

Prospects for weighing neutrinos in interacting dark energy models using joint observations of gravitational waves and γ -ray bursts

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In this work, we explore the capability of future gravitational wave (GW) standard siren observations to constrain the total neutrino mass in some typical interacting dark energy (IDE) models. We examine the combined potential of the third-generation ground-based GW detector network and a short γ -ray burst (GRB) detector similar to the THESEUS telescope for cosmological analysis. Our findings suggest that future GW standard siren observations could modestly refine the upper limit on the total neutrino mass, enhancing it by 5% to 10% over the existing limit given by the CMB+BAO+SN data. Additionally, the future observations are expected to significantly improve the constraints on Ω_m and H_0 . Moreover, the measurements could enhance the precision of the coupling strength β by 23% to 42%.

I. INTRODUCTION

The experiments involving solar and atmospheric neutrinos have demonstrated that neutrinos possess mass and exhibit substantial mixing among different species [1]. Despite this, directly measuring the absolute mass scale of neutrinos remains a formidable challenge for particle physics experiments. Neutrinos' mass has implications for the cosmic microwave background (CMB) and the universe's large-scale structure [2], making cosmological observations a crucial approach for determining their absolute mass.

Current principal cosmological observations fall into two categories. Firstly, there are observations related to the universe's expansion history, such as baryon acoustic oscillations (BAO) and type Ia supernovae (SN). Secondly, observations concerning the cosmic structure's growth, such as redshift space distortions, weak gravitational lensing, and galaxy clusters counts, are also pivotal. These observations help constrain the total neutrino mass, as extensively discussed in Refs. [3–36].

Recently, Ref. [36] explored the cosmological constraints on total neutrino mass within the framework of vacuum energy interacting with cold dark matter (Λ CDM), employing current observations. The study suggests that considering a direct interaction might alter the upper limit of the total neutrino mass, indicating that interactions between vacuum and cold dark matter could influence the cosmological measurement of neutrino mass. Nevertheless, current cosmological observations do not yet tightly constrain the total neutrino mass [37, 38]. Gravitational-wave (GW) standard siren obser-

ventions, however, show promise as a powerful cosmological probe to aid in constraining the total neutrino mass.

The era of multi-messenger astronomy was inaugurated by the observation of the binary neutrino star (BNS) merger event GW170817 and its electromagnetic (EM) counterparts [39, 40]. Analyzing the GW waveform from such events directly yields the absolute luminosity distance, known as a standard siren. Additionally, identifying the EM counterpart of a GW source allows for the measurement of its redshift. Establishing the relationship between cosmic distances and redshifts paves the way for probing the universe's expansion history. Thus, GW standard sirens are expected to significantly influence the estimation of cosmological parameters.

In the coming decades, third-generation (3G) ground-based GW detectors, such as Cosmic Explorer (CE) [41] and Einstein Telescope (ET) [42], are set to revolutionize the measurement of GW signals. These detectors are anticipated to form a network that will significantly enhance the capability to detect GW events. Reference Ref. [43] presents a preliminary discussion on using future GW standard siren observations to weigh neutrinos within interacting dark energy (IDE) models. This analysis is based on an assumption that 1000 standard sirens could be detected over a 10-year period by either the ET or CE. While this estimate aligns with projections in other studies [26, 35, 36], variations in the redshift distribution could impact cosmological parameter estimations.

This work aims to explore the potential of 3G standard siren observations to measure neutrino mass in IDE models. We will analyze the detection strategy employing a network comprising the ET, and two CEs—one in the US with a 40-km arm length and another in Australia with a 20-km arm length, collectively referred to as ET2CE. Additionally, we will consider observations of γ -ray bursts (GRBs) from a detector similar to THESEUS, enhancing our capability to ascertain cosmological parameters.

