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Does the Electron EDM Preclude Electroweak Baryogenesis ?

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Electroweak baryogenesis (EWBG) constitutes a theoretically compelling and experimentally testable mechanism for explaining the origin of the baryon asymmetry of the universe (BAU). New results for the electric dipole moment (EDM) of the electron place significant constraints on the beyond Standard Model CP-violation needed for successful EWBG. We show how new developments in EWBG quantum transport theory that include CP-violating sources first order in gradients imply more relaxed EDM constraints than implied by previous approximation formulations. Consequently, EWBG remains viable even in light of present EDM bounds. We also illustrate how these developments enable a more realistic treatment of CP-conserving interactions that can also have a decisive impact on the predicted BAU.

Explaining the origin of the baryon asymmetry of the universe (BAU) is a key unsolved problem at the interface of particle and nuclear physics with cosmology. Both the mechanism for baryogenesis as well as the early universe era in which it occurred remain unknown. A compelling possibility is electroweak baryogenesis (EWBG), which links the BAU to the spontaneous electroweak symmetry breaking (EWSB) and generation of elementary particle masses via the Higgs mechanism.[1–3] (For reviews, see *e.g.*, [4–6].) In principle, the Standard Model (SM) of particle physics contains the necessary ingredients for EWBG [7]: B-violation via electroweak (EW) sphaleron processes; C- and CP-violation in the electroweak sector; and out-of-equilibrium conditions in the guise of a first order electroweak phase transition (FOEWPT) to the present Higgs phase. In practice, the latter does not occur for a Higgs boson heavier than $\sim 70-80$ GeV [8–10], while the effects of CP-violation (CPV) in the Cabibbo-Kobayashi-Maskawa (CKM) matrix are too feeble to have generated the observed BAU, even for a sufficiently light Higgs boson [11–13].

Physics beyond the Standard Model (BSM) can remedy these shortcomings. An extended scalar sector can readily lead to a FOEWPT even for a 125 GeV Higgs boson (see [14] for extensive of references), while providing the efficient CPV. The requisite mass scale for these new particles ($\lesssim 700$ GeV) as well as the needed strength of their coupling to the Higgs boson generically puts them within the reach of future high energy collider searches and precision Higgs boson studies[14]. Results from the Large Hadron Collider do not preclude such an extended scalar sector, and it may require a future 100 TeV pp collider to provide a definitive test [14]. Next generation gravitational wave detectors, such as LISA, Taiji, and Tianqin, provide a complementary probe and could uncover a stochastic gravitational wave background arising from a FOEWPT [15–17].

Searches for the permanent electric dipole moments (EDMs) of atoms, molecules, and nucleons provide the most powerful probe of the BSM CPV needed for EWBG [18–20]. Theoretically, drawing quantitative inferences about EWBG viability from EDM search results requires performing robust computations of the early universe CPV dynamics. Here, we report on advances addressing this challenge and the corresponding implications for the EDM-EWBG connection.

The EWBG CPV dynamics occur during a FOEWPT that proceeds via nucleation of bubbles of broken electroweak symmetry, defined by regions of non-vanishing, spacetime varying scalar background fields $\varphi(x)$ (*i.e.*, the Higgs field). CPV-interactions at the bubble walls induce a non-zero density of left-handed SM fermions, n_L, that diffuses into the symmetric phase, biasing EW sphaleron transitions into creation of a net B+L number. The latter diffuses back inside the expanding bubbles, where EWSB

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quenches the sphalerons and preserves the BAU, assuming a sufficiently strong FOEWPT.

