Early dark energy and scalarization in a scalar-tensor model

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Abstract

We present a model in which the Gauss-Bonnet invariant holds the quintessence at a fixed point, respecting an initial Z_2 symmetry in the radiation-dominated era. This results in an early dark energy, which becomes significant around the matter-radiation equality era. However, due to Z_2 symmetry breaking, scalarization occurs, leading to a rapid reduction in the early dark energy density. The model then quickly behaves like the ΛCDM model. This scenario alleviates the Hubble tension and aligns with the assumption that the gravitational wave speed is infinitesimally close to the speed of light.

1 Introduction

Introducing a cosmological constant (Λ), into the Einstein equation provides a straightforward explanation for the observed acceleration in the later stages of our universe [1–6]. Although the Λ Cold Dark Matter model (ΛCDM) yields promising results, it encounters issues such as the cosmological constant problem [7], the coincidence problem [8–11], and the Hubble tension – the disparity between the Hubble parameter calculated by inverse distance ladder and low redshift measurements [12–15]. Consequently, one may consider alternative dark energy candidates, such as a slowly varying scalar field [16–27], to elucidate this late-time acceleration. A nearly constant scalar field with a potential $V(\phi)$ replicates the role of the cosmological constant in the background evolution, with energy density $\rho_\Lambda = V(\phi)$. However, the energy density of a component whose equation of state (EoS) parameter satisfies $w < -\frac{1}{3}$ dilutes more slowly than matter and radiation. Therefore, in the present era where matter and dark energy densities have comparable magnitudes, the relative dark energy density should have been negligible

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in the early Universe unless another mechanism, beyond the usual redshift, diminished the significant early dark energy density.

Early dark energy (EDE), which transiently becomes significant around matter-radiation equality, has recently been utilized to address the Hubble tension [28–37]. Models incorporating a dynamical scalar field as EDE have also been proposed to resolve the Hubble tension, as seen in [29–31]. EDE reduces the sound horizon r_s, leading to a larger present Hubble parameter value calculated as $H_0 \sim \frac{\theta_*}{r_s}$, where θ_* is the angular size on the last scattering surface determined from the first cosmic microwave background peak. An EDE, lowering the sound horizon by $\sim 7\%$, may alleviate the Hubble tension [33, 38–40]. Despite EDE garnering significant attention in recent years, the exploration of EDE predates the emergence of the Hubble tension problem, as evidenced by studies such as [41–46]. An EDE model with an equation of state (EoS) parameter $w = -1$ was employed in [44] to investigate the absorption of Cosmic Microwave Background (CMB) photons by the $21cm$ hyperfine transition of neutral hydrogen, a phenomenon reported by the Experiment to Detect the Global Epoch of Reionization Signature (EDGES) collaboration. EDE can lead to an earlier decoupling of gas temperature from radiation temperature. In [45], using a phenomenological parametrization of dark energy density across different eras, an upper bound for relative EDE density was suggested, with $\Omega_d < 0.06$. In another study [46], employing a parameterized energy model, the authors found an upper limit of $\Omega_d < 2.6\%$ during the radiation-dominated era and $\Omega_d < 1.5\%$ within the redshift range $z \in (100, 1000)$. It is essential to note that these papers distinguish between EDE and late dark energy based on a phenomenological parameterized approach, with both contributing separately to the total density.

An alternative approach to discussing cosmic positive acceleration involves modifying Einstein's theory of gravity by introducing geometric terms, such as the Gauss-Bonnet (GB) invariant, into the action. In four dimensions, the GB invariant only contributes a surface term and does not alter the Einstein equation. However, when coupled with exotic fields like quintessence, it induces significant and intriguing effects on cosmic evolution, particularly late-time acceleration and super-acceleration [47–56]. Recently, constraints placed on the gravitational wave speed have raised doubts about the direct influence of the GB model on cosmic evolution at low redshifts [57–62]. The scalarization of black holes and neutron stars, within the context of the scalar-GB model, has also garnered considerable attention. In a scalar-tensor model comprising a scalar field coupled to the GB invariant, neutron stars and black holes could exhibit scalar fields and behave differently compared to standard general relativity (GR). The scalarized solution may be triggered by a tachyonic instability [63–66].