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In our analysis of IDE models, we focus exclusively on the Λ CDM models, characterized by a vacuum energy equation of state parameter $w = -1$. In these models, the energy conservation equations for vacuum energy and cold dark matter are described as follows:

$$\dot{\rho}_{\text{de}} = Q, \quad (1)$$

$$\dot{\rho}_c = -3H\rho_c - Q, \quad (2)$$

where ρ_{de} and ρ_c represent the densities of dark energy and cold dark matter, respectively, H is the Hubble parameter, the dot denotes the derivative with respect to cosmic time t , and Q is the rate of energy transfer. In the field of IDE, one common assumption is that Q is proportional to the density of either dark matter or dark energy, i.e., $Q = \beta H\rho_c$ or $Q = \beta H\rho_{\text{de}}$ [44–53], where β is a dimensionless coupling parameter and H facilitates computational ease. Another assumption posits Q as $Q = \beta H_0\rho_c$ or $Q = \beta H_0\rho_{\text{de}}$ [32, 34, 54–58], with the Hubble constant H_0 used for dimensional consistency. In this work, we consider two forms of the energy transfer rate: $Q_1 = \beta H\rho_c$ and $Q_2 = \beta H_0\rho_c$. According to Eqs. (1) and (2), a positive β implies that cold dark matter decays into vacuum energy, a negative β suggests that vacuum energy decays into cold dark matter, and $\beta = 0$ indicates no interaction between them.

This paper is organized as follows. In Section II, we describe the methodology in our analysis. In Section III, we report the constraint results and make some relevant discussions. In Section IV, we make a conclusion for this work.

II. METHODOLOGY

For the Λ CDM model, the parameter space vector is $\{\omega_b, \omega_c, \tau, 100\theta_{\text{MC}}, \beta, \ln(10^{10}A_s), n_s\}$, where ω_b and ω_c are the present density of baryons and the cold dark matter, respectively, τ is the Thomson scattering optical depth due to reionization, θ_{MC} (multiplied by 100) is the ratio between the comoving sound horizon and the angular diameter distance at the decoupling epoch, β is the coupling parameter in the Λ CDM model, A_s is the amplitude of primordial scalar perturbation power spectrum, n_s is its power-law spectral index. When the case of neutrino mass is considered in the Λ CDM model, one extra free parameter $\sum m_\nu$ should be involved in the calculation.

Since we add neutrino mass into the Λ CDM model, the model considered in this paper is called the Λ CDM+ $\sum m_\nu$ model. For convenience, in this paper, we use Λ CDM1+ $\sum m_\nu$ and Λ CDM2+ $\sum m_\nu$ to denote the corresponding $Q_1 = \beta H\rho_c$ and $Q_2 = \beta H_0\rho_c$ models that involve neutrino mass, respectively. Thus, there are eight independent parameters in total for these Λ CDM+ $\sum m_\nu$ (Λ CDM1+ $\sum m_\nu$ and Λ CDM2+ $\sum m_\nu$) models.

It should be mentioned that, when we consider the situation of vacuum energy interacting with cold dark matter, then the vacuum energy is not a pure background any more, and in this case we must consider the vacuum energy perturbations. Note that, in the calculation of the dark energy perturbation evolution, there was a problem of perturbation instability appearing, namely, the cosmological perturbations of dark energy will be divergent in some parts of the parameter space, which ruins the IDE cosmology in the perturbation level. Under such circumstance, in order to avoid the perturbation instability problem in the Λ CDM model, we treat the vacuum energy perturbations in this work based on the extended parameterized post-Friedmann (PPF) approach [59–61] (for the original version of PPF method, see Refs. [62, 63]). For more information about the calculation of cosmological perturbations in the IDE models and the PPF approach, see Refs. [26, 52, 64, 65].

In this paper, we will simulate the GW standard siren data from the 3G GW detectors and future GRB detector, and use them to constrain neutrino mass in the Λ CDM models. We will investigate whether the GW standard sirens can improve the constraint on neutrino mass.

To show the constraining capability of the simulated GW standard sirens data, in this study, we consider two data combinations, CMB+BAO+SN (abbreviated as CBS) and CBS+GW, for comparison. For the CMB data, we use the CMB likelihood including the TT, TE, and EE spectra at $l \geq 30$, the low- l temperature commander likelihood, and the low- l SimAll EE likelihood, from the Planck 2018 data release [38]. For the BAO data, we use the measurements from the 6dFGS ($z_{\text{eff}} = 0.106$) [66], SDSS-MG ($z_{\text{eff}} = 0.15$) [67], and BOSS-DR12 ($z_{\text{eff}} = 0.38, 0.51, \text{ and } 0.61$) [68]. For the SN data, we use the Pantheon sample, which is comprised of 1048 data points [69].

