamma_n, \end{aligned} \quad (71)$$

where γ_n tends to zero as $n \rightarrow \infty$. Together with (67), it implies

$$R_0 + R_1 + 2\beta_n + \gamma_n \geq I(X_K B_{1K} B_{2K}; IJK)_{\hat{\rho}}. \quad (72)$$

By similar arguments,

$$R_1 + 2\beta_n + \gamma_n \geq I(X_K; IJ). \quad (73)$$

To complete the converse proof, we identify U , X , and $B_1 B_2$ with (I, J, K) , X_K , and $B_{1K} B_{2K}$, respectively. Observe that given (i, j, k) , the joint state of X_K and $B_{1K} B_{2K}$ is

$$\left(\sum_{x_k \in \mathcal{X}} p_{X_k|IJ}(x_k|i, j) |x_k\rangle\langle x_k|_{X_K} \right) \otimes \rho_{B_{1K}}^{(i,j)} \otimes \rho_{B_{2K}}^{(i,j)}, \quad (74)$$

where $p_{X^n|IJ}$ is the a posteriori probability distribution. Thus, there is no correlation between X , B_1 , and B_2 when conditioned on U , as required. $\square$

VII. SUMMARY AND DISCUSSION

We study coordination in three network models with classical communication links: 1) two-node network simulating a c-q state, 2) no-communication network simulating a separable state, and 3) a broadcast network simulating a c-q-q state. Our models are motivated by quantum-enhanced Internet of Things (IoT) networks in which the communication links are classical [33–36]. In our previous work [30], we considered entanglement coordination using quantum communication links, and our analysis therein was based on different tools compared to those used here.

Our findings generalize classical results from [32] and [13], and also quantum results from [27]. The no-communication and broadcast networks can easily be extended to m encoders and decoders, respectively. The results are relevant for various applications, where the network nodes could represent classical-quantum sensors [43], computers performing a joint computation task [11, 44], or players in a nonlocal game [45, 46]. Both this work and the previous work [30] can be viewed as a step forward in understanding coordination in networks that may comprise either classical or quantum resources.

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