Demonstration of system-bath physics on gate-based quantum computer

Pascal Stadler, Matteo Lodi, Andisheh Khedri, Rolando Reiner,

Kirsten Bark, Nicolas Vogt, Michael Marthaler, and Juha Leppäkangas

HQS Quantum Simulations GmbH, Rintheimer Straße 23, 76131 Karlsruhe, Germany

We demonstrate algorithmic cooling on IBM-Q devices. We utilize inherent qubit noise to simulate the equilibration of an interacting spin system towards its ground state, when coupled to a dissipative auxiliary-spin bath. The steady-state correlations in the system are defined by the system Hamiltonian and are stable as long as the algorithm can be executed. In particular, we demonstrate the relaxation of system spins to ferromagnetic and antiferromagnetic ordering, controlled by the definition of the Hamiltonian. We are able to perform simulated cooling for global systems of up to three system spins and four auxiliary spins.

I. INTRODUCTION

The quantum computers of today grapple with significant challenges posed by qubit noise and gate errors. Therefore, prior to achieving full quantum error correction, a crucial hurdle lies in discovering quantum algorithms that can effectively operate in the presence of noise. This involves either mitigating the noise or harnessing it as a resource. One intriguing approach involves mapping the effects of noise during a quantum mechanical time evolution onto processes within open quantum systems [1-3]. It can be argued that this should lead to the possibility to study almost any system in nature, since the very essence of nature is rooted in open systems. Notably, the study of finite or zero-temperature environments, often referred to as "baths", holds particular interest. Rather than randomizing the system, these baths seek to drive the system towards a new ordering. Many fundamental models of molecules and materials rely on such system-bath effects, including density-matrix embedding [4, 5], dynamical mean-field theory [6], and spinboson theories [7-9].

In addition, the exploration of open and thermally distributed quantum systems could pave the way for advancements in quantum machine learning [10]. Many models of quantum machine learning rely on achieving a thermal distribution of a spin system and explicitly require some type of bath. The typical examples here are quantum Boltzmann machines and quantum reservoir computing. In a quantum Boltzmann machine the general goal is to optimize the parameters of a system of coupled spins in a thermal Boltzmann distribution [11]. For quantum reservoir computing it has been explicitly proposed that the dissipative dynamics needed in reservoir computing can be achieved by considering the effects of natural noise in qubits [12].

The key ingredient of algorithms simulating interaction with finite- or zero-temperature baths (see for example Refs. [13–21]) is the implementation of non-unitary timeevolution with a dependency on the sign of the energy change. This directionality has, so far, been realized with the help of specifically engineered qubit reset gates [13, 17, 22, 23]. In our work, we successfully implement the needed non-unitary processes via utilization of inherent noise during the quantum computation.

It should be noted that, throughout the paper, we refer to "spins" as the degrees of freedom of the effective system that is simulated by a quantum computer, and refer to "qubits" to indicate the actual quantum computer. The time evolution of the spins is described by a a master equation which is not explicitly time-dependent. The quantum computer is going to time-evolve an initial state using an algorithm that mimics the master equation, while in fact rapidly applying various gates. Each spin is directly mapped to one qubit. However, what is important for our work is that the noise acting on a qubit can be transformed substantially through the application of gates when considered from the perspective of the spins. We discussed these effects at length in Ref. [3]. To a certain extent, we will make use of these transformations by creating an effective environment at infinite temperature. However, we will try to balance this with avoiding these transformations where possible, in order to achieve the directionality provided by qubit damping.

In this work, we follow the proposal of Ref. [16] and divide the available qubits into system and bath qubits, which represent the system we want to cool and a bath of auxiliary spins. The used algorithm is based on utilizing bath-qubit damping that occurs during the quantum computation. In this algorithm, the interactions within the system are simulated by standard Trotterization of the unitary time-evolution operator, whereas the energyabsorbing bath is simulated by combining the Trotterized time evolution of the auxiliary spins and the inherent damping of qubits representing them. The steady state of the system spins can be made to prefer populating lower-energy eigenstates, which in turn can then be altered by choosing different system Hamiltonians. We test the algorithm on IBM-Q quantum computers which are available via cloud access. We demonstrate the feasibility of this approach through the simulated relaxation of a system of interacting spins towards ferromagnetic or antiferromagnetic ordering. We are able to simulate global systems of up to three system spins and four auxiliary spins. We show that the steady-state correlations are stable under the inherent noise and last as long as the algorithm is being executed. Our work presents, to the best of our knowledge, a first demonstration of algorith-



FIG. 1. (a) We consider an open-system model where a quantum system of spins is cooled towards its ground state via coupling to auxiliary spins with dissipation. Cooling is a result of resonant interaction between the system and the auxiliary spins with damping. (b) On the gate-based quantum computer, the energy-level structure and the time evolution of the global system is implemented digitally, whereas the dissipation of the auxiliary spins is a result of bath-qubit damping. The simulated system is also connected to an additional environment causing random transitions between its energy levels and thereby heating. The origin of this is noise acting on system qubits, symmetrized by the used quantum algorithm (see Sec. III).

mic cooling on a publicly available quantum computer.

The outline of this paper is as follows. In Section II, we describe the open quantum system model we consider in this work. In Section III, we describe how it is implemented on a digital quantum computer using inherent noise. In Section IV, we present our results on running the algorithm on the IBM-Q devices. We conclude in Section V. In Appendix A, we discuss the modeled system-bath physics and the fundamental limitations of the cooling efficiency in terms of a bath spectral function. In Appendix B, we present our noise tomography performed on the IBM quantum computers. In Appendix C, we describe in more detail the circuits submitted to the IBM-Q devices. Finally, in Appendix D, we describe our numerical model of noisy quantum computing, used to fit and better understand the obtained results. The submitted jobs and the numerical simulations are also available online at Ref. [24] in a format compatible with the qoop toolkit [25].

II. SIMULATED OPEN QUANTUM SYSTEM

The open system considered is shown in Fig. 1(a). It consists of a system of interacting spins coupled to an auxiliary-spin bath. The auxiliary spins couple resonantly to the system, so that auxiliary spin energy splittings exist close to the relevant energy-level differences in the system. The auxiliary spins are also subjected to damping, which dissipates energy from the bath, and, through the resonant interaction, also provides cooling

for the system. The system is also connected to an additional environment, causing random transitions between its energy levels and thereby heating. We call the additional environment "background" since it directly acts on the system qubit and is not mediated through the bath qubit. Its interpretation in terms of a bath spectral function is given in Appendix A.

The Hamiltonian that describes our total (global) system can be written in the form

$$\hat{H} = \hat{H}_{\rm S} + \hat{H}_{\rm B} + \hat{H}_{\rm C} \,, \tag{1}$$

where $\hat{H}_{\rm S}$ describes the system spins, $\hat{H}_{\rm B}$ the auxiliaryspin bath, and $\hat{H}_{\rm C}$ the coupling between these two. The decoherence of systems spins and the auxiliary-spin bath is included within the Lindblad master-equation formulation, defined below.

In this work, we consider spin systems with longitudinal nearest-neighbor coupling $g_{ii'}$ and energy splittings $\epsilon_{s,i}$ ($\hbar = 1$),

$$\hat{H}_{\rm S} = -\sum_{i} \frac{\epsilon_{{\rm s},i}}{2} \hat{\sigma}_z^{{\rm s},i} + \sum_{\langle ii' \rangle} g_{ii'} \hat{\sigma}_z^{{\rm s},i} \hat{\sigma}_z^{{\rm s},i'} \,. \tag{2}$$

It should be noted that the approach is not restricted to this choice of the system Hamiltonian.