For the GW data, we use the simulation method as prescribed in Ref. [70]. Here, we provide only a brief overview. It is important to note that our approach includes a comprehensive calculation of the redshift distribution of GW-GRB events. This differs from the method used in previous studies, which assumed the detection of 1000 bright sirens over a 10-year observation period, as discussed in Refs. [43, 71–83].

Based on the star formation rate [84–86], the merger rate in the observer frame is

$$\mathcal{R}_{\text{obs}}(z) = \frac{\mathcal{R}_m(z)}{1+z} \frac{dV(z)}{dz}, \quad (3)$$

where $dV(z)/dz$ is the comoving volume element, \mathcal{R}_m is the BNS merger rate in the source frame that is given by

$$\mathcal{R}_m(z) = \int_{t_{\text{min}}}^{t_{\text{max}}} \mathcal{R}_f[t(z) - t_d] P(t_d) dt_d, \quad (4)$$

where \mathcal{R}_f is the cosmic star formation rate, which we adopt the Madau-Dickinson model of Ref. [87], $t(z)$ is

the universe's age at the time of merger, t_d is the time delay, t_{\min} and t_{\max} are the minimum and maximum delay time for a massive binary to evolve merger [84], here $t_{\min} = 20$ Myr and t_{\max} is the Hubble time. The overall normalization is fixed by requiring that the value of $\mathcal{R}_m(z=0) = 920 \text{ Gpc}^{-3} \text{ yr}^{-1}$ agrees with the local rate estimated from the O1 LIGO and the O2 LIGO/Virgo observation run [88], which is also in accordance with the latest O3 observation run [89]. $P(t_d)$ is the time delay distribution of the time delay t_d , and we follow and adopt the exponential form [84],

$$P(t_d) = \frac{1}{\tau} \exp(-t_d/\tau), \quad (5)$$

with an e-fold time of $\tau = 100$ Myr for $t_d > t_{\min} = 20$ Myr. We simulate a catalog of BNS merger for 10-year observation. For each simulated GW source, the location (θ, ϕ) , the polarization angle ψ , the coalescence phase ψ_c , and the cosine of the inclination angle ι are drawn from uniform distributions. The mass of neutron star (NS) is assumed to be a Gaussian distribution, and the center value of the NS mass is $1.33 M_\odot$ with a standard deviation of $0.09 M_\odot$, where M_\odot is the solar mass.

Under the stationary phase approximation (SPA) [90], the Fourier transform of the frequency-domain waveform for a detector network (with N detectors) can be written as [91, 92]

$$\tilde{\mathbf{h}}(f) = e^{-i\Phi} \mathbf{h}(f), \quad (6)$$

where Φ is the $N \times N$ diagonal matrix with $\Phi_{ij} = 2\pi f \delta_{ij} (\mathbf{n} \cdot \mathbf{r}_k)$, \mathbf{n} is the GW propagation direction, \mathbf{r}_k is the location of the k -th detector, and the $\mathbf{h}(f)$ is

$$\mathbf{h}(f) = \left[\frac{h_1(f)}{\sqrt{S_{n,1}(f)}}, \frac{h_2(f)}{\sqrt{S_{n,2}(f)}}, \dots, \frac{h_N(f)}{\sqrt{S_{n,N}(f)}} \right]^T. \quad (7)$$

We consider the waveform in the inspiralling stage for the non-spinning BNS system in this study, and we adopt the restricted Post-Newtonian (PN) approximation and calculate the waveform to the 3.5 PN order [93, 94]. The Fourier transform of the GW waveform of k -th detector can be expressed as

$$h_k(f) = \mathcal{A}_k f^{-7/6} \exp\{i[2\pi f t_c - \pi/4 - 2\psi_c + 2\Psi(f/2)] - \varphi_{k,(2,0)}\}, \quad (8)$$

where the detailed forms of $\Psi(f/2)$ and $\varphi_{k,(2,0)}$ could be found in Refs. [92, 93], and \mathcal{A}_k is the Fourier amplitude can be written as

$$\mathcal{A}_k = \frac{1}{d_L} \sqrt{(F_{+,k}(1 + \cos^2 \iota))^2 + (2F_{\times,k} \cos \iota)^2} \times \sqrt{5\pi/96} \pi^{-7/6} \mathcal{M}_{\text{chirp}}^{5/6}, \quad (9)$$

