

Optimal quantum strategy for locating Unruh channels

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Abstract

From the perspective of quantum information theory, the effect of Unruh radiation on a two-level accelerated detector can be modeled as a quantum channel. In this work, we employ the tools of channel-position finding to locate Unruh channels. The signal-idler and idler-free protocols are explored to determine the position of the target Unruh channel within a sequence of background channels. We derive the fidelity-based bounds for the ultimate error probability of each strategy and obtain the conditions where the signal-idler protocol is superior to the protocol involving idler-free states. It is found that the lower bound of the error probability for the signal-idler scheme exhibits clear advantages in all cases, while the idler-free scheme can only be implemented when the temperature of the two channels is very close and the number of initial states is insufficient. Interestingly, it is shown that the optimal detection protocol relies on the residual correlations shared between the emitted probe state and the retained idler modes.

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I. INTRODUCTION

The Unruh effect [1–4] is one of the most monumental achievements of quantum field theory in curved spacetime. It plays a crucial role in the understanding of vacuum fluctuations and the nature of quantum thermal effects. It was predicted that a uniformly accelerated observer will detect a thermal bath from expressing the vacuum state in terms of a different set of operator basis defined along the time-like killing vector in their locally accelerated coordinate system [5–9]. A variety of techniques have been employed to analyze this phenomenon including the response of a two level system, referred to as an Unruh-DeWitt (UD) detector [10–13], when it absorbs these thermal particles. Studying the Unruh effect from the perspective of quantum information theory could not only be helpful in understanding the Hawking effect [14–16], but also provides an explanation for the generation and degradation of entanglement in curved spacetime [17–19]. Direct observation of the Unruh effect is considered as one of the key experimental goals of contemporary physics [20–24]. However, a simple calculation shows that a 1 Kelvin Unruh temperature corresponds to an acceleration of the order of $\sim 10^{21}m/s^2$, which is extremely challenging to obtain [23, 24]. In this sense, the technical obstacles to the detection of Unruh radiation lead to the indistinguishability of Unruh channel in general relativistic background.

On the other hand, quantum channels can model various physical processes [25, 26], so the discrimination of different quantum channels [27–30] is a fundamental task in quantum information theory. The theory of channel-position finding (CPF) has been effectively used to determine the target channel with varying transmittance or induced noise within a range of background loss channels [28–30]. Recently, the advantages of quantum entanglement have been demonstrated in CPF, for example the thermal loss channel [25] and the amplitude damping channel [31, 32]. In this paper, we study the task of determining the location of Unruh channels, in which different accelerations would induce differentiated responses in the detectors [10, 11, 13, 33]. In the UD detector model, the divergence in temperature predictions among detectors with different acceleration simplifies the task from identifying the channel temperatures to identifying the detector’s accelerations.

Here we focus on the problem of CPF under the constraint that the sources considered are comprised of at most one photon. Two protocols will be considered: the signal-idler (SI) protocol and idler-free (IF) protocol. In fact, in the applications of quantum sens-

ing, the assistance of idler modes has been a crucial feature to achieve quantum enhanced performance [34, 35], but the IF channel identification schemes have also received a lot of attention because of their ability to eliminate quantum memory [29, 36]. We consider two scenarios: (i) The temperature of the target channel is zero, and it is located within a series of reference channels; (ii) the temperature difference between the target channel and the reference channel is particularly small. These two scenarios effectively encompass the potential background in which the target channel may exist. We establish fidelity-based bounds on the final error probability in the multiple channel discrimination problem and identify the quantum dominance involving various quantum sources. The main purpose of our study is to find the optimal strategy for locating the Unruh channels, and the optimal operating conditions for different strategies.

This paper is organized as follows: Sec. II outlines the model of Unruh channel location and calculates the detection error probabilities. In Sec. III, we compare the advantage of detection error probabilities between the SI protocol and the IF protocol, and Sec. IV presents the conclusions. Throughout the paper, we adopt the conventions $\hbar = G = c = \kappa_B = 1$.