The auxiliary spins are chosen to be non-interacting

$$\hat{H}_{\rm B} = -\sum_{j} \frac{\epsilon_{\rm b,j}}{2} \hat{\sigma}_z^{\rm b,j} \,. \tag{3}$$

This choice is motivated by the need to keep the bath Hamiltonian simple and thereby avoid unwanted transformations of the decoherence. Namely, we want to ensure that qubit damping maps to the damping of the auxiliary-spins in the simulated model (see Sec. III B). The coupling between the system and bath is chosen to be

$$\hat{H}_{\rm C} = \sum_{ij} \frac{v_{ij}}{2} \hat{\sigma}_x^{{\rm s},i} \hat{\sigma}_x^{{\rm b},j} \,. \tag{4}$$

The coupling induces transitions between eigenstates of the system, $[\hat{H}_{\rm S}, \hat{H}_{\rm C}] \neq 0$, and simultaneously excites the bath from its ground state.

The noise processes are included by introducing a Lindblad master-equation of the form

$$\begin{aligned} \dot{\hat{\rho}} &= \mathbf{i}[\hat{\rho}, \hat{H}] + \sum_{j} \gamma_{j}^{-} \left(\hat{\sigma}_{-}^{\mathbf{b}, j} \hat{\rho} \hat{\sigma}_{+}^{\mathbf{b}, j} - \frac{1}{2} \left\{ \hat{\sigma}_{+}^{\mathbf{b}, j} \sigma_{-}^{\mathbf{b}, j}, \hat{\rho} \right\} \right) \\ &+ \sum_{i} \gamma_{i}^{x} \left(\hat{\sigma}_{x}^{\mathbf{s}, i} \hat{\rho} \hat{\sigma}_{x}^{\mathbf{s}, i} - \hat{\rho} \right) + \sum_{i} \gamma_{i}^{y} \left(\hat{\sigma}_{y}^{\mathbf{s}, i} \hat{\rho} \hat{\sigma}_{y}^{\mathbf{s}, i} - \hat{\rho} \right) \\ &+ \sum_{i} \gamma_{i}^{z} \left(\hat{\sigma}_{z}^{\mathbf{s}, i} \hat{\rho} \hat{\sigma}_{z}^{\mathbf{s}, i} - \hat{\rho} \right) . \end{aligned}$$
(5)

where $\hat{\rho}$ is the density matrix of the global system, i.e., the system and auxiliary spins. In Eq. (5), the auxiliary spins are subjected to damping and the system spins to Pauli noise, which could be considered to be an effect of coupling of an infinite-temperature environment (background) to the system spins. It should be noted that we can also allow for Pauli noise on the bath qubits, which would effectively increase the bath temperature.

III. IMPLEMENTATION ON A NOISY QUANTUM COMPUTER

The goal is now to establish a quantum algorithm that time-propagates the simulated global system on a quantum computer, as described by the Lindblad master equation (5). As visualized in Fig. 1(b), the energy-level structure and the time evolution of the global system will be implemented digitally, while the simulated spin dissipation originates in qubit damping. The additional environment which causes heating is created by depolarizing noise acting on the system qubits and by the qubit damping, since the system-qubit damping has been manipulated using the system-noise symmetrization algorithm, as detailed below.

A. Coherent time evolution

Each simulated spin is directly mapped to one qubit on a quantum computer. The coherent time evolution of the spins, i.e., the commutator with \hat{H} , is implemented via Trotterization of the time-evolution operator [26]

$$e^{-\mathrm{i}\hat{H}t} \approx \left[\Pi_k e^{-\mathrm{i}\hat{H}_k\tau}\right]^m,\tag{6}$$

where the total simulated time t is divided into m Trotter time-steps τ , so we have $t = m\tau$. The chosen partial Hamiltonians \hat{H}_k satisfy $\hat{H} = \sum_k \hat{H}_k$ and correspond to spin splittings or interaction terms between two spins. The unitary operations $e^{-i\hat{H}_k\tau}$ are implemented using the available unitary gates \hat{U} on the quantum computer,

$$e^{-\mathrm{i}H_k\tau} = \Pi_l \hat{U}_{kl} \,, \tag{7}$$

and are chosen such that the simultaneously implemented incoherent time evolution is in the correct form, as detailed below.

B. Incoherent time evolution

The incoherent part of the Lindblad master equation (5) is realized by utilizing incoherent errors that occur during the time-propagation algorithm. Incoherent errors are modeled as non-unitary operations after the error-free gates, as indicated by the replacement

$$\hat{U} \to \mathcal{N}\mathcal{U}$$
. (8)

On the right-hand side, the unitary gate is represented as the superoperator \mathcal{U} and the effect of the incoherent error is described by the Kraus operator \mathcal{N} . It is conveniently expressed in the form

$$\mathcal{N} = e^{t_G \mathcal{L}_N} \,, \tag{9}$$

where \mathcal{L}_N is a Lindbladian (containing only the incoherent part) and t_G is the physical gate time.

We now map incoherent gate errors to an effective time-independent Linbladian, or, in other words: we determine how the noise behaves in the simulated system. The effective Lindbladian, in turn, should correspond to the incoherent part of Eq. (5). To do this, the sequence of noisy gates inside one Trotter step (the term inside the square brackets of Eq. (6)) is brought into the form

$$\Pi_k \Pi_l \left[\mathcal{N}_{kl} \mathcal{U}_{kl} \right] = \Pi_k \Pi_l \left[e^{t_{G,kl} \mathcal{L}_{N_{kl}}} \mathcal{U}_{kl} \right] \approx e^{\tau \mathcal{L}_{\text{eff}}} \,, \quad (10)$$

where to the lowest order in τ we have [3]

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_H + \sum_{kl} \frac{t_{G,kl}}{\tau} \bar{\mathcal{L}}_{N_{kl}} \,. \tag{11}$$

Here, the first term on the right-hand side corresponds to the coherent time evolution (commutator with \hat{H}), whereas the second term has all the noise collected during the computation. The Lindbladian \mathcal{L}_{eff} describes how the noise behaves in the simulated system.

To obtain Eq. (11), one has to commute all noise terms past all the coherent gates appearing after them [3, 16]. A simple illustrative example of commuting noise through unitary gates is a circuit with two gates $\hat{U}_2\hat{U}_1$ and noise,

$$\mathcal{N}_2 \mathcal{U}_2 \mathcal{N}_1 \mathcal{U}_1 = \mathcal{N}_2 \mathcal{N}_1' \mathcal{U}_2 \mathcal{U}_1 \,, \tag{12}$$

where the transformed noise is

$$\mathcal{N}_1' = \mathcal{U}_2 \mathcal{N}_1 \mathcal{U}_2^{-1} \,. \tag{13}$$

This unitary transformation of a noise superoperator implies, in practice, a unitary transformation of the "normal" noise operators in the Lindbladian of Eq. (9). An important example of this is the effect of the X-gates, which flip the definition of the qubit versus the spin states. In turn, this makes (for instance) physical qubit damping look like spin excitation in the simulated system, since

$$\hat{\sigma}_x \hat{\sigma}_- \hat{\sigma}_x = \hat{\sigma}_+ \,. \tag{14}$$

(The effect of several other gates is analyzed in Ref. [16].) Under the assumption of a small noise probability per gate, all such transformed Lindbladian terms can be collected together, and the final effective Lindbladian will be in the form of Eq. (11). Also note the rescaling of the noise operators by factors $t_{G,kl}/\tau$, i.e., by the relation between the physical gate times $t_{G,kl}$ and the simulated time-step τ . Important is that one is allowed to neglect



FIG. 2. Example of an open quantum system and a corresponding quantum algorithm. (a) A system spin couples to one auxiliary spin, with the coherent time evolution described by the Hamiltonian $\hat{H} = -\frac{\epsilon_s}{2}\hat{\sigma}_z^s - \frac{\epsilon_b}{2}\hat{\sigma}_z^b + v\hat{\sigma}_x^s\hat{\sigma}_x^b$. Additionally, spin damping and excitation (heating) acts on the system spin, whereas only damping acts on the auxiliary spin. (b) The corresponding quantum algorithm on a quantum computer with two-qubit gate U_{xx} , with damping acting on both qubits (gate noise). The X-gates surrounding every second Trotter step are used to effectively transform the system-qubit damping to system-spin heating (damping and excitation).

transformations due to commuting small-angle rotations, or complete decomposition blocks, since their effect is higher order in τ . An automated package doing this numerically for arbitrary circuits is available in Ref. [24].