By relating the three seemingly unrelated aforementioned topics, i.e. EDE, scalarization and the gravitational wave speed, this paper presents a model where the quintessence-GB coupling establishes conditions for an initial stable fixed-point solution, representing an EDE. The relative density of this EDE becomes significant around the matter-radiation equality era and then rapidly decreases to align with the usual ΛCDM model. This reduction is attributed to a tachyonic instability induced by radiation and matter dilution, triggering quintessence evolution and its emergence through a scalarization-like mechanism. We demonstrate that constraints on the gravitational wave speed associated with the GB coupling align with this scenario. Note that in our study, we assume that the gravitational wave speed is very close to the light speed and lays in the domain reported in [60].

The paper is structured as follows: Section 2 provides a detailed presentation of the model, emphasizing how the dilution of matter and radiation density initiates quintessence evolution through its coupling with the GB term, resulting in the aforementioned scenario. We explore stability conditions and investigate the impact of gravitational wave speed constraints on quintessence activation. To illustrate the possibility of the scenario, we present a specific example showing how the model works while fitting the observational data. In Section 3, we provide concluding remarks.

We use units $\hbar = c = 1$.

2 Early dark energy and scalarization in Z_2 symmetric Gauss-Bonnet model

We consider the following action describing a quintessence field ϕ coupled to the GB term [53]:

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2 R}{2} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V - \frac{1}{2}f\mathcal{G}\right) + S_m. \tag{1}$$

$M_P = 2.4\times 10^{18}GeV$ is the reduced Planck mass. The quintessence, ϕ, is coupled to the GB term through an even coupling function $f := f(\phi^2)$. The even quintessence potential is $V := V(\phi^2)$. Therefore the action has a Z_2 symmetry. The GB invariant is given by

$$\mathcal{G} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2. \tag{2}$$

We consider a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) space-time,

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \tag{3}$$

filled with the quintessence, cold dark matter, baryonic matter, and radiation. Friedmann equations read

$$\begin{aligned} 3M_P^2H^2 &= \rho_d + \rho_r + \rho_m \\ 2M_P^2\dot{H} &= -\rho_d - P_d - \rho_m - \frac{4}{3}\rho_r. \end{aligned} \tag{4}$$

H is the Hubble parameter, which in terms of the scale factor a, is given by $H = \frac{\dot{a}}{a}$. ρ_m and ρ_r are the sum of baryonic and cold dark matter, and radiation energy densities respectively. Theses barotropic fluids satisfy the continuity equations

$$\begin{aligned} \dot{\rho_m} + 3H\rho_m &= 0 \\ \dot{\rho_r} + \frac{4}{3}\rho_r &= 0. \end{aligned} \tag{5}$$

The effective dark energy density, and pressure are given by

$$\rho_d = \frac{1}{2}\dot{\phi}^2 + V + 12H^3\dot{f}, \tag{6}$$

and

$$P_d = \frac{1}{2}\dot{\phi}^2 - V - 8H^3\dot{f} - 4H^2\ddot{f} - 8H\dot{H}\dot{f}, \tag{7}$$

respectively. For the effective dark energy we have the following continuity equation:

$$\dot{\rho_d} + 3H(P_d + \rho_d) = 0, \tag{8}$$

which is equivalent to the following equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + V^{eff.}_{,\phi} = 0, \tag{9}$$

where we have defined [67]