II. THE MODEL OF UNRUH CHANNEL LOCATION

A. The scheme

In this paper, we employ the tools of CPF involving $N \geq 2$ boxes to locate the target Unruh channel position. As shown in Fig. (1), the boxes $\mathcal{C}_i (i = 1, 2, \dots, N)$ are modeled as Unruh channels with different temperatures. The characterization of each channel is presented in the Appendix A. The target channel $\mathcal{C}_{\mathcal{T}}$ occupies one box with acceleration q_i , while the other $N - 1$ boxes represent the reference channel $\mathcal{C}_{\mathcal{R}}$ with acceleration $q_{j \neq i}$. Identification of the target Unruh channel is a symmetric hypothesis testing problem where the task is to discriminate between N hypotheses given by [37, 38]

$$H_i : \mathcal{C}_i = \mathcal{C}_{\mathcal{T}}, \mathcal{C}_{j \neq i} = \mathcal{C}_{\mathcal{R}}. \quad (1)$$

At the transmitter, the initial state ρ^{in} injects into each of the boxes. Each channel is represented by an accelerated detector which interacts with its surroundings [13], and this detection process can be described as a quantum map for the quantum state, as detailed

in the Appendix A. The task of correctly identifying the target channel in a series of reference channels may be reduced to distinguishing the possible channel outputs $\mathcal{C}_{\mathcal{T}}(\rho)$ and $\mathcal{C}_{\mathcal{R}}(\rho)$ [28–30]. Suppose that the overall input state has a tensor product form over the N boxes, such that $\rho = \sigma^{\otimes N}$, and utilizing $M \gg 1$ identical transmissions of these types. We assume equiprobable hypotheses $p_i = N^{-1}$ for any i , and then compute the error probabilities $p_{\text{err}}^{N,M}(\rho)$. Obtaining an exact analytical bound of the error probability is challenging. However, the upper and lower bounds can be derived using the pretty good measurement (PGM) [38, 39]

$$p_{\text{err}}^{N,M}(\rho) \leq (N-1)F^{2M}(\mathcal{C}_{\mathcal{T}}(\rho), \mathcal{C}_{\mathcal{R}}(\rho)), \quad (2)$$

$$p_{\text{err}}^{N,M}(\rho) \geq \frac{N-1}{2N}F^{4M}(\mathcal{C}_{\mathcal{T}}(\rho), \mathcal{C}_{\mathcal{R}}(\rho)), \quad (3)$$

where $F(\rho, \sigma)$ is the Bures fidelity [40, 41]

$$F(\rho, \sigma) := \|\sqrt{\rho}\sqrt{\sigma}\|_1 = \text{tr} \sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}. \quad (4)$$

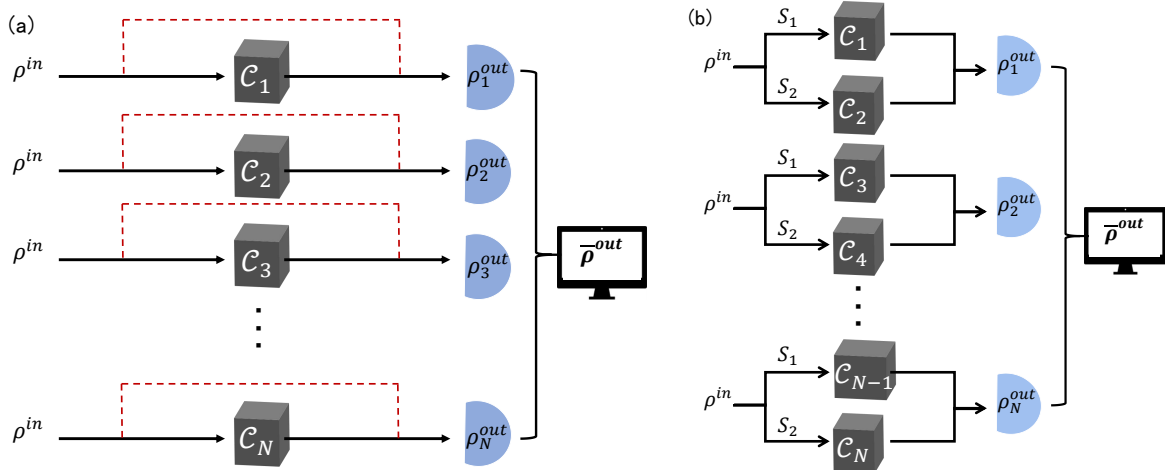


FIG. 1: Our setup for the CPF protocol that provides a benchmark for the general quantum channel. We assume N boxes, consisting of a target channel and $N - 1$ reference channels. Two different protocols for the CPF of Unruh channel are employed. In panel (a), we utilize the signal-idler protocol of CPF, while in panel (b), we employ the idler-free protocol of CPF for biphoton states.