where d_L is the luminosity distance of the GW source, $F_{+,k}$ and $F_{\times,k}$ are the antenna response functions of the k -th GW detector, $\mathcal{M}_{\text{chirp}} = (1+z)\eta^{3/5}M$ is the observed chirp mass, $M = m_1 + m_2$ is the total mass

of binary system with component masses m_1 and m_2 , and $\eta = m_1 m_2 / M^2$ is the symmetric mass ratio. In this study, we adopt the GW waveform in the frequency domain, then the time t is replaced by $t_f = t_c - (5/256)\mathcal{M}_{\text{chirp}}^{-5/3}(\pi f)^{-8/3}$ [92, 93], where t_c is the coalescence time.

In order to obtain the detection of GW events, we need to calculate the signal-to-noise ratio (SNR) for each GW event. The combined SNR for the detection network of N detectors is given by

$$\rho = (\tilde{\mathbf{h}}|\tilde{\mathbf{h}})^{1/2}. \quad (10)$$

The inner product is defined as

$$(\mathbf{a}|\mathbf{b}) = 2 \int_{f_{\text{lower}}}^{f_{\text{upper}}} \{\mathbf{a}(f)\mathbf{b}^*(f) + \mathbf{a}^*(f)\mathbf{b}(f)\} df, \quad (11)$$

where f_{lower} is the lower cutoff frequency ($f_{\text{lower}} = 1$ Hz for ET and $f_{\text{lower}} = 5$ Hz for CE), $f_{\text{upper}} = 2/(6^{3/2}2\pi M_{\text{obs}})$ is the frequency at the last stable orbit with $M_{\text{obs}} = (m_1 + m_2)(1+z)$, \mathbf{a} and \mathbf{b} are column matrices of the same dimension, and the star represents conjugate transpose. Here, we adopt the SNR threshold to be 12 in the simulation, and we assume a running period of 10 years and a duty cycle of 100% for the detector network.

For a GRB detected in coincidence with a GW signal, we require that the peak flux to be higher than the flux limit of the satellite. Based on the fitting results of GRB170817A [95], we adopt the Gaussian structured jet profile model

$$L(\theta_v) = L_c \exp\left(-\frac{\theta_v^2}{2\theta_c^2}\right), \quad (12)$$

where $L(\theta_v)$ is the luminosity per unit solid angle, θ_v is the viewing angle, and we assume the jet's direction is aligned with the binary orbital angular momentum, namely $\iota = \theta_v$. L_c and θ_c are the structure parameters defined by the sharpness of the angular profile. $L_c = L_p/4\pi \text{ erg s}^{-1} \text{ sr}^{-1}$ and $\theta_c = 4.7^\circ$, where L_p is the peak luminosity of each burst. For the distribution of the short GRBs, we assume the standard broken power-law model

$$\Phi(L_p) \propto \begin{cases} (L_p/L_*)^\alpha, & L_p < L_*, \\ (L_p/L_*)^\beta, & L_p \geq L_*, \end{cases} \quad (13)$$

where L_p is the isotropic rest frame peak luminosity in the 1 – 10000 keV energy range, L_* is a characteristic value separating the two regimes, and α and β are the characteristic slopes describing these regimes, respectively. In this paper, we use the values $L_* = 2 \times 10^{52} \text{ erg sec}^{-1}$, $\alpha = -1.95$, and $\beta = -3$, which is in accordance with Ref. [96]. In addition, we also assume a standard low end cutoff in luminosity of $L_{\min} = 10^{49} \text{ erg sec}^{-1}$. From the GW catalogue which has SNR larger than the threshold 12, we can select the GRB detection from the

BNS samples according to the probability distribution $\Phi(L_p)dL_p$.

