The Width of an Electron-Capture Neutrino Wave Packet

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We expand on the methodology outlined in previous work that predicted the width of an antineutrino wave packet emerging from a beta-decaying nucleus, to the case of a neutrino from electron capture decay. Based on this result, we also respond to a recent Beryllium Electron capture in Superconducting Tunnel junctions Experiment (BeEST) paper which utilizes this previous work in forming their measurement of the neutrino wave packet width. According to our interpretation, the direct limit on the neutrino wave packet width from electron capture decay ($e^- + {}^7\mathrm{Be} \rightarrow {}^7\mathrm{Li} + \nu_\mathrm{e}$) using the BeEST analysis should map to $\sigma_{\nu,x} > 6.2\,\mathrm{pm}$ while our theoretical prediction is $\sigma_{\nu,x} \sim 2.7\,\mathrm{nm}$.

I. INTRODUCTION

The coherent width of a neutrino wave packet is a quantum mechanical quantity that is expected to impact neutrino oscillation phenomenology at long baselines [3–9], through coherence loss between interfering mass eigenstates. This width is an emergent property, that is predictable from first principles [1, 10], with dependencies on the neutrino energy, initial flavor and production process. A recent paper from the BeEST collaboration [2] aims to set direct limits on the size of the neutrino wave packet emerging from electron capture via precision spectroscopy of the entangled recoil. The extremely precise energy resolution of superconducting tunnel junctions make them the ideal sensors for this type of measurement [11, 12], and since this is the first reported measurement of this kind, it represents an exciting advance in this area.

While it is difficult to imagine neutrino wave packet effects impacting oscillation observables in near-term neutrino experiments [9, 10], the prospect of sensitivity to the neutrino wave packet width based on measurements of the recoiling daughter nucleus, as in BeEST, motivates predictions for this quantity. Our previous work [1] presents an explicit calculation of the wave packet width of an antineutrino emerging from nuclear beta decay. We emphasize in that paper the role of the initial state delocalization scale in determining the wave packet width. Here, we apply this methodology to the case of electron capture. At present, the BeEST measurement extraction relies on misinterpretations of our previous work [1] for both calculations used to derive a neutrino coherent width limit based on the energy of the recoiling daughter nucleus in $e^- + {}^7\mathrm{Be} \rightarrow {}^7\mathrm{Li} + \nu_\mathrm{e}$ decay, and comparison between their measurement and our "prediction" of the neutrino width. This paper aims to point out the discrepancies we observe, and to provide improved estimates.

II. PREDICTED WIDTH OF AN ELECTRON CAPTURE NEUTRINO

The initial states in beta decay and electron capture are in fact quite different: in beta decay, a single nucleon that is localized within a nucleus transforms via weak decay into a final state consisting of an entangled system of electron, antineutrino, and nuclear recoil. In electron capture, two initial particles, atomic electron and parent nucleus, interact to yield a two particle final state of neutrino and daughter nucleus. There is a relative momentum uncertainty between the two initial-state particles in electron capture that corresponds to a distance uncertainty on atomic scales. Our prescription can be applied to electron capture, but it must account for both initial state particles and conserve energy and momentum in the process.

As a brief summary, our proposed prescription for calculating neutrino wave packet widths is as follows:

1. Write down the initial state, accounting for as many entangled microscopic degrees of freedom as desired. If the recipe is followed sufficiently carefully, there will be no ambiguity about which degrees of freedom / distance scales ultimately act to determine the neutrino wave packet width. Any extra recursive delocalization ultimately disappears when forming the reduced density matrix;

2. Conserve energy and momentum in the decay to form the final state density matrix for the full entangled system;

3. Trace out everything that is not the neutrino, to form the neutrino reduced density matrix. This will account for the localizing influence of everything except the neutrino that is entangled with it;

4. The full neutrino oscillation phenomenology can be obtained from the neutrino reduced density matrix. The quantity playing the role of the coherent wave packet width in this object is the off-diagonal width of the position-space density matrix.

We now proceed to follow this recipe for electron capture.