C. Tailoring the final algorithm

Whereas the coherent part of the Lindblad master equation (5) is implemented by standard Trotterization, the main difficulty lies in finding gates and gate decompositions that simultaneously reproduce the correct incoherent part also.

Our target simulation has an incoherent part, which takes the form of infinite-temperature background for the system, i.e., Pauli noise acting on the system spins, Eq. (5). There are two central motivations for this goal: (i) in this form, the system noise drives the system to a fully mixed state and allows for a physical interpretation of it as a hot background (or equivalently as a constant in the spectral function, see Appendix A), (ii) the effect of the auxiliary spins is now easy to detect. At infinite temperature, all Pauli operators or Pauli products we measure would be zero. However, since the auxiliary spins lead to cooling of the system, certain Pauli operators or Pauli products will be non-zero. Additionally, the algorithm we use simultaneously mitigates coherent gate-errors in the system, since it is similar to a spin-echo protocol.

To achieve our goal for hardware characterized by qubit damping, we perform X-flips (π -rotations) to every second Trotter circuit, so that system-qubit damping looks like an equal contribution of spin damping and excitation in the simulated system. In practice, this means that X-gates of system qubits are inserted at the beginning and the end of every second Trotter step. The flipped circuits have negated angles of system-qubit Zrotations, as shown in the example in Fig. 2(b). This approach is adequate for the considered form of the Hamiltonian and used hardware, but could be trivially generalized to symmetrization of other noise types, as listed in Table I. It should be noted that such an algorithm has similarities with randomized compiling [27, 28], which is an alternative way to reach the same goal.

| Original noise | X-flips | Y-flips | Z-flips |
|-----------------------------|---------------|-----------|-----------|
| $\hat{\sigma}^- \equiv Z^-$ | X and Y noise | Х, Ү | no effect |
| Y^{-} | X, Z | no effect | X, Z |
| X^{-} | no effect | Y, Z | Y, Z |

TABLE I. The effect of the symmetrization algorithm, with different possible spin-flip directions, to the form of the simulated noise. If the flip is performed around the axis that is different from the direction of the lowering operator, the original noise transforms into Pauli noise with two components.

Unlike the decoherence of the system spin, the auxiliary-spin noise must be predominantly damping, so that it drives the auxiliary spins towards their ground states, enabling the simulated cooling. A central requirement to be satisfied here is that we need to use hardware and gates that are characterized by qubit damping. According to our noise analysis (Appendix B), the inherent noise of IBM-Q devices is characterized by qubit damping and dephasing, as expected for superconducting quantum computers. However, we find that the application of two-qubit gates (CNOTs) is accompanied by fully randomizing noise to the control and target qubits. This result, especially for the CNOT control qubit, is, at first sight, surprising, but can be explained by a closer look at the implementation of the CNOT gates, which include an echoed cross-resonance protocol [29], similar to the Xflip protocol described above, effectively leading to symmetrized noise. Nevertheless, we find that the quantum algorithm can work also on IBM quantum computers, since damping during (unavoidable) qubit idling can be utilized. Qubit idling appears, for instance, when other qubits are acted on or when calling the identity gate, i.e., qubit sleep. In practice, this means that, for large global systems with unavoidable significant idling time (due to the form of the considered Hamiltonian), the effective Lindbladian \mathcal{L}_{eff} is of the desired form. For small global systems, however, we alter the algorithm by placing additional sleep phases in the Trotter circuit, as visualized in Fig. 3. This leads to the successful simulations presented in the next section.



FIG. 3. The Trotter circuit of the same global system as in Fig. 2, but the contribution of damping to the effective noise model is increased by additional idling at the end of the Trotter steps. This is needed if the gate noise is depolarization (or Pauli noise), which cannot be utilized for simulating system cooling. This type of algorithm was used in IBM-Q experiments involving in total two and four qubits respectively (Secs. IV A and IV B).

IV. RESULTS: SYSTEM-BATH ON IBM-Q

Here we present and analyze our results on running the system-bath quantum algorithm on the IBM-Q devices. In Sec. IV A, we start by confirming in the simplest setup that system-bath physics can be simulated using inherent noise as proposed. There we consider a global system consisting of one system spin and one auxiliary spin. In particular, we show that the steady-state can be made to prefer the ground state of the system or, if negating the energy of the bath, we can cause population inversion in the system. After learning the details of how to optimize circuits for the IBM-Q devices, we go on to test the approach for larger global systems. The runs for two system spins and two auxiliary spins are analyzed in Sec. IV B and the runs for three system spins and four auxiliary spins in Sec. IV C. In these experiments, we demonstrate that the steady-state of the system can be made to prefer ferromagnetic or antiferromagnetic spin ordering, depending on how the system Hamiltonian is defined. The circuits submitted to IBM-Q and the details of our numerical simulations are provided in Appendices C and D. They are also available online at Ref. [24].

A. One system and one bath qubit: simulated cooling and population inversion

We start by confirming that system-bath physics can be simulated using inherent noise as proposed. We study the case of a single system and a single auxiliary spin, with coherent time evolution given by

$$\hat{H} = -\frac{\epsilon_{\rm s}}{2}\hat{\sigma}_z^{\rm s} - \frac{\epsilon_{\rm b}}{2}\hat{\sigma}_z^{\rm b} + v\hat{\sigma}_x^{\rm s}\hat{\sigma}_x^{\rm b}, \qquad (15)$$

where we have set $\hbar = 1$ and, for simplicity, have also set the simulated time-step $\tau = 1$. We fix the system energy-splitting ϵ_s and the system-bath coupling v but sweep over the bath energy-splitting ϵ_b , i.e., the position of the peak in the effective bath spectral density (see Fig. 7 in the appendix). We then expect that the steadystate of the system can be changed by the energy of the auxiliary spin: if $\epsilon_{\rm s} = \epsilon_{\rm b} > 0$, the bath should absorb energy from the system, and if $\epsilon_{\rm b} \rightarrow -\epsilon_{\rm b}$, the system excited state should be preferred.