For the total uncertainty of the luminosity distance d_L , we consider the instrument error $\sigma_{d_L}^{\text{inst}}$, the weak-lensing error $\sigma_{d_L}^{\text{lens}}$, and the peculiar velocity error $\sigma_{d_L}^{\text{pv}}$, and thus the total error is given by

$$\sigma_{d_L} = \sqrt{(\sigma_{d_L}^{\text{inst}})^2 + (\sigma_{d_L}^{\text{lens}})^2 + (\sigma_{d_L}^{\text{pv}})^2}. \quad (14)$$

We first estimate the instrument error $\sigma_{d_L}^{\text{inst}}$ by using the Fisher information matrix. The Fisher information matrix of a network including of N independent GW detectors can be written as

$$F_{ij} = \left(\frac{\partial \tilde{\mathbf{h}}}{\partial \theta_i} \middle| \frac{\partial \tilde{\mathbf{h}}}{\partial \theta_j} \right) \quad (15)$$

where θ_i denotes nine parameters (d_L , $\mathcal{M}_{\text{chirp}}$, η , θ , ϕ , ι , t_c , ψ_c , ψ) for a GW event. The instrumental error of the parameter θ_i can be calculated by $\Delta\theta_i = \sqrt{(F^{-1})_{ii}}$, where F_{ij} is the total Fisher information matrix for the network of N interferometers.

The error caused by weak lensing is adopted from Refs. [97, 98],

$$\sigma_{d_L}^{\text{lens}}(z) = \left[1 - \frac{0.3}{\pi/2} \arctan(z/0.073) \right] \times d_L(z) \times 0.066 \left[\frac{1 - (1+z)^{-0.25}}{0.25} \right]^{1.8}. \quad (16)$$

The error caused by the peculiar velocity of the GW source is given by [99]

$$\sigma_{d_L}^{\text{pv}}(z) = d_L(z) \times \left[1 + \frac{c(1+z)^2}{H(z)d_L(z)} \right] \frac{\sqrt{\langle v^2 \rangle}}{c}, \quad (17)$$

where $H(z)$ is the Hubble parameter, c is the speed of light in vacuum, and $\sqrt{\langle v^2 \rangle}$ is the peculiar velocity of the GW source and we roughly use $\sqrt{\langle v^2 \rangle} = 500 \text{ km s}^{-1}$.

In this work, we wish to study the constraint on neutrino mass in the $\Lambda\text{CDM} + \sum m_\nu$ models from GW standard sirens. As the first attempt to explore the impacts of GW-GRB joint observation on the neutrino mass, we do not consider all the different cases of 3G GW observations, i.e., single ET, single CE, the CE-CE network (one in the US and another one in Australia), and the ET-CE-CE network (one ET detector and two CE-like detectors), but instead we only consider one case of ET-CE-CE network (abbreviated as GW hereafter) as a concrete example to analysis.

In what follows, we will use these observational data to place constraints on these $\Lambda\text{CDM} + \sum m_\nu$ models. We will use two data combinations, i.e., CBS and CBS+GW, to constrain the neutrino mass $\sum m_\nu$ and other cosmological parameters. we employ the modified version of the publicly available Markov-Chain Monte Carlo (MCMC) package CosmoMC [100] to perform the calculations, and we use the PPF package [59, 60] for the ΛCDM models to handle the perturbations of vacuum energy.

III. RESULTS AND DISCUSSION

In this section, we shall report the fitting results of these $\Lambda\text{CDM} + \sum m_\nu$ models and discuss the impacts of GW standard siren observation on constraining neutrino mass. The fitting results are given in Tables I and II as well as Figs. 1 and 2. In Table I, we quote $\pm 1\sigma$ errors for the parameters, but for the parameters that cannot be well constrained, e.g., the neutrino mass $\sum m_\nu$, we quote the 95.4% CL upper limits.

First, we discuss the effect of simulated GW data of ET2CE on constraining the total neutrino mass. In the case of CBS constraints, from Table I, we have $\sum m_\nu < 0.123 \text{ eV}$ for $\Lambda\text{CDM} + \sum m_\nu$, $\sum m_\nu < 0.150 \text{ eV}$ for $\Lambda\text{CDM1} + \sum m_\nu$, and $\sum m_\nu < 0.156 \text{ eV}$ for $\Lambda\text{CDM2} + \sum m_\nu$. After adding the GW data of ET2CE, namely, using the data combination CBS+GW, the constraint results become $\sum m_\nu < 0.103 \text{ eV}$ for $\Lambda\text{CDM} + \sum m_\nu$, $\sum m_\nu < 0.143 \text{ eV}$ for $\Lambda\text{CDM1} + \sum m_\nu$, and $\sum m_\nu < 0.140 \text{ eV}$ for $\Lambda\text{CDM2} + \sum m_\nu$. We find that, compared with the $\Lambda\text{CDM} + \sum m_\nu$ model, the constraint on neutrino mass $\sum m_\nu$ becomes slighter looser in these $\Lambda\text{CDM} + \sum m_\nu$ models. We show the two-dimensional marginalized posterior contours (68.3% and 95.4% CL) in the $\Omega_m - \sum m_\nu$ plane for the $\Lambda\text{CDM} + \sum m_\nu$ model and $\Lambda\text{CDM} + \sum m_\nu$ models in Fig. 1 (left panel for CBS and right panel for CBS+GW). From this figure, we can clearly see that once the interaction is considered in the model, then the upper limit value of $\sum m_\nu$ becomes larger.