The challenge in computing n_L entails solving – in the presence of $\varphi(x)$ – the quantum transport equations for Greens functions that encode information on particle densities. The mass of any particle that interacts with the $\varphi(x)$ varies with spacetime as it traverses the bubble wall, necessitating a continual re-definition of the mass eigenstates. Previous EWBG computations have employed various approaches to solving these transport dynamics[21–32]. For a given set of CPV parameters, the resulting BAU predictions can vary by an order of magnitude. The most optimistic typically result from the use of the "vev insertion approximation" (VIA) [33–37], whose theoretical consistency has been criticized recently in Refs. [26, 38, 39]. In the proposed alternative, semi-classical (SC) formulation [26, 38], the CPV source terms first arise at second order in gradients with respect to position along the bubble wall profile, leading to a significantly smaller BAU than in the VIA (for a review, see Ref. [23]). The corresponding implications of EDM limits for the viability of EWBG are plagued by the spread between the SC and VIA treatments.

In what follows, we argue that despite its theoretical shortcomings, the VIA as employed in earlier work can under-predict the magnitude of the BAU, in contrast to the conclusions drawn from the SC treatments. We do so by utilizing a consistent treatment of scalar field CPV that avoids the VIA inconsistencies and the SC approximations yet admits CPV sources first order in gradients[40, 41]. Employing a realistic EWBG model [42, 43] for concreteness, we solve the Kadanoff-Baym transport equations [44–49] using the vev resummation (VR) framework developed in Refs. [40, 41]. (See [41] for a detailed delineation of differences between the VR and SC frameworks.) For a given set of model parameters, the VR result for the BAU can be as large or even a few times larger than the VIA prediction. Consequently, EDM constraints on EWBG can be more relaxed than previously realized. We also provide a realistic, quantitative determination of the dependence of the BAU transport dynamics on model parameters – including those that enter the CP-conserving "collision terms" – a feature that has typically eluded earlier studies. While there remain open challenges pertaining to bubble wall dynamics [50–56], the results reported herein constitute a significant advance for assessing the EWBG-EDM interface.

We introduce general features of the scalar field transport dynamics before describing the concrete model illustration. Consider a model with two complex, electrically neutral scalar fields $H^0_{1,2}$ denoted by the "flavor space" vector $\eta \equiv (H^0_1, H^0_2)$. The two flavor components of η interact with scalar fields $\hat{\phi}_k(x)$, whose classical values $\varphi_k(x)$ define the bubble walls. The η-$\varphi_k(x)$ interactions lead to a mass-squared matrix having the generic form

$$M^2_\eta(x) = \begin{pmatrix} M^2_1(x) & R(x)\, e^{-i\,\alpha(x)} \\ R(x)\, e^{i\,\alpha(x)} & M^2_2(x) \end{pmatrix}\ , \qquad (1)$$

where $M_{1,2}(x)$, $R(x)$, $\alpha(x)$ depend on the model parameters and the spacetime-dependence of the $\varphi_k(x)$.

We solve for the neutral scalar Greens functions by first diagonalizing $M^2_\eta(x)$ at each spacetime point using a unitarity transformation $\hat{\eta} = U(x)\eta$, where the hatted fields correspond to the mass eigenstates with diagonal mass-squared matrix $\hat{m}^2(x)$. Evolution of the mass basis particle (anti-particle) density matrices f_m ($\bar{f}_m$) follows from Schwinger-Dyson (SD) equations for the scalar field Wightman functions $G^<_{ij}(x,y) \equiv \langle \hat{H}^\dagger_j(y)\hat{H}_i(x)\rangle$ and $G^>_{ij}(x,y) \equiv \langle \hat{H}_i(x)\hat{H}^\dagger_j(y)\rangle$, where $i,j \in \{1,2\}$. Following[40, 41], we transform to Wigner space co-ordinates $X = (x+y)/2$ and k, the wavenumber associated with the relative co-ordinate $x-y$, and reorganize the corresponding SD equations into the Kadanoff-Baym (KB) constraint and kinetic equations.

