Generalized Two-Particle Interference

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Two-photon interference is an interesting quantum phenomenon that is usually captured in two distinct types of experiments, namely the Hanbury-Brown-Twiss (HBT) experiment and the Hong-Ou-Mandel (HOM) experiment. While the HBT experiment was carried out much earlier in 1956, with classical light, the demonstration of the HOM effect came much later in 1987. Unlike the former, the latter has been argued to be a purely quantum effect. A generalized formulation of two-particle interference is presented here. The HOM and the HBT effects emerge as special cases in the general analysis. A realizable two-particle interference experiment, which is intermediate between the two effects, is proposed and analyzed. Thus two-particle interference is shown to be a single phenomenon with various possible implementations, including the HBT and HOM setups.

1 Introduction

Numerous fascinating phenomena, such as photon bunching and anti-bunching, were seen with the development of lasers and quantum optics [1]. Whether they are photons or neutral atoms, identical bosons in quantum optics have been revealing an increasing number of fascinating aspects of quantum mechanics [2]. Two-photon interference is a phenomenon which is at the heart of quantum optics [3]. Two experiments which beautifully unveil *two-particle interference*, are the Hanbury-Brown-Twiss (HBT) experiment [2] and the Hong-Ou-Mandel (HOM) experiment [4].

The Hanbury Brown-Twiss effect

The HBT effect was discovered much before the HOM effect, in classical radio waves. Later in 1956 it was demonstrated in classical light [5]. In the HBT experiment two particles emerge from two spatially separated sources A and B, and travel to separate, movable detectors at positions x_1 and x_2 (see Fig. 1). Individual detectors do not show any interesting effect, as expected. However, if one correlates the *intensity* of the two detectors, it shows an interference as function of the separation of the two detectors. This means that given one photon has landed at particular position, there are positions on which the other photon would never land. This is quite an unexpected behavior for independent photons.

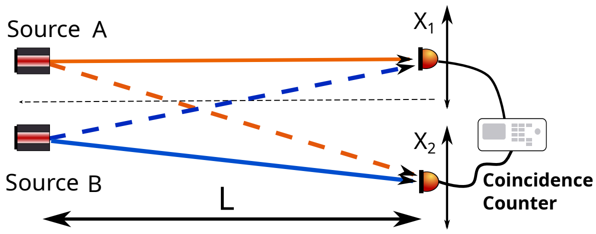

Figure 1: A schematic diagram for the Hanbury Brown-Twiss experiment. Independent particles from sources A and B travel and arrive at the two detectors at x_1 and x_2.

The phenomenon can be understood easily using classical waves. However, its applicability and meaning in quantum domain was widely debated and misunderstood [6]. People visualized that in order to show interference, the two photons, coming from independent sources, would need to "know" where to land! Now the HBT effect in the quantum domain is well understood [7, 8]. There is a crucial difference between the classical and quantum HBT effect. For classical waves, the HBT interference visibility can be at the most 1/2. However, in the quantum case the visibility can ideally be 1. The HBT effect has now been demonstrated using ultrcold atoms [9, 10, 11] and also with electrons [12].

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The Hong-Ou-Mandel effect

We briefly introduce the HOM experiment which was first reported in 1987 [13]. Two identical particles emerge from two spatially separated sources A and B (see Fig. 2). The two particles are split by the 50-50 beam-splitter BS, and reach the *fixed* detectors D_1, D_2. Since the two photons are independent, one would expect that half the time the two would land up in different detectors. However, it is observed that if the sources are tuned in such a way that the two photons arrive at the beam splitter at the same time, the two photons always go together to the same detector! In other words, the coincident count of detectors D_1 and D_2 shows a dip, and goes to zero in the ideal case. This is the famous "HOM dip." The HOM effect has been demonstrated for completely independent photons [14].

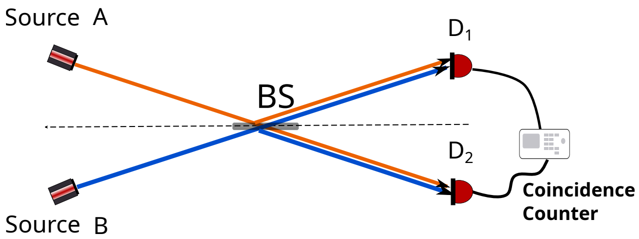

Figure 2: A schematic diagram for the Hong-Ou-Mandel experiment. Independent particles from sources A and B meet at the beam-splitter BS, and then arrive at the detectors D_1 and D_2.