Moreover, we also find that the GW data help reduce the upper limits of neutrino mass $\sum m_\nu$ by 16.26%, 4.67%, and 10.26% for the $\Lambda\text{CDM} + \sum m_\nu$ model, the $\Lambda\text{CDM1} + \sum m_\nu$ model, and the $\Lambda\text{CDM2} + \sum m_\nu$ model, respectively. When the GW data are considered, the constraints on $\sum m_\nu$ become slightly tighter in the $\Lambda\text{CDM} + \sum m_\nu$ model and the $\Lambda\text{CDM} + \sum m_\nu$ models (see also in Fig. 2). Thus, in the $\Lambda\text{CDM} + \sum m_\nu$ models, compared to the current observations, the GW standard siren observations from ET2CE can slightly improve the constraint on neutrino mass, which is in accordance with the conclusion in the previous study on the IDE models [43].

Next, we discuss how the GW data help improve the constraints on other parameters, i.e., Ω_m , σ_8 , and H_0 . In Table II, we show the absolute and relative errors of Ω_m , σ_8 , and H_0 from the CBS and CBS+GW data combinations. Comparing the results from two data combinations, we find that the constraints on Ω_m , σ_8 , and H_0 are improved by 73.53%–83.95%, 5.56%–30%, and 90%–92.68%, respectively, when considering the ET2CE data. Obviously, the GW data can indeed effectively improve the constraints on the parameters Ω_m and H_0 (see also in Figs. 1 and 2).

Finally, we present the constraint results of the coupling constant β . By using CBS data, we have $\beta = 0.0005 \pm 0.0013$ for $\Lambda\text{CDM1} + \sum m_\nu$ model, and $\beta = 0.0230^{+0.0470}_{-0.0480}$ for $\Lambda\text{CDM2} + \sum m_\nu$ model. It is shown

TABLE I. Fitting results for the $\Lambda\text{CDM}+\sum m_\nu$ models by using the CBS data and the CBS+GW data. We quote $\pm 1\sigma$ errors, but for the parameters that cannot be well constrained, we quote the 95.4% CL upper limits. Here, H_0 is in units of $\text{km s}^{-1} \text{Mpc}^{-1}$ and $\sum m_\nu$ is in units of eV.

Parameter	$\Lambda\text{CDM}+\sum m_\nu$ ($Q=0$)		$\Lambda\text{CDM1}+\sum m_\nu$ ($Q_1 = \beta H\rho_c$)		$\Lambda\text{CDM2}+\sum m_\nu$ ($Q_2 = \beta H_0\rho_c$)	
	CBS	CBS+GW	CBS	CBS+GW	CBS	CBS+GW
Ω_m	$0.3097^{+0.0062}_{-0.0067}$	0.3098 ± 0.0012	0.3078 ± 0.0081	0.3076 ± 0.0013	$0.3030^{+0.0160}_{-0.0180}$	$0.3024^{+0.0048}_{-0.0041}$
σ_8	$0.8124^{+0.0128}_{-0.0088}$	$0.8136^{+0.0117}_{-0.0086}$	$0.8140^{+0.0140}_{-0.0130}$	$0.8150^{+0.0130}_{-0.0110}$	0.8190 ± 0.0200	0.8200 ± 0.0140
H_0	$67.75^{+0.52}_{-0.48}$	67.75 ± 0.05	67.92 ± 0.65	67.92 ± 0.05	$68.06^{+0.81}_{-0.82}$	68.06 ± 0.06
β	0.0005 ± 0.0013	$0.0005^{+0.0009}_{-0.0011}$	$0.0230^{+0.0470}_{-0.0480}$	$0.0230^{+0.0260}_{-0.0290}$
$\sum m_\nu$	< 0.123	< 0.103	< 0.150	< 0.143	< 0.156	< 0.140