B. The signal-idler protocol

We first employ the SI strategy to locate the Unruh channels. Such protocol has been proved to be effective in discriminating Gaussian lossy channels [25, 26, 30–32]. As depicted in Fig. (1a), the initial state of the entire system is prepared in tensor product over all the boxes ($\otimes N$), where each signal S_i (black) box is entangled with an ancillary idler I_i (red). The input state for each box is a maximally entangled state

$$|\Psi^{in}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle). \quad (5)$$

The SI strategy involves only the signal probing the box, while the idler state is directly sent to the receiver for combination with the output. The associated quantum channel is expressed as

$$\mathcal{E}_i^N := \otimes_{j \neq i} (\mathcal{C}_{\mathcal{R}_j} \otimes \mathcal{I}_{I_j}) \otimes (\mathcal{C}_{\mathcal{T}_i} \otimes \mathcal{I}_{I_i}). \quad (6)$$

Upon the action of an Unruh channel only on the signal (S) mode while performing the identity on the reference idler (I) mode, we obtain the density operator of the output state

$$\rho^{out} = \frac{1-q}{2}(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|) + \frac{v^2}{2}|01\rangle\langle 01| + \frac{qv^2}{2}|10\rangle\langle 10|. \quad (7)$$

The fidelity between the two outputs under the Unruh channel with acceleration parameters q_1 and q_2 is found to be

$$F(\rho_{q_1}^{out}, \rho_{q_2}^{out}) = \frac{1}{4} [4(-1+q_1)(-1+q_2) + (1+q_1q_2)\nu^4] + \frac{1}{4} \left[\sqrt{-4(-2+q_1)q_1 - (1+q_1^2)\nu^4} \sqrt{-4(-2+q_2)q_2 - (1+q_2^2)\nu^4} \right]. \quad (8)$$

By inserting Eq. (8) into Eqs. (2)-(3), the error probability for the SI protocol is then lower and upper bounded by

$$p_{\text{err}}^{N,M}(\rho) \geq \frac{N-1}{2N} F(\rho_{q_1}^{out}, \rho_{q_2}^{out})^{4M}, \quad (9)$$

and

$$p_{\text{err}}^{N,M}(\rho) \leq (N-1) F(\rho_{q_1}^{out}, \rho_{q_2}^{out})^{2M}, \quad (10)$$

respectively.

C. The idler-free protocol

Then we consider the idler-free protocol without idler mode reservation. In this case, the two-mode state ρ^{in} serves as probes into two adjacent boxes, with the modes labeled

as signals S_1 and S_2 , as illustrated in Fig. (1b). While the optimal quantum strategy for various scenarios, such as quantum illumination [35, 42, 43], spectroscopy [44], and quantum readings [45], often involves an entangled idler-assisted protocol, the storage of the idler mode poses a challenging task. The character of IF protocols lies in their ease of implementation and the absence of considerations for memory construction. Therefore, investigating the IF strategy is valuable, as it will help us understand whether quantum superiority can still be achieved even without a quantum memory for storing the idler mode.

In the IF protocol setup, we take advantage of the complete entanglement exhibited with the Bell state in the multichannel array to achieve quantum advantage. For any CPF problem consisting of $N \geq 4$ (even) independent channels, the global quantum channel acting on the initial state is:

$$\mathcal{E}_i^{N/2} := \otimes_{j \neq i} (\mathcal{C}_{\mathcal{R}_j} \otimes \mathcal{C}_{\mathcal{R}_j}) \otimes (\mathcal{C}_{\mathcal{T}_i} \otimes \mathcal{C}_{\mathcal{R}_i}). \quad (11)$$