$$V^{eff.}_{,\phi} := V_{,\phi} + 12H^2(H^2 + \dot{H})f_{,\phi}. \tag{10}$$

The subscript $,\phi$ denotes derivatives with respect to ϕ. It is evident from (9) that the quintessence evolution is influenced by the GB invariant. We use this to introduce a scalarization scenario in which the system acquires a nontrivial quintessence solution through a tachyonic instability: We construct the model such that, during early eras, the GB invariant provides a stable fixed point solution :$\phi = 0$ respecting the Z_2 symmetry of the action. As $V_{,\phi}(0) = f_{,\phi}(0) = 0$, $\phi = 0$ is a trivial solution to (9). From (4), we find out that for this solution the Friedmann equations reduce to ordinary (non-modified) ones

$$\begin{aligned} 3M_P^2H^2 &= V(0) + \rho_r + \rho_m \\ 2M_P^2\dot{H} &= -\rho_m - \frac{4}{3}\rho_r. \end{aligned} \tag{11}$$

In this epoch dark energy density is given by $\rho_d = V(0)$, playing the role of a cosmological constant. As $V^{eff.}_{,\phi}(0) = 0$, this solution is stable (unstable) when $V^{eff.}_{,\phi\phi}(0) > 0(< 0)$, corresponding to the minimum (maximum) of the effective potential. Hence the quintessence stability at $\phi = 0$, depends on the sign of $V^{eff.}_{,\phi\phi}(0)$ given by

$$M^2_{eff.} \equiv V^{eff.}_{,\phi\phi}(0) = -\frac{2}{3M_P^4}(\rho_r + \rho_m - 2V(0))(\rho_r + \rho_m + V(0))f_{,\phi\phi}(0) + V_{,\phi\phi}(0). \tag{12}$$

We define the critical scale factor, denoted as a_c, by $M^2_{eff.}(a_c) = 0$ and $\frac{dM^2_{eff.}}{da}(a_c) < 0$. For $\phi = 0$ to be a stable solution before a_c, it is necessary to have $V^{eff.}_{,\phi\phi}(0) > 0$ for $a < a_c$, implying $f_{,\phi\phi}(0) < 0$. $M^2_{eff.}$ can be considered as the effective mass squared of small fluctuations around $\phi = 0$ for $a \leq a_c$. Additionally we aim to violate the stability throughout the evolution of the Universe to obtain a non trivial quintessence solution. This can be achieved by choosing $V_{,\phi\phi}(0) < 0$, such that for $a \geq a_c$, the quintessence becomes tachyonic: $M^2_{eff.} \leq 0$. In this situation $\phi = 0$ becomes an unstable point at the local maximum of the effective potential. Through a small fluctuation, the quintessence rolls down it effective potential, and its evolution begins and the system gets a non-trivial solution that no longer respects the Z_2 symmetry. The emergence of the scalar field also modifies the standard Friedmann equations from (11) to (4), where the GB term directly influences the expansion of the Universe. Note that to determine the behavior of the scalar field for $a > a_c$, we have to solve one of the Friedmann equations (4), and the equations of motions (5), (9).

As we have assumed $V_{,\phi\phi}(0) < 0$, the quintessence evolution along the effective potential decreases V, leading to a reduction in early dark energy (EDE) density. In general, describing this reduction through a process other than redshift, as we will explicitly show, is crucial for a model that considers non-negligible EDE: Using the relation

$$\frac{1-\Omega_d}{\Omega_d}\frac{\rho_d}{\rho_{d0}} = \left(\frac{1-\Omega_{d0}}{\Omega_{d0}}\right)\left(\frac{\Omega_{m0}a^{-3} + \Omega_{r0}a^{-4}}{\Omega_{m0} + \Omega_{r0}}\right), \tag{13}$$