Observing that there exists a hierarchy of length scales in the problem facilitates a tractable solution to the KB equations. We define the scale ratios: $\epsilon_w \equiv L_{\rm int}/L_w$, $\epsilon_{\rm coll} \equiv L_{\rm int}/L_{\rm mfp}$, and $\epsilon_{\rm osc} \equiv L_{\rm int}/L_{\rm osc}$, where $L_{\rm int} = |\vec{k}| \sim T^{-1}$ is the de Broglie wavelength with T being the temperture of the plasma; L_w is the wall thickness, which in many models is O$(10/T)$, so that $L_w >> L_{\rm int}$; $L_{\rm osc}$ is the length scale associated with "flavor" oscillations $H^0_1 \leftrightarrow H^0_2$; and $L_{\rm mfp}$ is the mean free path associated with gauge and scalar field interactions. For the scenarios of interest here, one finds $L_{\rm mfp} >> L_{\rm int}$ for perturbative values of the couplings, while CPV asymmetries are maximized for $L_w \sim L_{\rm osc}$ in the "thick wall" regime[40]. Thus, one has $\epsilon_{w,\rm coll,osc} << 1$ in the interesting region.

Expanding the constraint and kinetic equations to orders ϵ^0 and ϵ, respectively, yields the following quantum Boltzmann equations for the density matrices:

$$(u\cdot\partial_X + \vec{F}\cdot\nabla_k) f_m(\vec{k},X) = -\Big[i\omega_k + u\cdot\Sigma, f_m(\vec{k},X)\Big] + \mathcal{C}_m[f_m,\bar{f}_m](\vec{k},X) \qquad (2a)$$

$$(u\cdot\partial_X + \vec{F}\cdot\nabla_k) \bar{f}_m(\vec{k},X) = +\Big[i\omega_k - u\cdot\Sigma, \bar{f}_m(\vec{k},X)\Big] + \mathcal{C}_m[\bar{f}_m,f_m](\vec{k},X) \quad , \qquad (2b)$$

where $u^\mu = (1,\vec{v})$; $\vec{v} = \vec{k}/\bar{\omega}_k$; $\bar{\omega}_k = \sqrt{|\vec{k}|^2 + \bar{m}^2(x)}$; $\bar{m}^2 = (M^2_1 + M^2_2)/2$; $\vec{F} = \nabla_X \bar{\omega}_k$; $\omega_k = {\rm diag}\{\omega_{1k},\omega_{2k}\}$; $\omega_{ik} = \sqrt{|\vec{k}|^2 + {M_i}^2(x)}$; $\Sigma^\mu = U^\dagger\partial^\mu U$, and the "collision term" $\mathcal{C}_m$ is a functional of the f_m and $\bar{f}_m$.

Note that the terms in the LHS of (2a,2b) generalize the space-time derivative and force terms in classical Boltzmann equation. The "force" $\vec{F}$ is associated with the variation of the background fields which contribute to $\bar{m}(x)$. On the RHS, the commutator $-i[\omega_k, f_m]$ ($-i[\omega_k, \bar{f}_m]$) gives rise to (anti-)particle flavor oscillations and is identical in form to what appears in the familiar density matrix formalism for neutrino flavor oscillations. The commutators $[u\cdot\Sigma, f_m]$ and $[u\cdot\Sigma, \bar{f}_m]$ are the CPV sources. Due to the relative sign difference between the oscillation term and CPV source terms in Eqs. (2a, 2b), the CPV sources lead to a net number density (a.k.a., CPV asymmetry) for a given particle species. The collision term $\mathcal{C}_m$ embodies the effect of all interactions that lead to thermalization in the plasma, chemical equilibrium associated with particle species changing reactions, and diffusion ahead of the advancing bubble wall.