If both modes S_1 and S_2 pass through two Unruh channels with the same acceleration parameter q_1 , the final state at the output is

$$\rho_{q_1, q_1}^{out} = \frac{1}{2} \begin{pmatrix} Q_1^2 + \nu^4 & 0 & 0 & Q_1^2 \\ 0 & Q_1\nu^2 + Q_1q_1\nu^2 & 0 & 0 \\ 0 & 0 & Q_1\nu^2 + Q_1q_1\nu^2 & 0 \\ Q_1^2 & 0 & 0 & Q_1^2 + q_1^2\nu^4 \end{pmatrix}, \quad (12)$$

where $Q_1 = 1 - q_1$. If the two modes of the initial state ρ^{in} pass through two Unruh channels with acceleration q_1 and q_2 , the final state at the output takes the form:

$$\rho_{q_2, q_1}^{out} = \frac{1}{2} \begin{pmatrix} Q_1Q_2 + \nu^4 & 0 & 0 & Q_1Q_2 \\ 0 & Q_1\nu^2 + Q_2q_1\nu^2 & 0 & 0 \\ 0 & 0 & Q_2\nu^2 + Q_1q_2\nu^2 & 0 \\ Q_1Q_2 & 0 & 0 & Q_1Q_2 + q_1q_2\nu^4 \end{pmatrix}, \quad (13)$$

where $Q_2 = 1 - q_2$. The lower and upper bounds for the error probabilities of the correct channel pair, calculated based on the fidelity between the two output states mentioned, are as follows [30]:

$$\tilde{p}_{err}^{N, M}(\rho) \geq \frac{N-2}{2N} F(\rho_{q_1, q_1}^{out}, \rho_{q_2, q_1}^{out})^{4M}, \quad (14)$$

and

$$\tilde{p}_{err}^{N, M}(\rho) \leq \frac{N-2}{2} F(\rho_{q_1, q_1}^{out}, \rho_{q_2, q_1}^{out})^{2M}. \quad (15)$$

The objective of the CPF task is to determine the location of the target channel, rather than merely identifying the pair containing it. In an IF protocol, the first stage involves successfully identifying the correct pair, and the second stage entails engineering a secondary CPF protocol by combining the correct pair with two additional reference channels. This enables us to pinpoint the location of the target channel. To maintain the energy constraint, we choose to divide the total number of probes into two parts, generating $M/2$ probes for each stage of the IF strategy in the CPF process.

Considering this two-stage approach, there are two ways in which an overall error can occur. The first scenario involves misidentifying the pair where the target channel is located in the initial stage, resulting in a failure to accomplish the task of locating the target channel. The second scenario involves correctly identifying the pair in which the target channel is located in the first stage, but in the second stage, there is an incorrect identification of which of the two channels is the target channel. Therefore, we can utilize the relevant lower and upper bounds to derive the final error probability of the IF scheme as follows:

$$p_{\text{err}}^{N,M/2}(|\psi_2\rangle\langle\psi_2|) = \tilde{p}_{\text{err}}^{N,M/2}(|\psi_2\rangle\langle\psi_2|) + [1 - \tilde{p}_{\text{err}}^{N,M/2}(|\psi_2\rangle\langle\psi_2|)] \tilde{p}_{\text{err}}^{4,M/2}(|\psi_2\rangle\langle\psi_2|). \quad (16)$$

III. QUANTUM ADVANTAGE OF THE SIGNAL-IDLER STRATEGY

In the previous section, we utilized the two-energy level detector as a thermometer model and the theory of CPF for discriminating the Unruh temperature. As mentioned earlier, to find the most effective method for detecting the Unruh channel, we need to compare the upper and lower bounds of error probabilities associated with different protocols. Denoting the upper and lower bounds of the SI (IF) protocols as $p_{\text{err}}^{\text{SI,U}}$ and $p_{\text{err}}^{\text{SI,L}}$ ($p_{\text{err}}^{\text{IF,U}}$ and $p_{\text{err}}^{\text{IF,L}}$), we define the minimum guaranteed advantage (MGA) as the minimum performance enhancement achieved by a SI strategy over the IF one [41],

$$\Delta p_{\text{err}}^{\text{min}} := p_{\text{err}}^{\text{IF,L}} - p_{\text{err}}^{\text{SI,U}}. \quad (17)$$

If $\Delta p_{\text{err}}^{\text{min}} > 0$, the advantage of SI strategy is guaranteed. One can also define the maximum potential advantage (MPA) as follows:

$$\Delta p_{\text{err}}^{\text{max}} := p_{\text{err}}^{\text{IF,L}} - p_{\text{err}}^{\text{SI,L}}. \quad (18)$$

This represents the maximum potential improvement that quantum strategies can bring when the derived lower bound is fundamental.