TABLE II. The absolute and relative errors of cosmological parameters in the $\Lambda\text{CDM}+\sum m_\nu$ model and the $\Lambda\text{CDM}+\sum m_\nu$ models using the CBS and CBS+GW data combinations. Here, H_0 is in units of $\text{km s}^{-1} \text{Mpc}^{-1}$. For a parameter ξ , $\sigma(\xi)$ and $\varepsilon(\xi) = \sigma(\xi)/\xi$ represent its absolute error and relative error, respectively.

Parameter	$\Lambda\text{CDM}+\sum m_\nu$ ($Q=0$)		$\Lambda\text{CDM1}+\sum m_\nu$ ($Q_1 = \beta H\rho_c$)		$\Lambda\text{CDM2}+\sum m_\nu$ ($Q_2 = \beta H_0\rho_c$)	
	CBS	CBS+GW	CBS	CBS+GW	CBS	CBS+GW
$\sigma(\Omega_m)$	0.0065	0.0012	0.0081	0.0013	0.0170	0.0045
$\sigma(\sigma_8)$	0.0108	0.0102	0.0135	0.0120	0.0200	0.0140
$\sigma(H_0)$	0.50	0.05	0.65	0.05	0.82	0.06
$\sigma(\beta)$	0.0013	0.0010	0.0475	0.0275
$\varepsilon(\Omega_m)$	2.10%	0.39%	2.63%	0.42%	5.61%	1.49%
$\varepsilon(\sigma_8)$	1.33%	1.25%	1.66%	1.47%	2.44%	1.71%
$\varepsilon(H_0)$	0.74%	0.07%	0.96%	0.07%	1.20%	0.09%

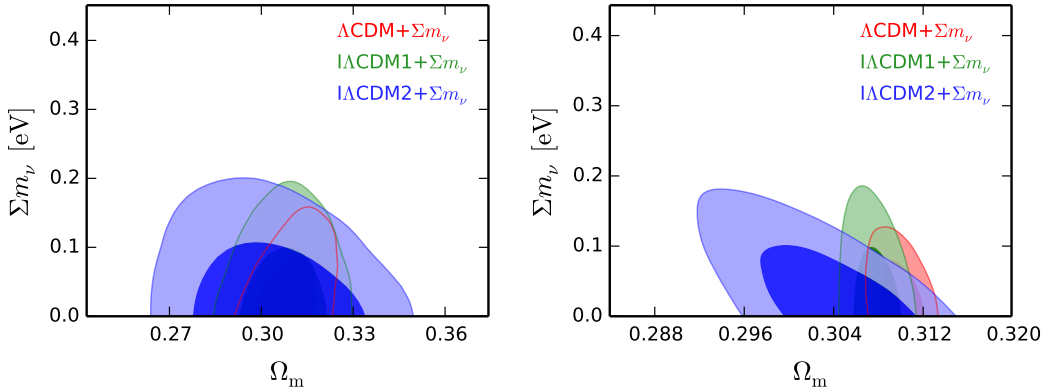


FIG. 1. Two-dimensional marginalized posterior contours (68.3% and 95.4% CL) in the $\Omega_m-\sum m_\nu$ plane for the $\Lambda\text{CDM}+\sum m_\nu$ models by using the CBS (left) and CBS+GW (right) data combinations.

that a positive value of β is slightly favored and $\beta > 0$ is only at 0.38σ and 0.48σ levels, respectively. By using CBS+GW data, we have $\beta = 0.0005^{+0.0009}_{-0.0011}$ for $\Lambda\text{CDM1}+\sum m_\nu$ model, and $\beta = 0.0230^{+0.0260}_{-0.0290}$ for $\Lambda\text{CDM2}+\sum m_\nu$ model. Thus we find that the error is slightly shrunk and now $\beta > 0$ is favored at 0.45σ and 0.79σ levels, respectively. For these two $\Lambda\text{CDM}+\sum m_\nu$ models, one can clearly see that, in the $\Lambda\text{CDM}+\sum m_\nu$