We now solve Eqs. (2a,2b) for the model of Refs. [42, 43], referred to henceforth as Two-Step EWBG. Baryogenesis occurs during the first of two successive electroweak symmetry-breaking (EWSB) transitions, wherein the $\varphi_k(x) \neq 0$ while the components of η admit no non-vanishing background field values. For renormalizable η-$\hat{\phi}_k(x)$ interactions, the emergence of a spacetime varying phase $\alpha(x)$ in Eq. (1) during the first step requires the presence of at least two non-vanishing $\varphi_k(x)$. Thus, one requires at least four scalar fields: the two components of η and the two $\hat{\phi}_k$.

A minimal realization entails a scalar sector consisting of two Higgs doublets $H_{1,2}$, a hypercharge $Y = 0$ real triplet Σ, and a SM gauge singlet S. All scalars are $\mathrm{SU}(3)_C$ singlets. The gauge and fermion sectors are unchanged from the SM. In order to model the impact of the latter, we introduce an additional scalar field A, whose dynamics implement all other flavor-diagonal thermalizing interactions in the plasma, such as those arising from gauge and Yukawa interactions. During the first EWSB transition, S and the neutral component of Σ obtain vacuum expectation values (vevs), v_s and v_σ, respectively, with corresponding field flucuations described by $s = S - v_s$ and $\sigma = \Sigma^0 - v_\sigma$. These vevs vary with spacetime, thereby providing the requisite two background fields $\varphi_k(x)$ with $k = 1, 2$. In the second transition, $(v_s,\, v_\sigma)$ relax to zero while the neutral components of the doublets obtain vevs, $v_{1,2}$, with $\sqrt{v_1^2 + v_2^2} = 246$ GeV. One may embed the model in a supersymmetric context[57, 58], with the corresponding additional superpartners augmenting the field content. For simplicity, we will consider the non-supersymmetric version.

For successful EWBG during the first step, this transition must be first order, a condition shown to be satisfied in both perturbative and non-perturbative (lattice) computations for suitable choices of the scalar potential parameters [59, 60]. CPV interactions between the $H_{1,2}$ and the (S, Σ) vevs catalyze generation of non-zero Higgs number densities, $n_{H_{1,2}}$. Yukawa interactions then transfer the latter into non-vanishing fermion number densities. Those associated with the left-handed fermions bias electroweak sphalerons into producing a non-zero B+L density that diffuses into the bubble interiors.

The scalar potential is $V(H_1, H_2, \Sigma, S, A) = V_H + V_\phi + V_{H\phi}$, where V_H is the CP-conserving Two Higgs Double Model (2HDM) potential [61–63], V_ϕ involves only the $\phi \equiv (S, \Sigma, A)$ fields, and the key "portal" interaction terms are contained in

$$V_{H\phi} \supset \frac{1}{2} H_1^\dagger H_2 \left(a_1 S^2 + a_2 \Sigma^2\right) + \text{h.c.} + \sum_{i=1,2} \left[y_1^{ii} S^2 + y_2^{ii} \Sigma^2 + y_3^{ii} A^2\right] H_i^\dagger H_i \quad . \tag{3}$$

The physical (rephasing-invariant) CPV phases are $\delta_S = \arg(a_1^* v_1 v_2^*)$ and $\delta_\Sigma = \arg(a_2^* v_1 v_2^*)$. A combination of these CPV phases and the (S, Σ) vevs induce the $M_i^2(x)$ as well as the $\alpha(x)$ in Eq. (1) and, thus, the CPV sources in the KB equations. The interactions in Eq. (3) also give rise to Higgs flavor off-diagonal collision terms, which we include in the computation. The A fields do not obtain vacuum expectation values and, thus, do not contribute to the spacetime-dependence in $M_\eta^2(x)$.