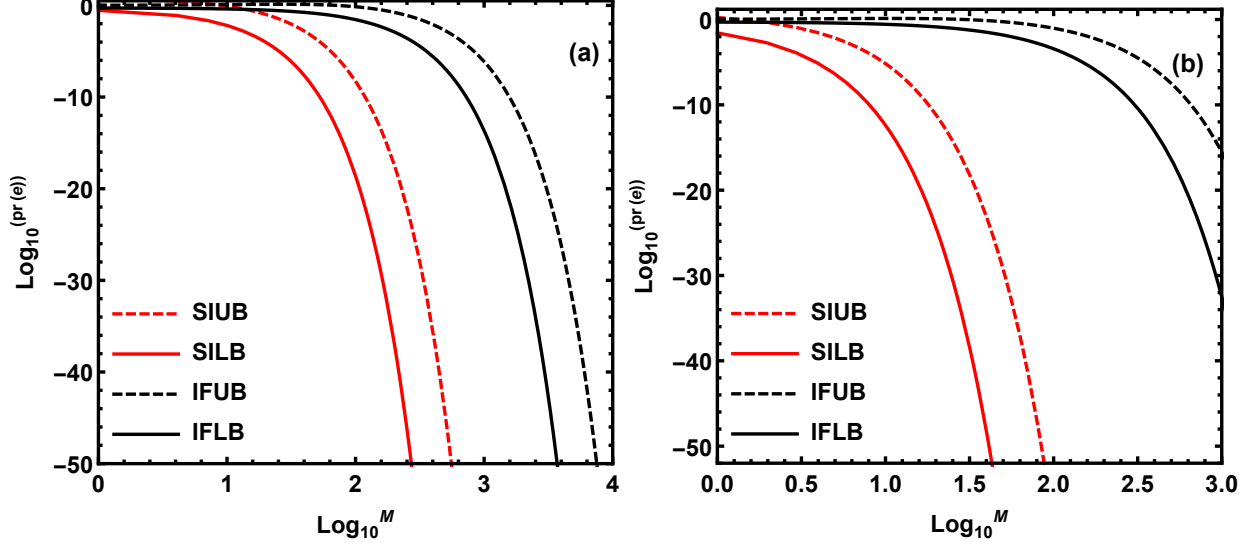


FIG. 2: (Color online) Quantum channel-position finding error probability $P_{err}^{N,M}$ with $N = 4$ versus the number of uses M for two types of protocols: a Bell biphoton state in both a signal-idler (red) and an idler-free (black) setup. The detection error probability of the target channel with zero acceleration, among reference channels with (a) low acceleration ($q_1 = 0.1$), and (b) high acceleration ($q_1 = 0.5$).

If the target channel is a zero acceleration channel within a series of Unruh reference channels, the probe scheme performance is illustrated in Fig. (2). It can be seen that for a given M , the error probability of $p_{err}^{SI,U}$ and $p_{err}^{SI,L}$ are both lower than $p_{err}^{IF,L}$. This result shows that the error probabilities bound in the SI protocol offer robust advantages, including both the MGA and MPA, along with a scaling advantage in the error exponent. Figs. (2a) and (2b) show that this conclusion remains valid regardless of whether the reference channel is cryogenic or high-temperature. We also find that the MGA function, which guarantees the advantage of the SI, increases as the number of copies of transmitted modes M increases.

If the single target channel is subjected to a nonzero acceleration $q_2 = q_1 + 0.01$, the temperatures of the target channel and the reference channel become very close, which makes it difficult to distinguish. Fig. (3) illustrates the detection error probabilities versus the number of modes M for the SI and IF protocols. When resolving two Unruh temperature channels, the SI scheme requires a larger number of copies M compared to the IF scheme, which exhibits both MPA and MGA. We can conclude that the IF protocols for channel localization are only feasible with a very low number of copy probes setting, in which $p_{err}^{SI,U}$