These results are confirmed in the runs shown in Fig. 4 performed on the IBM-Q Nairobi device. The selection of system and bath qubits, representing the system and auxiliary spins, is visualized in Fig. 4(a). In Figs. 4(b-c), we study the relaxation of the expectation value $\langle \hat{\sigma}_{z}^{s} \rangle$ towards the steady-state for four different bath energies ϵ_b . We start the simulation always from the state $|\uparrow_{\rm s},\uparrow_{\rm b}\rangle \equiv |\uparrow_4,\uparrow_5\rangle$, run the algorithm for a given number of Trotter steps m (simulated time $t = m\tau$), measure $\hat{\sigma}_z^{\rm s}$, and average over $N_{\rm m}=1024$ repetitions (for each m). As discussed in Sec. III C, we add two idling phases to each Trotter circuit to increase the spin damping: in Fig. 4(b) each idling phase lasts 1.6 μ s and in Fig. 4(c) 0.7 μ s. We confirm that the system steady-state prefers to populate the ground state or the excited state, depending on the sign of the implemented auxiliary-spin energy. The behavior is reproduced well by numerical simulations of the quantum computer (solid lines). We also find that the idle time between 1 μ s and 2 μ s in each Trotter step was the "sweet spot" in multiple experiments: using longer or shorter idle phases made the separation between the curves smaller. The total idle times were larger or comparable to T_1 decay times of the qubit (~ 400 μ s and ~ 200 μ s compared to ~ 100 μ s). It should, however, be noted that especially in the simulation of Fig. 4(c), the system has yet not fully reached the steady-state, which was due to the limitation of the overall circuit length set by the IBM-Q devices.

The desired effect of steady-state dependency on the bath energy is clearly observed, but would ideally be stronger. We find that an important limiting factor here is the strong depolarizing noise of two-qubit gates: ideally the noise after two-qubit gates would be qubit damping. We find that another important limiting factor here is the strong dephasing during idling: ideally the qubit coherence would be T_1 -limited. The main limiting factor is, however, the presence of system noise. If low-noise qubits would be available for the system, we would expect that the maximal steady-state variation between ± 1 can be approached.

B. Two system and two bath qubits: ferromagnetic and antiferromagnetic steady-states

In the next simulation, we increase the size of the global system and consider two system spins coupled to two auxiliary spins. To address certain limiting factors, we ideally select bath qubits with longer T_1 -times compared to those of the system qubits. The conditions for this selection were not available at the Nairobi IBM-Q device, but they were available at IBM-Q Lagos. The



FIG. 4. Time-evolving a system-bath model consisting of one system spin coupled to one auxiliary spin on the IBM-Q Nairobi device. (a) The selection of the system (qubit 4) and bath (qubit 5) on the IBM-Q Nairobi device, representing the system and the auxiliary spin. (b) The measured (dots) and numerically simulated (solid lines) expectation values of system $\hat{\sigma}_z^s$ when starting from the state $|\uparrow_s,\uparrow_b\rangle$ for four different auxiliary-spin energies ϵ_b , while fixing the system energy to $\epsilon_s = 1.0$. The system steady-state prefers populating the ground state (expectation value above zero) or the excited state (expectation value below zero), depending on the sign of ϵ_b . The measured data corresponds to an average over 1024 measurements, whereas the numerical simulation corresponds an average over infinite measurements. We use idling phases of 1.6 μ s in each Trotter circuit to enhance qubit damping (Fig. 3). We use a system-bath coupling of v = 0.1 and a Trotter time-step of $\tau = 1.0$. (c) The same experiment as in (b), but now performed with idling phases of 0.7 μ s, leading to a reduced simulated noise and a slower relaxation towards the steady-state.



FIG. 5. Time-evolving a system-bath model consisting of two system spins coupled to two auxiliary spins on the IBM-Q Lagos device. (a) The selection of the system qubits and bath qubits on the IBM-Q Lagos device. (b) The measured (dots) and numerically simulated (solid lines) expectation values of the system operator $\hat{\sigma}_z^{s,1} \hat{\sigma}_z^{s,2} \propto \hat{H}_S$, revealing the preferred spin ordering in the steady-state. We always start the simulation from all spins being in the state $|\uparrow\rangle$ and vary the inner-system spin-spin coupling g. The interaction with the auxiliary-spin bath makes the system prefer lowest-energy system eigenstates in the steady-state, which are defined by the sign of the coupling g. When we set the coupling g = 0, the measured steady-state value is close to zero. In this simulation we use two (additional) idling phases of 0.7 μ s in each Trotter circuit, similarly to Fig. 3. (c) The same simulation as (b), but here performed without the additional idling phases. Here we also find a significant separation of the steady-states, since unavoidable idling of qubits already exists during the implementation of spin-interactions in the quantum algorithm. In both simulations, the measured data corresponds to an average over ≈ 16000 measurements and the numerical simulation to an average over infinite measurements, and we have system-bath couplings v = 0.2, bath energies $\epsilon_b = 1.0$, and a Trotter time-step of $\tau = 1.0$.

selection of system and bath qubits on the IBM-Q Lagos device is shown in Fig. 5(a). The coherent time evolution

is described by the Hamiltonian

$$\hat{H} = g\hat{\sigma}_{z}^{s,1}\hat{\sigma}_{z}^{s,2} - \frac{\epsilon_{b}}{2}\hat{\sigma}_{z}^{b,1} - \frac{\epsilon_{b}}{2}\hat{\sigma}_{z}^{b,2} + v\hat{\sigma}_{x}^{s,1}\hat{\sigma}_{x}^{b,1} + v\hat{\sigma}_{x}^{s,2}\hat{\sigma}_{x}^{b,2}$$
(16)

The coupling between the system spins is longitudinal, the system-bath coupling is transverse, and the systemspin splittings are set to zero. The eigenstates of the system Hamiltonian $\hat{H}_{\rm S} = g\hat{\sigma}_z^{\rm s,1}\hat{\sigma}_z^{\rm s,2}$ are the ferromagnetic orderings $|\uparrow_{\rm s,1},\uparrow_{\rm s,2}\rangle \equiv |\uparrow_3,\uparrow_5\rangle$ and $|\downarrow_{\rm s,1},\downarrow_{\rm s,2}\rangle$ with the eigenvalue g, and the antiferromagnetic orderings $|\uparrow_{\rm s,1},\downarrow_{\rm s,2}\rangle$ and $|\downarrow_{\rm s,1},\uparrow_{\rm s,2}\rangle$ with the eigenvalue -g. The energy difference between these two orderings is 2g. To optimize the energy-absorption efficiency by the bath, we set both auxiliary-spin energies to $\epsilon_{\rm b} = 2|g|$. We then expect that, depending on the sign of the implemented g, one of the two orderings to be preferred in the steadystate, i.e., ferromagnetic for g < 0 and antiferromagnetic for g > 0.

In Fig. 5(b-c), we show that the expected behavior is obtained on the IBM-Q device. We plot the result for the measured system operator $\hat{\sigma}_z^{s,1} \hat{\sigma}_z^{s,2}$ during simulated relaxation to the steady-state. The measured average value of $\langle \hat{\sigma}_z^{s,1} \hat{\sigma}_z^{s,2} \rangle$ reveals which spin ordering is preferred. We start the simulation always from all spins being in the state $|\uparrow\rangle$ and run the algorithm until the steady-state is reached. In Fig. 5(b) we use 0.7 μ s idling phases in each Trotter circuit and in Fig. 5(c) we do not use any sleep phases. We perform $N_{\rm m} \approx 16000$ measurements at each time-step, reducing the measurement uncertainty considerably (~ $1/\sqrt{N_{\rm m}}$). We confirm the expected shift of the average of $\langle \hat{\sigma}_z^{\rm s,1} \hat{\sigma}_z^{\rm s,2} \rangle$ to the direction which reduces the system energy: for q < 0 the ferromagnetic ordering is preferred and for q > 0 the antiferromagnetic ordering is preferred. On the other hand, when we set the coupling to q = 0, the measured expectation value lies in between these results, close to zero. Notable is that we find a significant separation of the steady-state values also without the additional idling phases, see Fig. 5(c). This is due to the unavoidable idling of qubits during execution of the quantum algorithm. This occurs during execution of CNOT gates, with gate-times varying between 0.1 μ s and 0.5 μ s. We also observe that the separation of the curves in the steady-state is usually largest for (additional) idling phases between 0.5 μ s and 1.0 μ s.