In the way that the two effects have been described above, and also in the way they came about historically, the two are quite distinct. While HBT effect was originally seen in classical waves, the HOM effect is believed to be completely quantum. Since at the quantum level both the effects are rooted in the indistinguishability of identical particles, one might wonder if there is a deeper common origin of both. That is the issue we address in this investigation, and demonstrate that indeed there is a single two-particle interference phenomenon underlying both.

2 Generalized n-port interferometer

The connection between the HBT and HOM experiments can be understood by drawing an analogy with the connection between a single particle two-slit interference and the Mach-Zehnder interferometer [15]. The two-slit interference and the Mach-Zehnder interference are essentially the same. The only difference is that while in the Mach-Zehnder interferometer, a beam is split into two distinct beams, in the two-slit interference experiment the beam emerging from one slit eventually spreads over an infinite number of positions on the screen. We believe something similar happens in the HOM and HBT experiments. Particles from one source are split into two distinct beams by the beam-splitter in the HOM experiment. On the other hand, particle emerging from a single source in the HBT experiment, spread over a continous set of positions when they reach the screen.

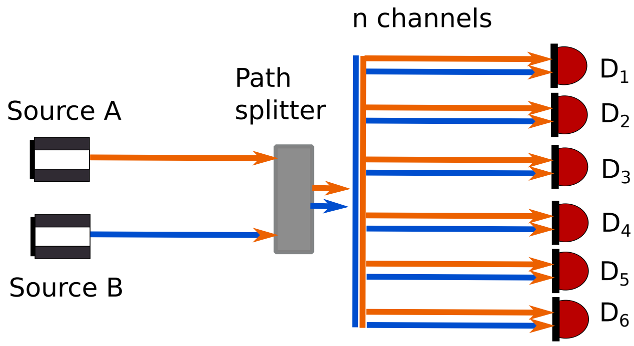

Figure 3: Schematic diagram for a *generalized* n-port two-particle interference experiment. Independent particles from sources A and B are split into n common channels by the path-splitter, and then arrive at detectors D_1 to D_n.

In order to understand two-particle interference, we assume a general scenario where there are two sources and a particle from a particular source is split into n channels. The same happens with the particle coming from the other source. The channels are the same for both the particles (see Fig. 3). The two particles coming to different channels may pick up different phases. Particle emnating from source A has a state $|\psi_A\rangle$, and that from source B, a state $|\psi_B\rangle$, such that the combined two particle state before entering the path-splitter is

$$|\Psi_0\rangle = \tfrac{1}{\sqrt{2}}(|\psi_A\rangle_1|\psi_B\rangle_2 + |\psi_A\rangle_2|\psi_B\rangle_1), \qquad (1)$$

where the subscripts on the kets denote the particle label. Now each particle gets split into an equal superposition of n output channels. Each channel j ends up at unique detector $|D_j\rangle$. Thus the effect of the path-splitter on the two initial states is given by

$$\mathbf{U_{PS}}|\psi_A\rangle = \frac{1}{\sqrt{n}}\sum_{i=1}^{n} e^{i\theta_i}|D_i\rangle$$
$$\mathbf{U_{PS}}|\psi_B\rangle = \frac{1}{\sqrt{n}}\sum_{j=1}^{n} e^{i\phi_j}|D_j\rangle, \quad (2)$$

where θ_k, ϕ_k are the phases picked up by the particles in arriving in channel k, from source A and B, respectively.