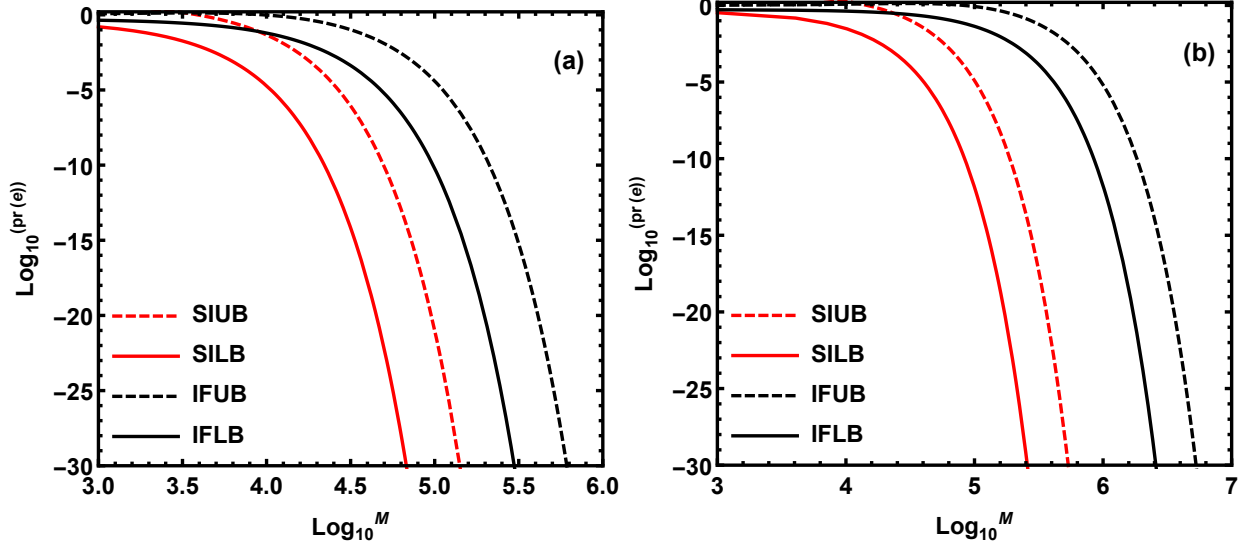


FIG. 3: (Color online) Quantum channel-position finding error probability $P_{err}^{N,M}$ with $N = 4$ versus the number of uses M for two types of protocols: a Bell biphoton state in both a signal-idler (red) and an idler-free (black) setup. The detection error probability of the target channel with nonzero acceleration ($q_2 = q_1 + 0.01$), among reference channels with (a) low acceleration ($q_1 = 0.1$), and (b) high acceleration ($q_1 = 0.5$).

is greater than $p_{err}^{IF,L}$. This demonstrates that a lower probability of detection error can be achieved by increasing the number of copy states. Comparing Fig. (3a) and Fig. (3b), we observe that both schemes perform well in locating a range of cryogenic reference channels, given that high temperatures tend to attenuate the initial quantum correlation.

Based on the analysis, we observe that the SI scheme exhibits significant advantages in the CPF task of Unruh channels, particularly when employing a large number of copy states. Upon performing the calculations, it becomes evident that the residual quantum correlation of the final state in the SI scheme surpasses that of the IF scheme, enhancing its efficiency in distinguishing between different channels. In other words, the similarity between the signal and idler states reduces the error probability of the CPF task. However, it is worth noting that the IF protocol eliminates the need for idler assistance to achieve quantum advantages in some of the most relevant discrimination scenarios, thereby relaxing practical requirements for prominent quantum sensing applications.

IV. CONCLUSION

This paper has presented performance comparisons among various channel position schemes for the CPF problem, considering channels with different temperatures. Our detection scheme is conducted under energy constraints, specifically utilizing an average of only one photon per channel for detection of the entire channel array. We consider the geometric characterization of the Unruh channel in Appendix A, where each channel is represented as an operator resulting from the interaction of an UD with its environment. We investigate the task of determining the location of two or more given quantum channels by exploring the SI and IF protocols. The objective is to pinpoint the position of a target Unruh channel within a sequence of reference channels.

In the task of locating between two Unruh channels, we calculated the output fidelity of CPF to test multiple quantum hypotheses. This provides upper and lower bounds on the error probability, even in cases of small differences in channel temperature and the number of probe states is insufficient. When performing the task of distinguishing the zero-temperature channel and the Unruh channel, we stressed out that the SI strategy outperforms the IF strategy, exhibiting both MGA and MPA across entire value regions. When resolving the target channel and reference channel with very close temperatures, the IF scheme is effective only in a scenario with a very low number of copy probes. These findings not only demonstrate that augmenting the number of copy probes can exponentially enhance the efficiency of the detection strategy, but also offer a theoretical framework for laboratories to employ diverse detection protocols for detection tasks. We hope that our results will stimulate further research on the discrimination of quantum operations. Our main lesson is that the residual feeble quantum correlation may offer an enormous performance advantage despite its being used in an entanglement-breaking scenario.