C. Three system and four bath qubits: stabilized long-range correlations

In the last simulation, we increase the size of the global system to the maximum allowed by the used IBM-Q devices: we divide the global system into three system spins and four auxiliary spins, as shown in the insert of Fig. 6(a). The coherent time evolution is described by the Hamiltonian

$$\begin{aligned} \hat{H} &= g_{12}\hat{\sigma}_{z}^{\text{s},1}\hat{\sigma}_{z}^{\text{s},2} + g_{23}\hat{\sigma}_{z}^{\text{s},2}\hat{\sigma}_{z}^{\text{s},3} \\ &- \frac{\epsilon_{b}}{2} \left(\hat{\sigma}_{z}^{\text{b},1} + \hat{\sigma}_{z}^{\text{b},2} + \hat{\sigma}_{z}^{\text{b},3} + \hat{\sigma}_{z}^{\text{b},4} \right) \\ &+ v\sigma_{x}^{\text{s},1} \left(\hat{\sigma}_{x}^{\text{b},1} + \hat{\sigma}_{x}^{\text{b},2} \right) + v\hat{\sigma}_{x}^{\text{s},3} \left(\hat{\sigma}_{x}^{\text{b},3} + \hat{\sigma}_{x}^{\text{b},4} \right) , \quad (17) \end{aligned}$$

modeling longitudinal nearest-neighbor coupling in the system and transverse nearest-neighbor coupling between the system and bath. For simplicity, we have again set the system energies ϵ_s to zero. The eigenstates of the system Hamiltonian [the first line on the right-hand side of Eq. (17)] are product states of $|\uparrow_{s,i}\rangle$ and $|\downarrow_{s,i}\rangle$ of system spins *i*. The lowest-energy eigenstate is degenerate and is determined by the signs of the couplings $g_{ii'}$: it has opposite spin-directions for neighboring spins with positive coupling, and vice versa for the negative coupling. The degeneracy is due to the invariance of the energy when flipping all spin-directions simultaneously. The lowest eigenenergy is always $E_s = -|g_{12}| - |g_{23}|$.

For optimizing the energy absorption by the auxiliaryspin bath, the system energy-changes under transitions caused by the bath are essential. Here, the transition is always a flip of system spin 1 or 3 with a corresponding energy change of $\pm 2|g_{12}|$ or $\pm 2|g_{23}|$. The sign is defined by the flip direction. Since we consider here the case of equal-magnitude system couplings, $|g_{12}| = |g_{23}| = g$, it is then optimal to choose all auxiliary-spin energies to be $\epsilon_{\rm b} = 2g$ to maximize the energy absorption efficiency by the bath.

In Fig. 6(a-d), we plot the result for the measured $\langle \hat{\sigma}_z^{{\rm s},i} \hat{\sigma}_z^{{\rm s},i'} \rangle$ expectation value during the relaxation to the steady-state for all four different sign combinations of the couplings g_{12} and g_{23} . Each plot includes three curves, corresponding to the three possible system-index pairs (i, i'). We again start the simulations from all spins being in the state $|\uparrow\rangle$ and run the algorithm until the steady-state is reached. We perform $N_{\rm m} \approx 16000$ measurements at each time-step. We always observe a steady-state value with the expected sign, i.e., a sign that reduces the average system energy. For example, for an antiferromagnetic Hamiltonian, the nearest-neighbor spins prefer pointing to opposite directions, which also means the next-nearest neighbor spins prefer pointing to the same direction, which is indeed measured in Fig. 6(a). Also, changing the sign of one of the nearest-neighbor couplings causes a switch in the corresponding spin ordering, also resulting in that the next-nearest-neighbor spins now prefer pointing to the opposite directions, as also measured in Fig. 6(b-c) The measured effect is strongest between nearest-neighbor spins, but it is also observed between next nearest-neighbor spins, confirmed by the fact that the measured mean values were always larger than the standard deviation when averaging over the last 15 Trotter steps. The same result is reproduced also by the numerical modeling of the experiment and can be understood by simple reasoning: if the probability for coherence between system spins 1 and 2 is p, which is well below 1, and the probability for coherence between system spins 2 and 3 is also p, then the indirect probability for the coherence between spins 1 and 3 can be expected to be $p^2 \ll 1$. Note that we did not need to introduce any additional sleep phases to the Trotter circuits (to increase simulated bath damping), since significant qubit idling is inevitable already without them, due to the pres-



FIG. 6. Time-evolving a system-bath model consisting of three system spins coupled to four auxiliary spins on the IBM-Q Nairobi device. (a) The insert shows the selection of the system qubits and bath qubits on the IBM-Q Nairobi device. (a-d) The measured (dots) and numerically simulated (solid lines) expectation values of the system correlations $\langle \hat{\sigma}_z^{s,i} \hat{\sigma}_z^{s,i'} \rangle$ during relaxation to a steady-state for all (four) different sign-combinations of inner-system couplings g_{12} and g_{23} . We always start the simulation from the state where all spins are in the state $|\uparrow\rangle$. The auxiliary spins absorb energy from the system and the steady-state prefers reduced system energy, which leads to finite steady-state values of the spin-spin correlations. The measured data corresponds to an average over ≈ 16000 measurements, whereas the numerical simulation corresponds to an average over infinite measurements. We use inner-system couplings $|g_{12}| = |g_{23}| = 0.5$, system-bath couplings v = 0.2, and a Trotter time-step of $\tau = 1.0$. No sleep phases (additional idling times) are introduced to the Trotter circuits.

ence of multiple non-commuting spin-spin interactions in the Hamiltonian.

In this specific realization of the open-system model. the steady-state correlations between the distant system spins remained weak. In another application, we may want to realize a regime where the long-range coherence is strong. In the presented realization, one important limiting factor for long-range coherence was the absence of the auxiliary spins connected to the center systemspin, see Fig. 6(a). Ideally, we would then use hardware which provides a tri-diagonal array of qubits. Another important limiting factor, here, is the strong depolarizing noise of the two-qubit gates. Additionally, qubit dephasing plays a role, meaning that T_1 -limited coherence would be ideal. The main limiting factor for the long-range coherence is, however, the presence of system noise, meaning that for modeling long-range correlation effects, the system qubits need to have higher quality.

V. CONCLUSION AND DISCUSSION

We have demonstrated the utilization of inherent qubit noise in digital quantum simulation by mapping its effect to physical processes. In particular, we have shown that qubit damping can be exploited to create the key effect of a finite-temperature bath, namely non-unitary time evolution with its direction defined by energy changes. This digital quantum simulation creates steady-states that prefer to populate low-energy eigenstates of the simulated system. The created steady-state is characterized by non-local qubit-qubit correlations, which are robust against the noise, since their creation is based on utilizing it. The state can be maintained, in principle, as long as the algorithm is being executed. The exact form of the correlations can be engineered by the definition of the Hamiltonian and the details of the gate decompositions and the time-propagation algorithm. Our work presents,

to the best of our knowledge, a first demonstration of finite-temperature system-bath physics (and algorithmic cooling) on a publicly available quantum computer. The circuits we have developed and run on the IBM-Q devices are available online at Ref. [24], together with the shown numerical simulations of noisy quantum computation.