The final two-particle state at the detectors is then given by

$$|\Psi_f\rangle = \mathbf{U_{PS}}\frac{1}{\sqrt{2}}(|\psi_A\rangle_1|\psi_B\rangle_2 + |\psi_A\rangle_2|\psi_B\rangle_1),$$
$$= \frac{1}{n\sqrt{2}}\sum_{i=1}^{n} e^{i\theta_i}|D_i\rangle_1 \sum_{j=1}^{n} e^{i\phi_j}|D_j\rangle_2$$
$$+\frac{1}{n\sqrt{2}}\sum_{j=1}^{n} e^{i\theta_j}|D_j\rangle_2 \sum_{i=1}^{n} e^{i\phi_i}|D_i\rangle_1. \quad (3)$$

There are two kinds of terms in the product of the two sums. One are the diagonal terms involving just one channel, and the other are the "cross terms" involving two channels. Latter ones potentially give rise to interference. The final state then has the following form

$$|\Psi_f\rangle = \frac{\sqrt{2}}{n}\sum_{i=1}^{n} e^{i(\theta_i+\phi_i)}|D_i\rangle_1|D_i\rangle_2$$
$$+\frac{1}{n\sqrt{2}}\sum_{j\neq i}(e^{i(\theta_i+\phi_j)} + e^{i(\theta_j+\phi_i)})|D_i\rangle_1|D_j\rangle_2. \quad (4)$$

In order to proceed any further we need some information on n and the various phases θ_i, ϕ_k.

2.1 General n: A simple case

For an arbitrary value of n, let us assume that all phases for the particle coming from source A are zero, i.e., $\theta_i = 0$ for $i = 1, \ldots, n$. For particle coming from source B, $\phi_i = 0$ for odd i, and $\phi_i = \pi$ for even i. Now it is easy to see that the phase factor in the cross-term

$$e^{i(\theta_i+\phi_j)} + e^{i(\theta_j+\phi_i)} = \begin{cases} 2 & (i,j \text{ both odd or both even}) \\ 0 & (\text{in } i,j \text{ one is even, one odd}) \end{cases} \quad (5)$$

So the terms where one among i, j is even, and the other odd, disappear. This represents destructive interference. The detection results can be summarized as follows.

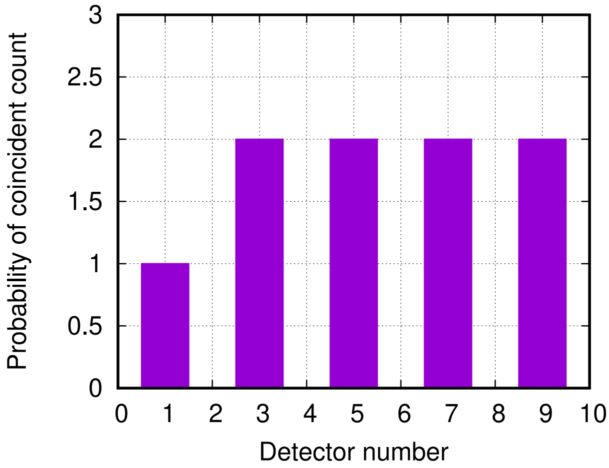

Figure 4: Probability of coincident count of detector 1 with various detectors, in units of $2/n^2$. The probability of coincident count with even number detectors is zero. They constitute the dark fringes of the interference pattern.

1. Probability of both particles landing at i'th detector $= |_1\langle D_i|_2\langle D_i|\Psi_f\rangle|^2 = \frac{2}{n^2}$
2. Probability of particles landing at both odd or both even detectors $= |_1\langle D_i|_2\langle D_j|\Psi_f\rangle|^2 = \frac{4}{n^2}$. Such terms represent the bright fringes, with two two-particle amplitudes adding up.
3. Probability of one particle landing at odd and one at even detector $= |_1\langle D_i|_2\langle D_j|\Psi_f\rangle|^2 = 0$. Such terms represent the dark fringes, with two two-particle amplitudes destroying each other.

If one plots the probability of a coincident count of a particular detector with various detectors, one would get an interference pattern, with alternate detectors showing zero coincident count (see Fig. 4). This general analysis can be used to study various real two-particle interference experiments. We do that in the ensuing analysis.

2.2 $n = 2$: The HOM Experiment

In the preceding analysis if we put $n = 2$, it can exactly describe the HOM experiment. In Fig. 2 if we assume that the *lower* surface of the mirror is half-silvered, then the photons coming from source A reach the detectors D_1, D_2 without any phase change. However, the photons coming from source B pick up a phase of π in reaching D_2. The analysis for a general n can then be applied here directly. We notice that there are cross terms involving only one even and one odd detector. Consequently in the coincident counts there will be only one dark fringe, and no bright fringe. That is precisely what is seen in the HOM experiment. The coincident count between the two detectors goes to zero.