To solve Eqs. (2a, 2b) we choose the couplings y_a^{ii} ($a = 1, 2,\ i = 1, 2$) so as to yield $\bar{m}^2$ x-independent, implying that $\vec{F} = 0$ in our set up. Doing so allows a direct comparison with the results in Ref. [41]; we will investigate the impact of $\vec{F} \neq 0$ in future work. We then make additional simplifying assumptions relevant to the collision integrals $\mathcal{C}_m[f_m, \bar{f}_m]$ outlined in [41, 43]. We also consider a type I 2HDM in which only one of the Higgs doublets has Yukawa interactions with the third generation up-type quarks that are in chemical equilibrium. For the interactions of the Higgs particles with the fields A, we assume the corresponding rates Γ_A are large (small) compared to the weak sphaleron and Yukawa (strong sphaleron) interaction rates. We thus obtain $n_L = (4c_T + 5c_Q) n_{H_1}$, where $c_{T,Q}$ are functions of statistical factors k_j relating the number density for a given species n_j to its chemical potential μ_j. In addition, consider planar bubble walls so that physical quantities depend only on the comoving coordinate $z = X + v_w t$, the distance to the wall, with v_w the wall velocity.

The Higgs number density n_{H_1} is obtained by (i) solving the quantum Boltzmann equations (2a,2b); (ii) integrating the difference of mass-basis densities matrices to obtain the mass-basis number density $\hat{n}$; and (iii) inverting $\eta = U^\dagger \hat{\eta}$ to obtain the flavor basis density for H_1 as $n_{H_1} = \left[U(X)\, \hat{n}(X)\, U(X)^\dagger\right]_{11}$. The baryon number density is then given by

$$n_B = -3 \frac{\Gamma_{\text{ws}}}{v_w} \int_{-\infty}^{0} dz\, n_L(z) \exp\left(\frac{15}{4} \frac{\Gamma_{\text{ws}}}{v_w} z\right), \tag{4}$$

where we have integrated over the region of unbroken EW symmetry in which Γ_{ws} is unsuppressed.

To obtain a numerical solution to Eqs. (2a,2b), which comprise a system of eight coupled integro-differential equations (the f_m and $\bar{f}_m$ are 2×2 matrices in the mass basis), we observe that the $(f_m, \bar{f}_m)$ depend on the momentum variables $k \equiv |\vec{k}|$ and $\cos\theta_k \equiv \hat{k} \cdot \hat{w}$, with $\hat{w}$ being the normal to the wall. Moreover, the collision terms couple $(f_m, \bar{f}_m)$ for different k and $\cos\theta_k$. Since the density distributions experience Boltzmann suppression for larger k, we can truncate the momentum k with a given maximum limit $k_{\rm max}$. To make the problem tractable, we discretize k and $\cos\theta_k$ into N_k and N_θ bins within the ranges $0 < k < k_{\rm max}$, $\quad -1 < \cos\theta_k < 1$, and we take the central values of each bin. The Boltzmann equations then yield a system of $8\times N_k\times N_\theta$ coupled first order ordinary differential equations with boundary conditions, which we solve with the "relaxation method"[64]. Far from the wall ($z\to\pm\infty$), the $(f_m,\bar{f}_m)$ approach their equilibrium forms: $f_m^{\rm eq}(\mathbf{k},z) = {\rm diag}\left(n_B[\omega_{1\mathbf{k}}(z)], n_B[\omega_{2\mathbf{k}}(z)]\right)$. Since the collision terms bring the density matrices to equilibrium in the positive time direction, we only need to impose the thermal-equilibrium boundary conditions in the negative (positive) time directions for the right-moving (left-moving) modes.

We will compare our results to those obtained in the VIA. The latter framework treats the (S,Σ^0) vevs as perturbative insertions, and otherwise utlizes flavor basis Greens functions. Note that flavor non-diagonal collision terms arising from interactions between the $H_{1,2}$ with s and σ and arising from the first line in Eq. (3), are absent in the VIA treatment. When comparing our results with those of the VIA computation, we follow the methods used in Ref. [43]. For the $v_s(x)$ and $v_\sigma(x)$ profiles we adopt the forms in Ref. [43], along with the corresponding profile parameter values as well as wall velocity, $v_w = 0.05$. The benchmark parameter choices are: $a_1 = 3.0$; $a_2 = 1.5$; $\sin\delta_1 = 0.1$; $\sin\delta_2 = 0$; $y_3^{11} = 0.75$; $y_3^{22} = 1.0$; and the thermal masses of H_1, H_2, S, Σ and A fields at FOEWPT temperature $T = 123\,{\rm GeV}$ as $m_{H_1} = 1.5T$; $m_{H_2} = 1.32T$; $m_s = m_\sigma = 0.8T$; $m_A = 0.12\,T$; and diffusion constant $D_H = 110/T$ as in [43].