where $\Omega_i = \frac{\rho_i}{3M_P^2H^2}$ represents the relative density, and the subscript "0" denotes the present time specified by $a = 1$. According to [68], we can set $\Omega_{m0} \simeq 0.32, \Omega_{r0} \simeq 8.4 \times 10^{-5}$. For the early universe, where $a \ll 1$, (13) implies $\frac{1-\Omega_d}{\Omega_d}\frac{\rho_d}{\rho_{d0}} \gg 1$. For example by taking $a \lesssim 10^{-4}$ (before matter radiation equality), we obtain $\frac{1-\Omega_d}{\Omega_d}\frac{\rho_d}{\rho_{d0}} \sim 5 \times 10^{10}$. If the dark energy were only a cosmological constant, i.e., $\rho_d = \rho_{d0}$, its contribution in the total energy at a would be $\Omega_d \simeq 10^{-11}$, which is negligibly small. Even for a dark energy with an effective EoS parameter $w_d^{ef} < -\frac{1}{3}$ we would have $\Omega_d \lesssim 3 \times 10^{-4}$. Therefore, restricting dark energy dilution to the redshift results in a negligible EDE. Hence, to reconcile non-negligible EDE and late Dark energy, an additional mechanism, beyond the redshift, is required to reduce the significant EDE density by several orders of magnitude. In our scenario, this reduction is achieved through the activation of the quintessence via the discussed scalarization. For this purpose, *we choose the potential as a decreasing function of* ϕ^2. Unlike models where quintessence activation drives cosmic acceleration, in our case, its activation leads to a reduction in dark energy density. Such a formalism is necessary in models which alleviate the Hubble tension by using a temporarily significant EDE around the matter-radiation equality.

It is important to note that, owing to the presence of the friction term, the evolution of the scalar field does not begin immediately after $a = a_c$. Therefore, for the desired scenario of having significant dark energy and its subsequent reduction around the matter-radiation equality era, as sought in alleviating the Hubble tension through EDE, the critical scale factor a_c must be within the radiation-dominated era. The critical scale factor, a_c, is obtained by solving the equation

$$V_{,\phi\phi}(0) = \frac{2}{3M_P^4}(\rho_{r0}a_c^{-4} + \rho_{m0}a_c^{-3} - 2V(0))(\rho_{r0}a_c^{-4} + \rho_{m0}a_c^{-3} \\ +V(0))f_{,\phi\phi}(0), \tag{14}$$

where ρ_{i0} is the energy density at the present time ($a_0 = 1$). As evident from (14), smaller values of a_c result in tinier values of $f_{,\phi\phi}$, which, as will see, is in favor of the GWS constraint. By considering the perturbed space time

$$ds^2 = -dt^2 + a^2(t)(t_{ij} + \delta_{ij})dx^i dx^j, \tag{15}$$

one can obtain a second-order action for divergenceless and traceless t_{ij}, and obtain GWS. For the model (1) which is a special case of Horndeski's theories, the gravitational wave speed (GWS) is obtained as [69, 70]

$$c_T^2 = \frac{4f_{,\phi\phi}\dot{\phi}^2 + 4f_{,\phi}\ddot{\phi} - M_P^2}{4Hf_{,\phi}\dot{\phi} - M_P^2} = \frac{4\ddot{f} - M_P^2}{4H\dot{f} - M_P^2}. \tag{16}$$

According to [60], this speed is constrained at the late time for the redshift $z < 0.009$ as:

$$-3 \times 10^{-15} \leq \frac{c_T}{c} - 1 \leq 7 \times 10^{-16}, \tag{17}$$

where c is the light speed. To satisfy (17), generally we must take $4H\dot{f} \ll M_P^2$ and $\ddot{f} \ll M_P^2$, which implies that a nearly constant ϕ with infinitesimal couplings to the GB term are required. For a quadratic f a tiny coupling favours that a_c is in the radiation era. A nearly constant quintessence at the late time is realized if the potential becomes nearly flat: $V(\phi) \simeq \Lambda$, $V_{,\phi}^2 \ll H^2 V$, $V_{,\phi\phi} \ll H^2$. In such a situation the quintessence slowly rolls $\frac{1}{2}\dot{\phi}^2 \ll V(\phi)$. Note that $4H\dot{f} \ll M_P^2$ and $\ddot{f} \ll M_P^2$ results in that the GB term has no direct influence in cosmic evolution in the late time. This can be seen from the Friedmann equations, rewritten as [71]

$$\begin{aligned}
&3(M_P^2 - 4H\dot{f})H^2 = \frac{1}{2}\dot{\phi}^2 + V + \rho_r + \rho_m \\
&2(M_P^2 - 4H\dot{f})\dot{H} = -\dot{\phi}^2 - (M_P^2 - 4H\dot{f})(c_T^2 - 1) \\
&-\rho_m - \frac{4}{3}\rho_r .
\end{aligned} \tag{18}$$