2.3 $n = 4$: An Extended HOM Experiment

In the HOM experiment, a particle from a particle source, is split by the beam-splitter into a superposition of two parts, one reaching D_1 and the other reaching D_2. Let us visualize an extended version of this experiment where the particles coming from both the sources are split into a superposition of four parts each. A realizable setup which implements this scheme is shown in Fig. 5. The combination of three beam-splitters

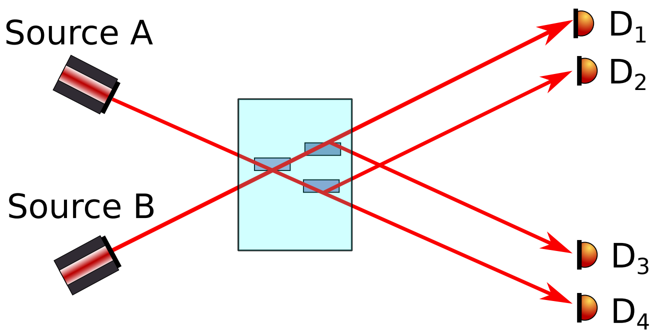

Figure 5: Schematic diagram for a 4-port two-particle interference experiment. Independent particles from sources A and B are split into 4 common channels by a combination of 3 beam-splitters, and then arrive at detectors D_1, D_2, D_3, D_4.

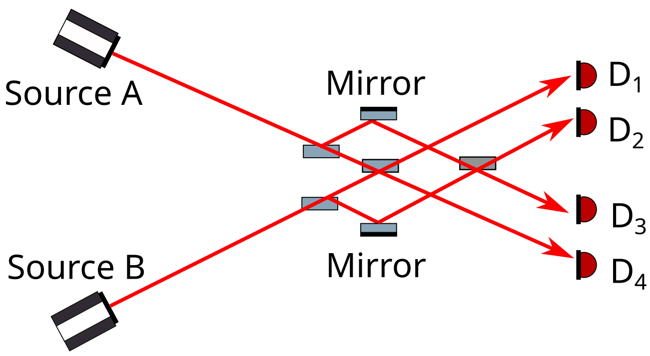

Figure 6: Schematic diagram for an alternate 4-port two-particle interference experiment. Independent particles from sources A and B are split into 4 common channels by a combination of 4 beam-splitters, and then arrive at detectors D_1, D_2, D_3, D_4.

plays the role of a 4-port path-splitter. The effect of the path-splitter on the particles coming from source A and B, can be described as

$$\begin{aligned} \mathbf{U_{PS}}|\psi_A\rangle &= \tfrac{1}{2}(|D_1\rangle + |D_2\rangle + |D_3\rangle + |D_4\rangle) \\ \mathbf{U_{PS}}|\psi_B\rangle &= \tfrac{1}{2}(|D_1\rangle - |D_2\rangle + |D_3\rangle - |D_4\rangle) \end{aligned} \tag{6}$$

Now if the initial state before the two particles enter the path-splitter is given by (1), the final state at the four detectors turns out to be

$$\begin{aligned} |\Psi_f\rangle &= \mathbf{U_{PS}}\tfrac{1}{\sqrt{2}}(|\psi_A\rangle_1|\psi_B\rangle_2 + |\psi_A\rangle_2|\psi_B\rangle_1), \\ &= \tfrac{\sqrt{2}}{4}\Big(|D_1\rangle_1|D_1\rangle_2 + |D_1\rangle_1|D_3\rangle_2 + |D_2\rangle_1|D_2\rangle_2 \\ &\quad +|D_2\rangle_1|D_4\rangle_2 + |D_3\rangle_1|D_3\rangle_2 + |D_3\rangle_1|D_1\rangle_2 \\ &\quad +|D_4\rangle_1|D_4\rangle_2 + |D_4\rangle_1|D_2\rangle_2\Big). \end{aligned} \tag{7}$$