cases (with $Q_1 = \beta H\rho_c$ and $Q_2 = \beta H_0\rho_c$), $\beta = 0$ is favored within the 1σ significance range, no matter if the GW data is taken into account. Thus, in the two $\Lambda\text{CDM}+\sum m_\nu$ models, there is no evidence of a nonzero interaction. Moreover, comparing the values of β from the two data combinations, we also find that the accuracy of β is increased by 23.08% ($Q_1 = \beta H\rho_c$) and 42.11% ($Q_2 = \beta H_0\rho_c$) when adding the GW data of ET2CE in

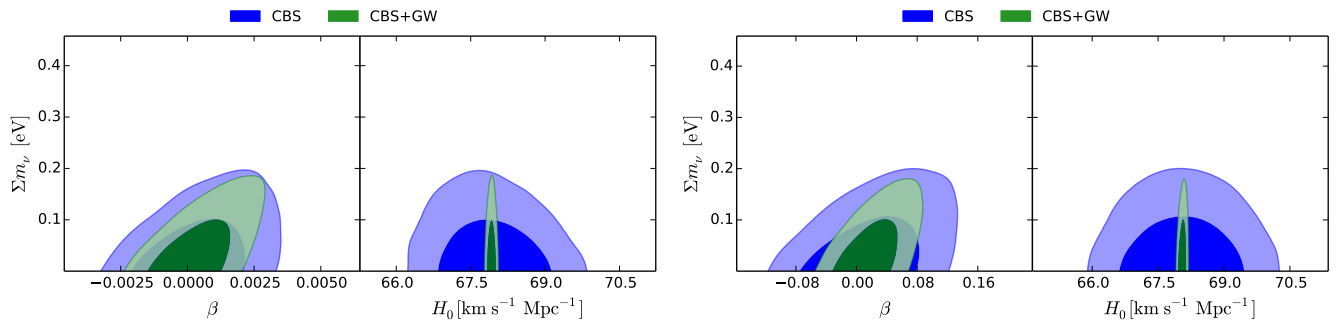


FIG. 2. Two-dimensional marginalized posterior contours (68.3% and 95.4% CL) in the $\sum m_\nu$ - β and $\sum m_\nu$ - H_0 planes for the Λ CDM1+ $\sum m_\nu$ ($Q_1 = \beta H \rho_c$) (left), and Λ CDM2+ $\sum m_\nu$ ($Q_2 = \beta H_0 \rho_c$) (right) models by using the CBS and CBS+GW data combinations.

cosmological fit (see also in Fig. 2). This indicates that the GW data of the ET2CE can also improve the constraint accuracies of coupling parameter β .

IV. CONCLUSION

In this paper, we investigate how GW standard sirens from 3G ground-based GW detectors can constrain the total neutrino mass ($\sum m_\nu$) in interacting vacuum energy models with energy transfer forms $Q_1 = \beta H \rho_c$ and $Q_2 = \beta H_0 \rho_c$. We use an extended version of the PPF approach to compute the vacuum energy perturbations within these interacting scenarios. We focus on the synergy between 3G GW detectors and a GRB detector for multi-messenger observations, specifically employing the ET2CE network as a representative for GW discussions. To evaluate the impact of GW data on the constraints of $\sum m_\nu$, we also consider existing CMB+BAO+SN data for comparison and combination.

Our analysis reveals that GW data can reduce the upper limits of $\sum m_\nu$ by 16.26%, 4.67%, and 10.26% for the Λ CDM+ $\sum m_\nu$ model, the Λ CDM1+ $\sum m_\nu$ model,

and the Λ CDM2+ $\sum m_\nu$ model, respectively. Thus, GW standard siren data from ET2CE offer a modest improvement in constraining $\sum m_\nu$ compared to CBS alone. For the derived parameters Ω_m and H_0 , incorporating GW data into the cosmological fit substantially enhances the precision: the accuracy of Ω_m improves by 73.53%–83.95%, and that of H_0 by 90%–92.68%. These significant enhancements underscore the value of including GW data from ET2CE. Moreover, based on the combined constraints from CBS and CBS+GW data, we find that GW data from ET2CE also significantly refine the accuracy of the coupling strength β .

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