Smallness of the quintessence-GB coupling also helps to the stability of the model, against ghost and Laplacian instabilities caused by scalar and tensor perturbations, which requires that the following inequalities hold [71]:

$$\begin{aligned}
&q_T := M_P^2 - 4f_{,\phi}\dot{\phi}H > 0 \\
&q_s := 2(q_T + 24H^4 f_{,\phi}^2) > 0 \\
&c_s^2 = \frac{2q_T - 16H^4 f_{,\phi}^2(2 + 6w + c_T^2)}{q_s} > 0.
\end{aligned} \tag{19}$$

$w = -1 - \frac{2}{3}\frac{\dot{H}}{H^2}$ is the Universe effective equation of state.

A recent motivation to consider the EDE is to alleviate the Hubble tension. This can done by an EDE component which becomes significant around the matter-radiation equality era and then decreases quickly through a mechanism other than the redshift, similar to what happens in our proposal. This decreases the sound horizon and results in a larger value for the present Hubble parameter through the relation $H_0 = \frac{\theta_*}{r_s}$, where θ_* is the angular size on the last scattering surface determined from the first cosmic microwave background peak. At the last scattering, the comoving sound horizon denoted r_s, is derived as [46], [31]

$$\begin{aligned}
r_s &= \int_{z_*}^{\infty} \frac{c_s(z)}{H(z)} dz = \int_{z_*}^{\infty} \frac{c(z)}{\sqrt{\frac{1}{3M_P^2}\sum_i \rho_i}} dz \\
&= \frac{1}{H_0}\int_{z_*}^{\infty} dz \frac{c(z)}{\sqrt{\Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \frac{\rho_d}{3M_P^2 H_0^2}}},
\end{aligned} \tag{20}$$

$\Omega_m = \Omega_{dm0} + \Omega_{b0}$, where Ω_{b0} and Ω_{dm0} are relative densities of dark matter and baryonic matter at $a = 1$, respectively. z_* is the redshift of the last scattering, and $c_s(z)$ is the sound speed in the baryon-photon fluid, given by [72]:

$$c_s(z) = \frac{1}{\sqrt{3}}\left(\frac{3}{4}\frac{\Omega_{b0}}{\Omega_{r0}}\frac{1}{1+z} + 1\right)^{-\frac{1}{2}} \tag{21}$$

Adding an EDE, increases $H(z)$, and consequently decreases r_s. This ameliorates the Hubble tension [38]. If the EDE density increases effectively the energy density as $\sum \rho \to (1+\gamma)^2 \sum \rho$ in (20), then $H \to (1+\gamma)H$, and r_s decreases by $\frac{100\gamma}{1+\gamma}\%$, e.g for $\gamma = 0.075$, r_s decreases by $\simeq 7\%$ [38].

We continue our study with a specific example to illustrate how the model works. We choose the potential:

$$V = V_0 \exp(-\frac{1}{2}\mu^2\phi^2) + \Lambda, \tag{22}$$

which is a decreasing function of ϕ^2, consisting of a constant Λ playing the role of a cosmological constant energy density at the late time, and a steep part which decreases rapidly for $\mu^2\phi^2 \gtrsim 1$. We adopt a quadratic coupling:

$$f = -\frac{1}{2}\alpha^2\phi^2. \tag{23}$$

We employ dimensionless parameters

$$\begin{aligned} &\hat{H} = \frac{H}{H^*},\ \hat{t} = H^* t,\ \hat{\mu} = M_P\mu,\ \hat{\alpha} = H^*\alpha,\ \hat{\rho} = \frac{\rho}{M_P^2 H^{*2}}, \\ &\hat{V}_0 = \frac{V_0}{M_P^2 H^{*2}},\ \hat{\Lambda} = \frac{\Lambda}{M_P^2 H^{*2}} \end{aligned} \tag{24}$$

where H^* is a mass scale. We set the initial conditions in the radiation dominated epoch at $a = 1/37500$ as

$$\hat{\phi} = 0,\ \hat{\phi}' = 10^{-16},\ \hat{\rho}_m = 10^{13},\ \hat{\rho}_r = 10^{14} \tag{25}$$

a prime denotes derivative with respect to the dimensionless time $\hat{t}$. We select the following parameters

$$\hat{\alpha} = 10^{-7},\ \hat{\mu} = 30,\ \hat{\Lambda} = 0.04,\ \hat{V}_0 = 4 \times 10^8 \tag{26}$$

We have selected the parameters and initial conditions such that initially the relative EDE is insignificant $\Omega_d \simeq 10^{-6}$, where we have denoted the relative density by $\Omega_i \equiv \frac{\rho_i}{3M_P^2 H^2}$. Note that the parameters and initial conditions satisfy (14). (14) implies that a_c has to be taken in the radiation dominated era to have a very small $\hat{\alpha}$. Note that the small value of $\hat{\alpha}$ along with the slowness of the quintessence rolling at the late time which is a consequence of the chosen potential, ensures compliance with the GWS constraint. $\hat{\Lambda}$ is chosen such that the model gives the correct dark energy density at our present era.

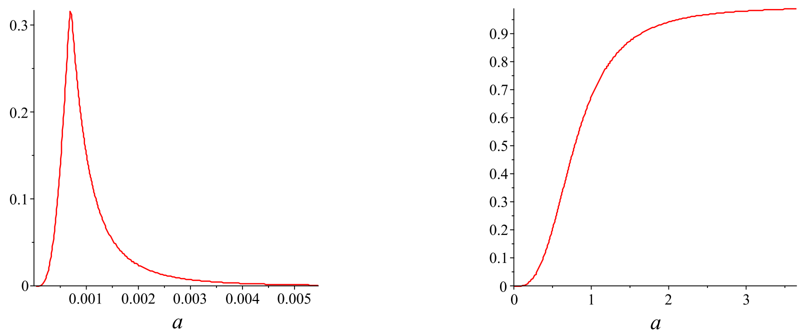
Using the Friedmann equation (4), the continuity equations (5), and the equation of motion (9), we can depict numerically the behavior of the system. We specify by $a = 1$, $(z = 0)$, our present time. The parameters are adjusted by confronting the results at $a = 1$ with [68]. The evolution of the quintessence is depicted in Fig.(1)

(a) Beginning of the evolution (b) Late time evolution

Figure 1: Quintessence evolution versus the scale factor

Initially, we set ϕ to be equal to zero. This represents a trivial solution for the quintessence equation of motion. During the radiation era, the quintessence becomes tachyonic, but its evolution and emergence actually commence around the time of matter-radiation equality, thanks to the friction term. At the stable point $\phi = 0$, we have $V(0) = V_0 + \Lambda$, which serves as an early cosmological constant energy density. When $\mu^2\phi^2 \gg \ln(\frac{V_0}{\Lambda})$, the potential becomes nearly flat: $V(\phi) \simeq \Lambda$, and it takes on the role of the actual cosmological constant. It is only between these two stages that the GB term may directly impact the Friedmann equations.

The relative density of dark energy is depicted in Fig.(2), showing that the EDE is negligible for large redshifts in the radiation era. It increases until the quintessence becomes dynamic due to the tachyonic instability, and then due to the steep potential decreases quickly and behaves as the cosmological constant in ΛCDM model. In this figure we have $\Omega_d(a = 1) = 0.68$, which is compatible with the value reported in [68], on the base of ΛCDM model from Planck CMB power spectra in combination with CMB lensing (TT,TE,EE+lowE+lensing 68% limit), as $\Omega_d = 0.6847 \pm 0.0073$.