Acknowledgments

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Appendix A: The characterization of Unruh channel

In this appendix, we discuss the channel characterization of Unruh radiation on the accelerated detectors. Consider a two-level semi-classical UD detector [10, 11], in which the detector follows a classical world line, while its degrees of freedom are quantum. For a two-qubit system involving Alice and Rob, the detectors carried by Alice remain static, while Rob's detector undergoes uniform acceleration a along the x -direction, and its duration of motion is denoted as Δ . The world line of Rob is described as

$$t(\tau) = a^{-1} \sinh a\tau, x(\tau) = a^{-1} \cosh a\tau, y(\tau) = z(\tau) = 0, \quad (19)$$

where τ is the detector's proper time.

The initial state of the total system (detector+field) is given by

$$|\Psi_{-\infty}^{AR\phi}\rangle = |\Psi_{-\infty}^{AR}\rangle \otimes |0_M\rangle, \quad (20)$$

where $|\Psi_{-\infty}^{AR}\rangle = \alpha|0_A\rangle|1_R\rangle + \beta|1_A\rangle|0_R\rangle$ denotes the initial state shared by Alice's (A) and Rob's (R) detectors, with $|\alpha|^2 + |\beta|^2 = 1$. $|0_M\rangle$ represents that the external scalar field is in Minkowski vacuum.

The total system Hamiltonian can be expressed as

$$H = H_A + H_R + H_{KG} + H_{\text{int}}^{R\phi}, \quad (21)$$

where $H_A = \Omega A^\dagger A$, $H_R = \Omega R^\dagger R$, and Ω represents the detectors' energy gap. H_{KG} represents the Hamiltonian for the free Klein-Gordon field. The accelerated detector Rob is coupled to a scalar field $\phi(x)$ through the interaction Hamiltonian [2]

$$H_{\text{int}}^{R\phi}(t) = \epsilon(t) \int_{\Sigma_t} d^3\mathbf{x} \sqrt{-g} \phi(x) [\psi(\mathbf{x})R + \psi^*(\mathbf{x})R^\dagger], \quad (22)$$

where $g \equiv \det(g_{ab})$ and $\mathbf{x}$ represent the coordinates defined on the Cauchy surface $\Sigma_{t=\text{const}}$ associated with a suitable time-like isometry. The smooth function ϵ describes the coupling action between the detector and field, enabling the detector to remain active for a proper time interval Δ . $\psi(\mathbf{x})$ is a smooth compact support complex-valued function indicating that the detector only interacts with the field in the neighborhood of its world line.

In the interaction picture, we consider the first-order perturbation, and the final state $|\Psi_t^{AR\phi}\rangle$ is

$$|\Psi_t^{AR\phi}\rangle = [I - i(\phi(f)R + \phi(f)^\dagger R^\dagger)] |\Psi_{-\infty}^{AR\phi}\rangle, \quad (23)$$

where

$$\phi(f) = i[a_{RI}(\bar{\lambda}) - a_{RI}^\dagger(\lambda)]. \quad (24)$$

Here $a_{RI}(\bar{\lambda})$ and $a_{RI}^\dagger(\lambda)$ represent annihilation and creation operators for the λ mode. By inserting Eq. (24) into Eq. (23), we obtain

$$|\Psi_t^{AR\phi}\rangle = |\Psi_{-\infty}^{AR\phi}\rangle + \alpha|0_A\rangle|0_R\rangle \otimes (a_{RI}^\dagger(\lambda)|0_M\rangle) + \beta|1_A\rangle|1_R\rangle \otimes (a_{RI}(\bar{\lambda})|0_M\rangle). \quad (25)$$