Fig. 1 shows the resulting VR and VIA profiles $n_L(z)$ as a function of the distance normal to the bubble wall. The pronounced structure near $z = 0$ reflects the variation in the bubble profiles near the wall center and the corresponding impact on the CPV sources involving $u\cdot\Sigma$ entering the RHS of Eqs. (2a,2b). Importantly, the VR diffusion tail ($z < 0$) is significantly enhanced as compared to the VIA result. As the resulting value of n_B entails integrating over this tail as in Eq. (4) we expect the VR to yield a larger baryon asymmetry.

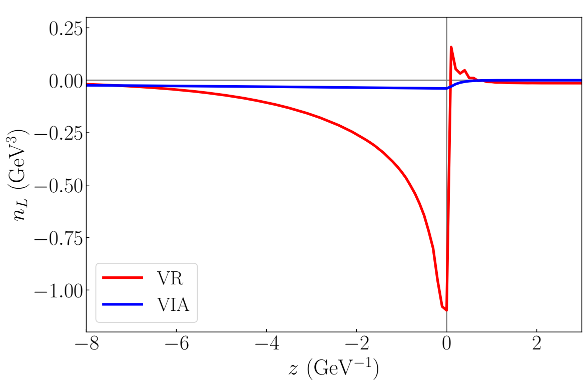

FIG. 1. Left handed quark density $n_L = (4c_T + 5c_Q)n_{H_1}$ for the VIA (blue) and VR (red) approaches. The bubble exterior (interior) corresponds to $z < 0$ ($z > 0$).

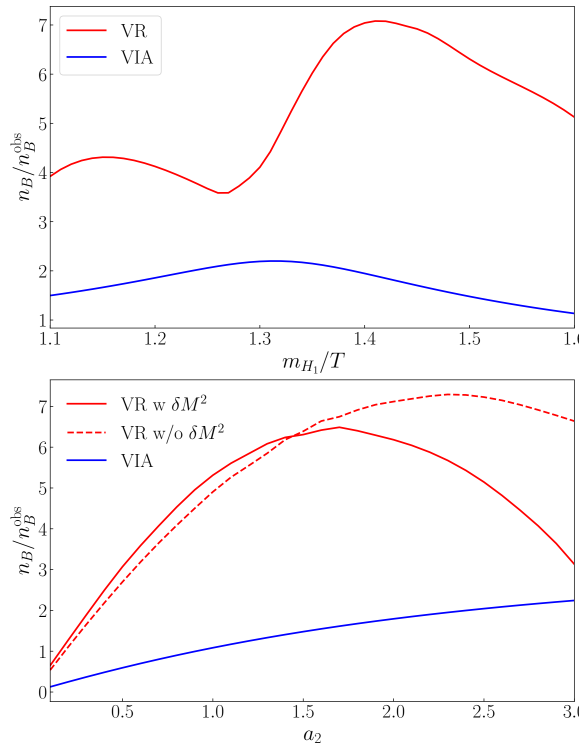

FIG. 2. The obtained BAU n_B as a function of m_{H_1} (with fixed $m_{H_2} = 1.32T$) (top) and the portal coupling a_2 (bottom) for VR and VIA approaches.