The final goal of this work is to be able to perform either useful simulation of materials and molecules by solving relevant system-bath models [4–7], or to map open quantum systems to relevant quantum machine learning algorithms. A tool to translate arbitrary systembath models to noise-utilizing quantum simulations is "bath mapping", described in Ref. [16]. The final quantum algorithm in the bath mapping is analogous to the one presented in this paper, with the additional level of Hamiltonian-parameter determination so that the target spectral function is simulated as well as possible.

Finally, long-range multi-party correlations and entanglement, in particular, pose major difficulties for classical simulation methods. Therefore, for useful applications of system-bath models, devices will need a large number of qubits. We concluded that, for achieving stronger long-range steady-state correlations, certain criteria for hardware properties should be reached. We found that qubit architectures which are at least bilinear or trilinear, T_1 limited qubits, and low-noise system qubits would be necessary. Ideally, variable-angle twoqubit gates would be available on the hardware, such as the variable iSWAP gates on superconducting quantum computers [17], as they would be particularly useful [16], since they can be expected to be characterized by damping noise instead of fully depolarizing noise. It can also be expected that the use of control-Z gate decompositions with tunable qubits and couplers [17] would perform better than the CNOT gate decompositions with fixed-frequency qubits available in this work. The main obstacle for achieving low-temperature quantum simulations is, however, the presence of noise in the system qubits, which, in the used algorithm, mapped to the presence of an infinite-temperature background environment. Therefore, achieving long-range multi-party correlations and entanglement would be most efficiently reached by having system qubits (not bath qubits) of substantially better quality.

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Appendix A: Description using bath spectral function

We will now roughly sketch how the cooling and heating effects can interplay and what are the fundamental limitations of the cooling efficiency in this setup. For this we write down a spectral function for the bath, describing transitions between energy-levels of the system induced by the auxiliary-spin bath and the background. The bath spectral function describes how the bath absorbs (positive energies) and emits (negative negative) energy as a function of the energy changes of the system ω . (Note that we have put $\hbar = 1$ and thereby work in units where energies are equal to frequencies.) In the case of one auxiliary spin, the spectral function is of the simple form

$$S(\omega) = \gamma_{\text{background}} + v^2 \frac{\gamma_-}{(\gamma_-/2)^2 + (\omega - \epsilon_{\text{b}})^2}, \quad (A1)$$

which consists of two parts: the constant $\gamma_{\text{background}}$ (which will be proportional to the system-qubit noise) and the Lorentzian at the simulated auxiliary-spin energy ϵ_{b} , with the subscript *b* indicating *bath*, with broadening defined by the damping rate of the auxiliary spin γ_{-} , and area $2\pi v^2$ defined by the system-bath coupling *v* [see Eq. (4)]. Most important for our purposes is that the spectral function is not symmetric around the zero energy, i.e., the total bath can absorb energy more than it emits. The form of the bath spectral function is visualized in Fig. 7.

For couplings $v \leq \gamma_{-}$, we can make a rough estimate for the system steady-state populations (considering now a single-spin system), which approximately satisfy

$$\frac{p_1}{p_0} \approx \frac{S(-\epsilon_{\rm s})}{S(\epsilon_{\rm s})},$$
 (A2)

where $p_{0(1)}$ is the population of the state $|\uparrow\rangle (|\downarrow\rangle)$ with energy $\epsilon_s/2$ ($-\epsilon_s/2$), ith the subscript s indicating system. Assuming that $\epsilon_{\rm b} > 0$, the system is then more probable to populate the lower energy eigenstate. Note that if we instead simulate $\epsilon_{\rm b} < 0$, the auxiliary-spin bath drives the system to population inversion. In Fig. 7, we have plotted the bath spectral function of Eq. (A1) corresponding to the coupling $v = \gamma_{-}$ and $\gamma_{\text{background}} = 4\gamma_{-}$, the latter corresponding to the use of the symmetrization algorithm (see Sec. III) with homogeneous qubit damping. Here, the maximum of the auxiliary-qubit peak equals the constant background rate $\gamma_{\text{background}}$. According to Eq. (A2), we then have $p_1/p_0 \approx 1/2$, assuming $\epsilon_{\rm b} = \epsilon_{\rm s} \gg \gamma_{-}$, which is close to the exact solution. An analysis for couplings beyond the validity of Eq. (A2) gives that the maximal population difference is $p_1/p_0 \gtrsim 1/3$ achieved in the parameter regime $\epsilon_{\rm b}, v \gg \gamma_{-}$. It should be noted that the full ground-state population $(p_1/p_0 = 0)$ can be trivially approached in the limit of vanishing system noise ($\gamma_{\text{background}} \rightarrow 0$).

Appendix B: Noise on IBM-Q

The quantum circuits need to be tailored for the available gates and the device noise. In this Appendix, we present our simplified noise tomography performed on the IBM quantum computers. Here we do not aim to full knowledge of the noise characteristics which would need performing a full quantum process tomography (see



FIG. 7. The form of the bath spectral function in the opensystem model (solid lines). We consider the case of one auxiliary spin with damping (a) $\gamma_{-} = 0.05\epsilon_{\rm b}$, (b) $\gamma_{-} = \epsilon_{\rm b}$, systembath coupling $v = \gamma_{-}$, and the use of noise-symmetrization algorithm (see Sec. III) on a hardware with homogeneous qubit damping rates. The spectral function describes how the environment of the system can absorb (positive energies) and emit (negative energies) energy.

for example Refs. [30, 31]), but rather probe the most essential information for our purposes, i.e., the presence of qubit damping and the type of coherent gate errors.

1. Qubit idling

During idling, the noise of superconducting qubits is expected to be damping and dephasing. The strengths of these processes can be retrieved from the calibration data (T_1 and T_2 times), but can also be probed independently by running corresponding quantum circuits and measurements. The results may be different, since after the calibration the properties of the quantum computers will change in time.

A probe of the qubit decay rate via state initialization, use of identity (sleep) gates, and performing qubit measurements is shown in Fig. 8(a). Here we initialize the qubit to the eigenstate of $\hat{\sigma}_x$ and first monitor the expectation value of $\hat{\sigma}_z$. We manifest a decay of the qubit population to its ground state. The decay rate we obtain from the fitting is consistent with the T_1 obtained from the calibration data. In Fig. 8(b), we perform the same protocol but measure $\langle \hat{\sigma}_x \rangle$. We observe spurious coherent oscillations, which can be fitted by a presence of coherent Z-error of angle $\delta \phi = -0.03$ (qubit 4) and $\delta \phi = 0.01$ (qubit 5) for each identity gate (corresponding to 35 ns qubit-sleep). This is most probably due to a frequency shift $\delta\epsilon$ of the qubit after the calibration [32, 33], which effectively introduces a permanent qubit Hamiltonian term $\delta \epsilon \hat{\sigma}_z/2$. Such coherent errors change slowly in time but can often be considered to be constants over



FIG. 8. Spectroscopy of single-qubit errors on IBM-Q with circuits whose expectation value differ from 1 only due to errors. (a) Probing qubit damping on IBM-Q during qubit sleep. We initialize the qubit to the eigenstate of $\hat{\sigma}_x$ and "run" different lengths of sleeping gates per Trotter step and measure the expectation value of $\hat{\sigma}_z$ as a function of applied Trotter steps. The decay rate is consistent with the IBMQ-Q given $T_1 \sim 100 \ \mu$ s (solid lines). (b) Probing dephasing of two different qubits by now measuring $\langle \hat{\sigma}_x \rangle$. We confirm that idling noise roughly fits to the given $T_2 \sim 80 \ \mu$ s. We also see coherent oscillations, which are most probably due to change of the qubit frequency after the calibration, leading to an effective R_z gate with angle $\delta \phi = -0.03$ (qubit 4) and $\delta \phi = 0.01$ (qubit 5) per identity gate (corresponding to 35 ns qubit-sleep). (c) Observation of overrotations of the X-gate on IBM-Q. The oscillations correspond to overrotation of $\delta \phi = -0.01$ (qubit 5) and $\delta \phi = 0.003$ (qubit 4) per \sqrt{X} -gate. We also observe weak damping of the oscillations, due to the incoherent error. (d) Observation of overrotations of \sqrt{X} -gate on IBM-Q.

one experiment (as observed here). The oscillations decay towards zero due to qubit dephasing (finite T_2 -time), with significant contribution coming from pure dephasing (instead of qubit damping).