(a) Early universe (b) Late time

Figure 2: Relative dark energy density versus the scale factor

The deceleration parameter, $q = -1 - \frac{\dot{H}}{H^2}$ is depicted in Fig.(3), showing that, the Universe entered the positive acceleration phase at $z \simeq 0.6$. In the present era, $q(a=1) = -0.521$ which corresponds to an effective EoS parameter $w(a=1) = -0.6807$ for the Universe. This is compatible with [68], where dark energy EoS parameter is reported as $w_d = -1.03 \pm 0.03$ (note that in our era $w \simeq \Omega_d w_d$).

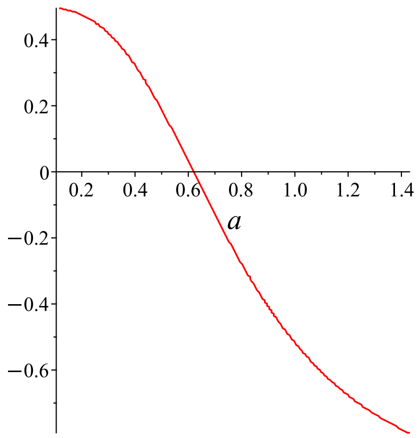


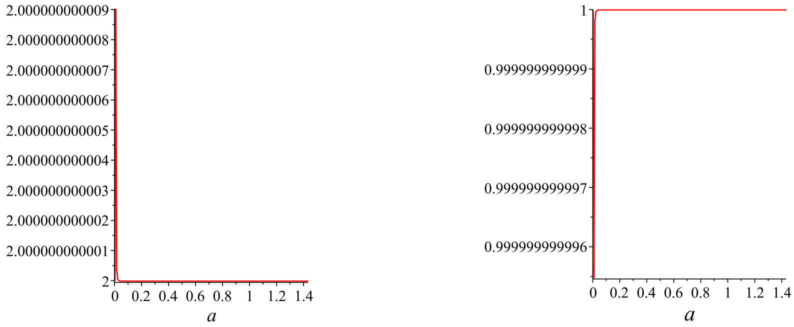
Figure 3: Deceleration parameter in terms of the scale factor

To obtain an estimation of the sound horizon one can numerically solve the set of equations (4),(5),(9) along with the additional equation

$$\frac{d\psi}{dt} = \frac{1}{\sqrt{3}a}\frac{1}{\sqrt{\frac{3\rho_b}{4\rho_r}+1}}, \tag{27}$$

which is the same as $\frac{d\psi}{dz} = -\frac{c_s(z)}{H(z)}$, where c_s is given by (21). In this manner one obtains $\psi(z)$, from which the sound horizon is derived as $r_s = \psi(\infty) - \psi(z_*)$. Repeating the same computation while ignoring EDE one obtains $r_s^{\Lambda CDM}$. Taking the last scattering redshift at $z_* = 1100$, for the above example we obtain $\frac{r_s^{\Lambda CDM} - r_s}{r_s^{\Lambda CDM}} \simeq 0.06$. This is compatible with the results reported by SHOES team $H_0 = 72.1 \pm 2.0 km/s/MPC$, and reported in [68] (TT,TE,EE+lowE+lensing+BAO 68% limit) $H_0 = 67.66 \pm 0.42 km/s/MPC$.