Notice that the terms $|D_1\rangle_1|D_3\rangle_2$ and $|D_3\rangle_1|D_1\rangle_2$

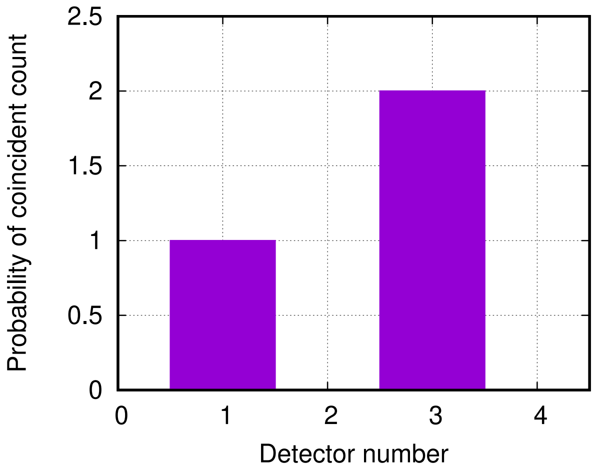

Figure 7: Probability of coincident count of detector 1 with all four detectors, in units of $1/8$.

both contribute to the same coincident count, i.e., between detector 1 and 3. In the HOM experiment, the probability of both the particles landing at a particular detector is $1/2$, so the total probability of both particles landing at D_1 or at D_2 add up to 1. That is because there is no other possibility. However, in our extended 4-port HOM experiment, the probability of both particles landing at a particular detector is not $1/4$, rather it is $1/8$. So the total probability of both the particles landing up at the same detector doesn't add up to 1. That is becuase there are other possibilities, e.g., of one particle going to D_1 and the other to D_3

If one plots the probability of a coincident count of (say) D_1 with various detectors, one would get an interference pattern, with two dark fringes (see Fig. 7). One would notice that although this case is an extension of the HOM experiment, the result has some similarity with the HBT experiment where one obtains an interference by doing coincident counts at detectors at varying positions. A 4-port interferometer can also be set up using an alternare arrangement (see Fig. 6), but the analysis is identical to what has been presented here.

2.4 $n \to \infty$: The HBT Experiment

Next we investigate if the general n-port interferometer can capture the HBT experiment. In an HBT experiment two particles emerge from two sources A and B, localized at positions $x = \pm x_0$, respectively. The particles travel along the y-axis and are finally detected at a continuous set of positions x_1 and x_2 by two movable detectors (see Fig. 1). Essentially a particle emerging from a source is split into a continuous set of infinite number of channels which end up at a continuous set of detector positions. The n-channel path-splitting described by (2) should then be modified to take into account the continuous detector positions. In order to normalize the probability in this continuous case, a position dependent probability should be assigned to each channel, which is essentially $|\psi(x)|^2 dx$, $\psi(x)$ being the wavefunction of the particle in the detection plane. The phases picked up by the channels are naturally position dependent. The path-splitting can then be summarized as

$$
\begin{aligned}
\mathbf{U_{PS}}|\psi_A\rangle &= \int \psi(x) e^{i\theta_x} |x\rangle dx \\
\mathbf{U_{PS}}|\psi_B\rangle &= \int \psi(x) e^{i\phi_x} |x\rangle dx,
\end{aligned} \tag{8}
$$

where θ_x, ϕ_x are the phases picked up by the particles coming from sources A and B, respectively, when they reach a position x in the detection plane. Here it has been assumed that $\psi(x)$ is approximately the same for both the particles since $L \gg 2x_0$ (see Fig. 1). If λ represents the real or de Broglie wavelength of a particle, and it travels a distance L along y-axis to reach the detector position x, the phases acquired are given by $\theta_x = 2\pi x_0 x/\lambda L$ and $\phi_x = -2\pi x_0 x/\lambda L$ [8]. We do not specify the form of $\psi(x)$ here - typically it is a Gaussian envelope. The final two-particle state can then be written as

$$
\begin{aligned}
|\Psi_f\rangle &= \mathbf{U_{PS}} \tfrac{1}{\sqrt{2}} (|\psi_A\rangle_1 |\psi_B\rangle_2 + |\psi_A\rangle_2 |\psi_B\rangle_1), \\
&= \tfrac{1}{\sqrt{2}} \int \psi(x) e^{i\theta_x} |x\rangle_1 dx \int \psi(x') e^{i\phi_{x'}} |x'\rangle_2 dx' \\
&\quad + \tfrac{1}{\sqrt{2}} \int \psi(x) e^{i\theta_x} |x\rangle_2 dx \int \psi(x') e^{i\phi_{x'}} |x'\rangle_1 dx'.
\end{aligned} \tag{9}
$$