The Bogoliubov transformations between the Rindler operators and the operators annihilating the Minkowski vacuum can be expressed as follows [10, 11]:

$$\begin{aligned} a_{RI}(\bar{\lambda}) &= \frac{a_M(\overline{F_{1\Omega}}) + e^{-\pi\Omega/a}a_M^\dagger(F_{2\Omega})}{(1 - e^{-2\pi\Omega/a})^{1/2}}, \\ a_{RI}^\dagger(\lambda) &= \frac{a_M^\dagger(F_{1\Omega}) + e^{-\pi\Omega/a}a_M(\overline{F_{2\Omega}})}{(1 - e^{-2\pi\Omega/a})^{1/2}}, \end{aligned} \quad (26)$$

where $F_{1\Omega} = \frac{\lambda + e^{-\pi\Omega/a}\lambda \circ w}{(1 - e^{-2\pi\Omega/a})^{1/2}}$, $F_{2\Omega} = \frac{\overline{\lambda \circ w} + e^{-\pi\Omega/a}\bar{\lambda}}{(1 - e^{-2\pi\Omega/a})^{1/2}}$, and $w(t, x) = (-t, -x)$ represents the wedge reflection isometry.

Then we obtain the reduced density matrix of the detector's state by tracing out the degrees of freedom of the external field

$$\begin{aligned} \rho_t^{AR} &= \left\| \Psi_t^{AR\phi} \right\|^{-2} \text{Tr}_\phi \left| \Psi_t^{AR\phi} \right\rangle \left\langle \Psi_t^{AR\phi} \right| \\ &= \begin{pmatrix} \mathcal{C} & 0 & 0 & 0 \\ 0 & |\alpha|^2 \mathcal{A} & \alpha\beta \mathcal{A} & 0 \\ 0 & \alpha\beta \mathcal{A} & |\beta|^2 \mathcal{A} & 0 \\ 0 & 0 & 0 & \mathcal{B} \end{pmatrix}, \end{aligned} \quad (27)$$

where

$$\begin{aligned} \mathcal{A} &= \frac{1 - q}{(1 - q) + \nu^2(|\alpha|^2 + |\beta|^2 q)}, \\ \mathcal{B} &= \frac{\nu^2 |\beta|^2 q}{(1 - q) + \nu^2(|\alpha|^2 + |\beta|^2 q)}, \\ \mathcal{C} &= \frac{\nu^2 |\alpha|^2}{(1 - q) + \nu^2(|\alpha|^2 + |\beta|^2 q)}, \end{aligned} \quad (28)$$

with the parametrized acceleration $q \equiv e^{-2\pi\Omega/a}$. The effective coupling between the detector and the scalar field is $\nu^2 \equiv \|\lambda\|^2 = \frac{\epsilon^2 \Omega \Delta}{2\pi} e^{-\Omega^2 \kappa^2}$ [8, 10, 11, 13], where $\Omega^{-1} \ll \Delta$ is necessary for the validity of the above definition. In the present work the coupling parameter is

constrained to $\nu^2 \rightarrow 0$ to ensure the validity of the perturbative approach. Notably, q is a monotonic function of acceleration a , and $q \rightarrow 0$ corresponds to zero acceleration. These facts suggest that the Unruh effect can be interpreted as a noisy quantum channel.

The dynamics of open quantum systems can be characterized as follows. The evolution from the detectors initial state $\rho_{-\infty}^{AR}$ to the final state ρ_t^{AR} can alternatively be expressed as:

$$\rho_t^{AR} = U(t)\rho_{-\infty}^{AR}U^\dagger(t), \quad (29)$$

where $U(t)$ is the propagator of the joint system dynamics from the initial time to the final time. The object of interest is the subsystem R , whose state at all times t is governed according to the standard quantum mechanical prescription by the following quantum dynamical process:

$$\rho_t^R = \text{Tr}_B [U(t)\rho_{-\infty}^{AR}U^\dagger(t)]. \quad (30)$$

The quantum dynamical process can be described by a quantum map denoted as:

$$\mathcal{C}_\rho = \sum_j^\infty M_j \rho_0 M_j^\dagger, \quad (31)$$

where ρ_0 is an arbitrary initial state, and M_j is an operator for different dynamical evolution processes. Based on the above analysis and calculation, the operators M_ν^R acting on Rob can be characterized by the following Choi Matrix:

$$M_1^R = \begin{pmatrix} \sqrt{1-q} & 0 \\ 0 & \sqrt{1-q} \end{pmatrix}, \quad M_2^R = \begin{pmatrix} 0 & 0 \\ v\sqrt{q} & 0 \end{pmatrix}, \quad M_3^R = \begin{pmatrix} 0 & v \\ 0 & 0 \end{pmatrix}. \quad (32)$$

If we are considering the subsystem ρ_t^A , with M_μ^A represents identical because the detector of Alice remains static and is switched off.

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