2. Single qubit gates

The available single qubit gates on IBM-Q are \sqrt{X} , X, and virtual R_z gates. A study of the gate \sqrt{X} -gate is shown in Fig. 8(c). Here the "Trotter circuit" consists

of four \sqrt{X} -gates, which generate all together an identity up to a global phase (an operation $-\hat{I}$). In the absence of errors, the expectation value should not differ from the initial value. However, we observe damped oscillations. The oscillation period can be fitted by X-overrotations of angle $\delta \phi = -0.01$ (qubit 5) and $\delta \phi = 0.003$ per \sqrt{X} -gate. The damping is due to an incoherent error. The results of the spectroscopy of X-gates are similar and are shown in Fig. 8(d)

The overrotations could be explained with the model of off-resonance rotations [34] (ORR), which in turn could



FIG. 9. Spectroscopy of two-qubit errors on IBM-Q with circuits whose expectation value differ from 1 only due to errors. (a) Circuit with 2 CNOT gates. We measure $\hat{\sigma}_z$ as a function of applied Trotter steps. (b) Observation of strong incoherent and coherent error of the CNOT gate for both the control and target qubit on the IBMQ Nairobi device. The incoherent noise drives the qubits towards equal population of the qubit states. The coherent oscillations can be reproduced by additional X-or Y-rotations performed with angle $\delta \phi = -0.05$ on the control qubit after each CNOT operation. The overrotation on control leads to overroation of the target. (c) Same run as in (b) but on IBMQ Quito. We observe an incoherent error on the control and both incohorent and coherent error on the target. The coherent oscillations can be reproduced by additional X-rotations on the target with angle $\delta \phi = -0.07$, after each CNOT.

be due to the influence of higher levels of the qubits. However, if the resulting overrotation due to ORR over the full cycle of four \sqrt{X} gates is $\delta\phi$, then the magnitude of coherent errors in Y- and Z-directions after a single \sqrt{X} -gate is $\sim \sqrt{\delta\phi}$. This number is consistent with other analyses of the IBM-Q single-qubit error when fitted using ORR errors [35]. However, such error would result in a very low fidelity for the single \sqrt{X} -gates (of the order $\delta\phi$) and is therefore not expected to be the dominating source of the observed error.

The observed oscillations are weakly damped. The damping is due to an incoherent error after each \sqrt{X} -gate, whose magnitude is consistent with inherent decay and dephasing. The decay is towards zero and thereby the form of the effective noise after four \sqrt{X} -gates is Pauli noise. The exact form of the noise after one \sqrt{X} -gate is not solved in this measurement. We note that its magnitude is small when compared to the incoherent noise of CNOT gates, but cannot be neglected completely.

3. Two-qubit gates

The gate decompositions in this work were based on using CNOT gates, available on the IBM-Q devices. The CNOT gates were the main source of incoherent error in the experiments. They also came often with significant coherent error.

We have probed the coherent and incoherent errors by Trotter circuits consisting of two CNOT gates, see Fig. 9. Again, the expectation value should not deviate from 1 without errors. For a damping noise after each CNOT, the result should also not change. On the other hand, in the presence of coherent errors and damping, the steady-state should still be biased towards the qubit ground state. We instead observe a fast decay of the qubit populations towards a steady-state where $\langle \hat{\sigma}_z^i \rangle \approx 0$. This means that there is roughly an equal amount of qubit damping and excitation, both in the control and in the target. At first sight, this result is not expected for the control qubit, which should rather stay closer to the ground state. However, since during the implementation of the CNOT gate the control qubit is operated twice by X-gates [29], the effective noise gets symmetrized to an equal contribution of damping and dephasing, exactly in the same way as in our noise-symmetrization algorithm [see Eq. (14)].

We also observe coherent oscillations (coherent error) on both gubits (control and target) with rather high frequency. There are many possible origins for this behavior. Firstly, during the gate [29], both the control and the target are subject to large-angle single-qubit Xrotations, which can come with overrotations and ORR errors. Also, the Hamiltonian of the cross-resonance gate has a rich structure of different unwanted terms [29]. whose magnitudes will change in time after the calibration. Also the effect of spurious qubit-frequency shifting may result in similar errors, due to the long gate times of CNOTs. All together, an exact theoretical modeling of the error would be highly cumbersome, and therefore we use a simplified coherent error model where each CNOT gates comes with additional single-qubit rotations (X or Y). In Fig. 9(b), we observe a situation where it is enough to add rotations only to the control to make also the target oscillate with the observed frequency. On the other hand, in Fig. 9(c), we see an opposite situation, where oscillations exist in the target qubit, caused by coherent error only in the target qubit. We then observe that also the type of coherent error may vary between different runs.

4. Mapping coherent errors to Hamiltonian

Coherent noise can be mapped to Hamiltonian disorder in the simulated model [36]. Perhaps the simplest example is a constant Z-error, due to a spurious qubitfrequency shift [as measured in Fig. 8(b)]. This can directly transform to shifts of simulated spin energies (except when appearing inside decomposition blocks, as discussed below). We observe this effect in multiple runs of the system-bath algorithm as shifts of the actually implemented auxiliary spin energies. In particular, long idling times can come with large shifts in the auxiliary spin energies. Note that X-gates between Trotter steps cancel out coherent Z-errors, but cannot be applied to bath qubits the algorithm to work.

More complicated interplay between coherent errors and gate decompositions occurs, for example, when coherent errors appear inside CNOT-decompositions. However, when writing down coherent errors in the discussed (noise-after-gate) form, it can also here be straightforward to map them to Hamiltonian terms. For example, an overrotation of control qubit rotation $R_x(q\tau)$ by angle $\delta\phi$ in the implementation of XX interactions (two CNOTs surrounding control-qubit rotation $R_x(q\tau)$ leads to the shift $g \to g + \tilde{\phi}/\tau$ in the effectively implemented XXcoupling. Especially the implementation of small Hamiltonian terms (or small Trotter time-steps τ) is then very fragile against such errors, since the size of the Hamiltonian disorder scales like $1/\tau$ relative to the correct Hamiltonian. Also, target Z-rotations inside the CNOTs (for example due to qubit-frequency jumps, see Sec. B1) induce ZZ-interaction. It should be noted that also many others types of disorder terms can emerge. For example, coherent Y-errors of a control qubit between CNOTs leads to effective YX-coupling, and so on.