The probability amplitude of detecting one particle at x_1 and the other at x_2 is then given by

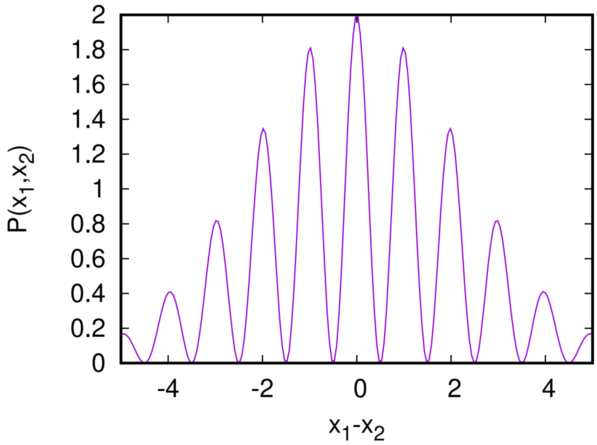

Figure 8: Probability density (unnormalized) of coincident detection at positions x_1 and x_2, against the detector separation $x_1 - x_2$ (in the units of the fringe width). Here x_1 is fixed at 0 and x_2 is varied.

$$
\begin{aligned}
\Psi_f(x_1, x_2) &= \frac{\psi(x_1)\psi(x_2)}{\sqrt{2}} \Big[e^{\frac{i2\pi x_0 (x_1 - x_2)}{\lambda L}} \\
&\quad + e^{\frac{-i2\pi x_0 (x_1 - x_2)}{\lambda L}} \Big] \\
&= \sqrt{2} \psi(x_1)\psi(x_2) \cos\left(\tfrac{2\pi x_0 (x_1 - x_2)}{\lambda L} \right).
\end{aligned} \tag{10}
$$

The probability density of a coincident detection at positions x_1, x_2 is then given by

$$
P(x_1, x_2) = |\psi(x_1)\psi(x_2)|^2 \left[1 + \cos\left(\tfrac{4\pi x_0 (x_1 - x_2)}{\lambda L} \right) \right] \tag{11}
$$

In expression (11) it is easy to see that there are values of detector separation $x_1 - x_2$ for which probability is zero. Those are the dark fringes, and they are separated by $\lambda L / 2x_0$. That is essentially the HBT effect. The probability density of a coincident detection is plotted in Fig. 8, choosing $\psi(x)$ to be a Gaussian function.

3 Discussion and Conclusion

In the preceding section we formulated a general n-port two-particle interferometer. It produces a generalized two-particle interference with n detectors. For $n = 2$ it reduces to the HOM experiment. For $n = 4$ it represents an extended HOM experiment. This extended HOM experiment can be realized without much difficulty. In the limit $n \to \infty$ when the fixed detectors are replaced by two movable detectors in continuous space, the generalized interferometer reduces to the HBT experiment. Interestingly a multiport two-particle interferometer has very recently been realized, in a somewhat different context [16].

Our study reveals that two-particle interference is a single common phenomena, with HOM and HBT experiments being its two specific cases, among many possible ones. An earlier result showed that a common duality relation exists, between the interference visibility and particle distnguishability, for both HOM and HBT effects, indicating a common origin for both [15]. This duality relation was experimentally confirmed too [17]. The present work demonstrates the equivalence of the HBT and HOM effects in a more rigorous manner. It was earlier believed that the HOM effect is a purely quantum effect whereas the HBT effect is possible for classical waves too, although with a maximum visibility 1/2. However, it has now been demonstrated that the HOM effect can also be realized with classical states [18, 19]. So the final message is that the two-particle interference should be viewed as a single phenomenon with a variety of potential implementations, such as the HOM and HBT configurations.

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