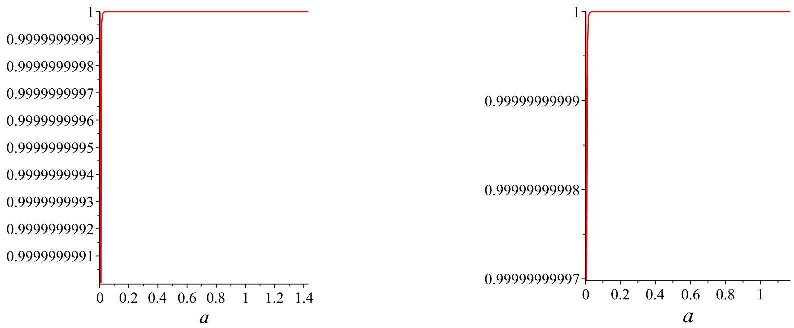
$\hat{q}_s := \frac{q_s}{M_P^2}$, and $\hat{q}_T := \frac{q_T}{M_P^2}$ are depicted in Fig.(4), showing the model is free from ghosts: $\hat{q}_s > 0$, and $\hat{q}_T > 0$.



(a) q_s versus the scale factor (b) q_T versus the scale factor

Figure 4: q_T and q_s versus the scale factor

We have also depicted $c_s^2 > 0$ as shown in Fig.(5), and also $c_T^2 > 0$ which imply that the model have no Laplacian instability.



(a) c_s^2 in terms of the scale factor (b) c_T^2 in terms of the scale factor

Figure 5: c_T^2 and c_s^2 versus the scale factor

These figures demonstrate that the parameters closely align with their ΛCDM counterparts, $\hat{q}_T = 1$, $\hat{q}_s = 2$, $\hat{c}_s^2 = \hat{c}_T^2 = 1$, except for a brief interval where the EDE becomes significant. However, as they remain positive, this deviation does not result in instability. Finally $1 - c_T$ is depicted in Fig.(6),

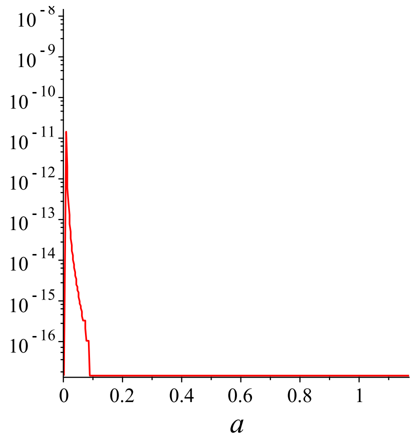


Figure 6: $1 - c_T$ in terms of the scale factor

which shows that the model respects the constraint (17) at low redshifts.

3 Conclusion

We introduced a scalar-tensor model consisting of a quintessence, ϕ, with an even potential $V(\phi^2)$ coupled to the Gauss-Bonnet (GB) invariant through an even function $f(\phi^2)$. We showed provided that $f_{,\phi\phi}(0) < 0$, the quintessence is trapped in the stable fixed point $\phi = 0$ for scale factors less than a critical value $a < a_c$ resulting in a standard cosmological model with an initial cosmological constant. As the universe expands the relative dark energy density increases, and the quintessence effective mass squared, which depends on density, decreases. Provided that $V_{,\phi\phi}(0) < 0$, at $a = a_c$ the quintessence becomes tachyonic and gains a non trivial solution, causing a rapid (for a steep potential) decrease in relative initial dark energy density. This is somehow similar to the scalarization where the scalar field gains a nontrivial solution when becomes tachyonic, and by its emergence the cosmological model deviates from the standard one derived from general relativity (GR). The GWS constraint quoted in [59], favors a vanishingly small GB-quintessence coupling $f_{,\phi\phi}(0)$ which for a quadratic coupling is satisfied by taking a_c in the radiation dominated era. The constraint also favors a very slowly rolling quintessence at the late time, therefore we have chosen a potential which becomes eventually flat. In this way the constructed model describes a dynamical dark energy component which becomes significant in matter radiation equality era and then decreases rapidly and behaves as a late time cosmological constant, which is much less than the initial cosmological constant. This model suggests a mechanism by which the dark energy could have a higher early value than what is obtained from the redshift calculation alone, but decreases rapidly after the matter radiation equality era and so may also alleviate the Hubble tension problem.

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