A table implying which types of Hamiltonian terms do single-qubit coherent errors in different spin directions induce is given in Table II.

| Gate, qubit | Х | Υ | Ζ |
|-------------|------------------------|------------------------|---------------|
| CNOT, 0 | XX | ΥX | ΖI |
| CNOT, 1 | IX | $\mathbf{Z}\mathbf{Y}$ | ZZ |
| CZ, 0 | $\mathbf{X}\mathbf{Z}$ | \mathbf{YZ} | \mathbf{ZI} |
| CZ, 1 | $\mathbf{Z}\mathbf{X}$ | ΖY | IZ |

TABLE II. Coherent errors between CNOT or CZ gates lead to effective Hamiltonian disorder. The shown (Clifford) table implies which types of Hamiltonian terms emerge from singlequbit coherent errors. The considered errors can appear either in the control (0) or target (1) qubit and can be rotations around arbitrary spin axes.

Appendix C: Submitted circuits

In this appendix, we give more detailed information on the circuits run on the IBM-Q devices. The gate set of IBM-Q on the device we run is \sqrt{X} , X, CNOT, virtual R_z , and identity.

As outlined in the main text, symmetrization of the system qubit was achieved by adding X gates to the circuits. This procedure is depicted in Fig. 10(a), which illustrates the symmetrized circuit for a single system qubit and one bath qubit.

For the cases of two and three system qubits, the corresponding circuits are presented in Fig. 10(b-c). These figures focus on the Trotterized circuits, with the symmetrization process being analogous to that of the single system-bath qubit pair.

Additionally, we have made the circuit descriptions available in a serialized JSON-format compatible with the QoQo toolkit [24, 25]. The JSON files, being serializable, can be integrated into scripts for direct use. The JSON files should enhances the reproducibility of the experimental results. The supplied QoQo files are compatible with various platforms, including the QuEST [37] simulator, and can be executed on hardware through interfaces such as qoqo-qiskit or qoqo-for-braket, offering flexibility in the choice of simulation or experimental execution environments.

Appendix D: Numerical model

In this Appendix, we provide a description of the numerical model used to simulate the experimental results. The model is constructed based on quantum circuits and the corresponding calibration data, which have been recorded for all experimental runs.

The model presented integrates both incoherent noise and coherent errors. As described in the main text, the noise model includes spin depolarizing noise, which affects all qubits while a gate is applied. Additionally, during idle periods, the qubits are subject to both damping and dephasing processes. To address coherent errors into our simulations, we have explored various approaches such as addition error-gates $(R_x(\theta), R_y(\theta))$ and $R_z(\theta)$) after or before every gate. Given the lack of precise knowledge regarding the nature and magnitude of coherent errors, we have decided to adjust the parameters within the spin-Hamiltonian (rather than adding coherent errors to the circuits) when doing the fitting of the experiment. This motivated by the fact that coherent gate errors lead to variations on the Hamiltonian, as discussed in Appendix B4. The same approach is used for the rate of the incoherent noise, which can change after the calibration (we keep the form of the noise as described above). The adaption of parameters is aimed at refining the simulation to more closely mirror the experimental results. Through this approach, we have achieved a satisfactory correlation between the simulation outcomes and the experimental data.

The renormalization of Hamiltonian parameters and noise rates differ for the three cases we consider in the main manuscript. In Fig. 5 we achieved a high degree



FIG. 10. Circuit that we run on IBM-Q. (a) Circuit corresponding to the Hamiltonian with one system spin (0) and one auxiliary spin (1). We show the Trotter circuit in the symmetrized form, where X-gates are added after idling gates. In (b) and (c), we only show the Trotter circuit (the symmetrization is done analogously). (b) The circuit corresponding to two (1 and 3) system and two auxiliary spins (0 and 2) (c) The circuit corresponding to three system (1, 3 and 5) and four auxiliary spins (0, 2, 4 and 6).

of agreement between numerical simulation and experiment using directly the calibration data and the noise rates from IBM-Q (with converting given gate errors to depolarising noise as implied by universal formulas derived in Ref. [38]). In Figs. 4 and 6, it was necessary to adjust the parameters of the simulated spin-Hamiltonian and the noise rates to better align with the experimental results. The magnitude and type of changes in the Hamiltonian parameters is very much consistent with the magnitude and type of observed coherent errors in spectroscopy presented in Appendix B. In the following, we present the numerical simulations both with and without these modifications.

A comparison showing the original and adapted parameters is given in Table III.



FIG. 11. Time-evolution of a system-bath model consisting of one system spin coupled to on auxiliary spin. The left column (a,c) shows the numerical simulation of a Trotter time-evolution with calibration data obtained on the day the experiment was conducted. The right column (b,d) present the experimental results and corresponding numerical simulations previously illustrated in Fig. 4. Here we included coherent errors by shifting parameters of the spin Hamiltonian and adapted the noise rates. Notable in (c) is the oscillation period of the experiment, which is slower compared to the simulation, a clear sign of coherent error. The simulation in (b) and (d) correspond to a coupling is v = 0.058 in contrast to v = 0.1 in (a) and (c). Such effective change is consistent with the general magnitude and type of coherent errors observed in analysis presented in Appendices B3 and B4. Additionally, in (b) and (d) additional R_z rotations were introduces on the bath qubits, originating most probably in collected coherent Z-error during the idling phases (qubit frequency shift, see Appendix B1). This error vanishes in the system, due to the symmetrization algorithm. The additional angles in (b) are specified as 0.31 (case $\epsilon_b = 0.3$), $0.25 \ (\epsilon_b = -0.3), \ 0.25 \ (\epsilon_b = -1.0)$ and in (d) $0.13 \ (\text{case} \ \epsilon_b = -1.0)$. Such effective change is consistent with a coherent error collected during additional qubit idling during the used 42 (a,b) or 20 (c,d) idling gates (the error being less than 0.01 per idling gate), see Appendix B1. If angles are not mentioned, we did not add R_z rotations on bath qubits. It should be mentioned than in several experiments (not presented in this manuscript) we have observed in a similar experiment coherent Z-errors larger than 1, corresponding to coherent Z-error larger than 0.02 per idling gate. The rate of damping, dephasing the depolarising noise associated with CNOT gates are in (b,d) scaled down by a factor of 0.5.



FIG. 12. Time-evolution of a system-bath model consisting of three system and four bath qubits. The figure shows the time-evolution with calibration data form day of running the experiments and have to compared with the Fig. 6.

| g_{13} | g_{35} | v | noise scaling | bath shift |
|--------------|-----------|-----------|---------------|------------|
| (a) 0.6 (1) | 0.6(1) | 0.16(0.2) | 1(1) | 0.55(0) |
| (b) -1 (-1) | 1(1) | 0.15(0.2) | 1.3(1) | 0.17(0) |
| (c) $0.5(1)$ | -0.5(-1) | 0.17(0.2) | 1.4(1) | 0.17(0) |
| (d) -1 (-1) | -0.3 (-1) | 0.15(0.2) | 1.3(1) | 0.07(0) |

TABLE III. Parameters for the model simulate in Fig. 6. The parameters in bracket are the original parameters of the system-bath model. The noise scaling gives the magnitude of the used noise in comparison to the calibration data. The bath shift corresponds to the value of coherent Z-error collected over one Trotter circuit (assumed here to be a constant for all bath qubits, to reduce the amount of fitting parameters). Such effective change is consistent with the general magnitude and type of coherent errors of individual gates observed in Appendix B, and with the relatively long depth of the circuits. In particular, the large shift in the ZZ-interactions q may have contributions from a direct ZZ-error during the application of the CNOT gates, spurious ZZ-interaction during idling of the used fixed-frequency qubits [29], and spurious Z-error of a target qubit between two CNOTs.