This expectation is born out as illustrated in Fig. 2 (top), where we show the value of n_B as a function of m_{H_1} with all other parameters fixed as above. For both the VR and VIA, the increase in n_B for m_{H_1} near m_{H_2} reflects the resonant enhancement as discussed in Refs. [35, 40, 41]. At the maximum, the VR asymmetry is more than four times larger in magnitude than the VIA value. The double peak structure of VR arises due to a vanishing of the CPV sources $[u\cdot\Sigma, f_m]$ and $[u\cdot\Sigma,\bar{f}_m]$ for $m_{H1} = m_{H2}$[40]. Thermal mass corrections induce a slight shift the location of the dip minimum. The VR/VIA enhancement away from this degeneracy point is surprising, as earlier work had suggested the VIA significantly over-estimated the asymmetry.

Figure 2 (bottom) gives the dependence of n_B on the flavor non-diagonal portal coupling a_2, illustrating the impact of flavor non-diagonal interactions that enter the VR treatment via the CPV source and CP-conserving collision term. The VIA includes only the former. Naïvely, one might anticipate increasing $|a_2|$ would lead to a monontonically increasing n_B, owing to correspondingly stronger CPV sources. This expectation is consistent with the VIA curve (blue). In the VR approach, however, for sufficiently large $|a_2|$ the asymmetry begins to decrease, even though the magnitudes of the CPV sources continue to grow. This decrease results from increasingly important damping effects from the CP-conserving collision terms, resulting in closer alignment of the $H_{1,2}$ number densities. An additional suppression at large a_2 arises due to flavor non-diagonal thermal mass corrections in the symmetric phase, δM^2 (dashed red curve). Clearly, a realistic asymmetry computation requires full inclusion and consistent treatment of the CP-conserving interactions, as facilitated by the VR framework.

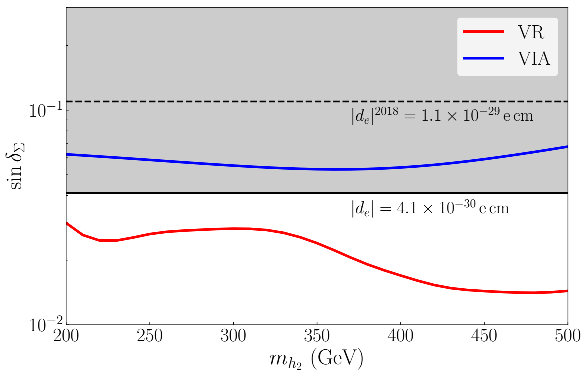

FIG. 3. Constraints on the CPV phase δ_Σ as a function of the physical $T = 0$ mass m_{h_2} with the other parameters fixed. The solid red (blue) band gives the VR (VIA) prediction. The shaded region above the solid (dashed) black line is excluded by the current (previous) electron EDM limit [65] ([66]).

In Fig. 3 we show the BAU as a function of the CPV phase δ_Σ and m_{h_2}, the physical mass of H_2 at $T = 0$, and compare with the corresponding constraints from experimental limits on d_e. The latter arises in this model from the two-loop "Barr-Zee" graphs [43]. The present bound $|d_e| < 4.1 \times 10^{-30} e\,\mathrm{cm}$ excludes the shaded region above the solid black line. For reference, we also show the previous d_e bound (dashed black line). The VR and VIA BAU results are indicated by the red and blue lines, respectively. Importantly, according to the VR computation, this EWBG source remains viable even in light of the new d_e bound. In contrast, the VIA computation – and by inference the alternative SC approach – would imply that that model is ruled out.

We expect that application of the VR formulation to other models with scalar field CPV sources will also yield more relaxed EDM constraints on EWBG than would be inferred from SC and even VIA treatments. Moreover, it implements a state-of-the art treatment of collision, damping, and flavor oscillation dynamics (both thermal and non-thermal), facilitating a robust confrontation between EWBG theory and experiment. An analogous treatment of fermion field CPV sources will appear in forthcoming work.

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