We will consider an initial state comprised of an electron and a nucleus, with momentum-space wave function

$$\Phi_i = \int d^3P \int d^3p \ \Psi(\vec{P})\psi(\vec{p})|N(\vec{p}_N)\rangle \otimes |e(\vec{p}_e)\rangle. \tag{1}$$

Here the coordinates chosen are the sum and difference of the momenta,

$$p = p_{e/\nu} - p_N, \quad P = p_{e/\nu} + p_N. \tag{2}$$

A note is in order about this coordinate choice. For the initial state, an alternative coordinate choice would be the center-of-mass and reduced-mass coordinates

$$q = \mu \left(\frac{p_e}{m_e} - \frac{p_N}{m_N} \right), \quad Q = p_e + p_N. \tag{3}$$

This coordinate system has the convenient feature that variable q satisfies the Schrödinger equation with mass μ, and it is the one that is typically used to find the electron orbitals of the atom in the center of mass frame. On the other hand, since in the final state the neutrino is relativistic, these coordinates eventually become a burden. It is important to observe, however, that in the center of mass frame where $p_e = -p_N$, there is an equivalence $q = p$ and $Q = P$. As such, we can assert that the center of mass wave function $\phi(p)$ will be well approximated by $\phi(q)$, as long as we restrict our calculations to frames where the neutrino carries far more of the energy than the nucleus. The frames we are interested in will always satisfy this requirement.

The electron capture process is expressed in terms of initial N_i and final N_f nuclei as

$$N_i + e \to N_f + \nu_e. \tag{4}$$

For a sufficiently long-lived nucleus, we can assume energy and momentum to be conserved in the decay, for practical purposes. For a sufficiently short lifetime, an intrinsic energy uncertainty is also required by the energy-time uncertainty principle, but this will be a small contribution for the ^{7}Be system. Thus we can schematically consider this decay as being mediated by the following quantum operator, which conserves momentum in the decay, and also that energy will be conserved in the decay amplitude,

$$\mathcal{O} = \int d^3p_i d^3p_e d^3p_f d^3p_\nu \ a_{N_i}^{p_i} a_e^{p_e} a_{N_f}^{p_f \dagger} a_\nu^{p_\nu \dagger} \delta^3(p_i + p_e - p_f - p_\nu). \tag{5}$$

Operator 5 conserves the center of mass momentum of the whole system, but changes the relative momentum between the lepton and nucleus. Thus the final state system wave function Φ_f must be expressible as

$$\Phi_f = \int d^3P \int d^3p \ \Psi(\vec{P}) \left[\phi(\vec{p})|N_f(\vec{p}_{N_f})\rangle \otimes |\nu(\vec{p}_\nu)\rangle \right]. \tag{6}$$

Where Ψ is the same as it was before (the total delocalization wave function of the center of mass in momentum space, which we assume to be a Gaussian with width $\sigma_P = 1/2\sigma_X$) and ϕ is a new center of mass wave function that is derivable from ψ through kinematic considerations.

For capture from the K shell the relative momentum wave function is spherically symmetric $\psi(\vec{p}) = \psi(p)$. Assuming nothing in the initial state to be spin-polarized, the final state will also be spherically symmetric in the center of mass frame $\phi(\vec{p}) = \phi(p)$. Since the nuclei are both non-relativistic, energy conservation requires that

$$\left(m_{N_i} + \frac{p_{N_i}^2}{2m_{N_i}} \right) + \sqrt{p_e^2 + m_e^2} = \left(m_{N_f} + \frac{p_{N_f}^2}{2m_{N_f}} \right) + |p_\nu|. \tag{7}$$

If we consider this expression in the center of mass frame then we may write

$$p_e = -p_{N_i} = p_i, \quad p_\nu = -p_{N_f} = p_f, \tag{8}$$

$$p_i^2 \left(\frac{1}{2m_e} + \frac{1}{2m_{N_i}} \right) + M = \frac{p_f^2}{2m_{N_f}} + |p_f|. \tag{9}$$

Here $M = m_{N_f} - m_{N_i} - m_e$. The nucleus is much heavier than both the electron mass and the relevant kinetic energies in the problem, so we can to a reasonable degree of approximation neglect the $1/m_N$ terms, to give the following relation between initial and final center of mass momenta,

$$|p_f| = M + \frac{p_i^2}{2m_e}. \tag{10}$$

This approximation is equivalent to the statement that in the center of mass frame where momentum in the decay is shared equally, the electron or neutrino must carry essentially all of the kinetic energy in the initial or final state, respectively. A more accurate relation can be obtained using the quadratic solution to Eq. 9 and following the steps below, adding complexity but with negligible impact on the final answer.

We consider here only the K shell electron capture, so the relevant initial wave function is approximately the 1S electron wave function with atomic number $Z = 4$,

$$\psi(\vec{x}) = \frac{Z^{3/2}}{a_0^{3/2}} \frac{1}{\sqrt{\pi}} e^{-Zr/a_0}, \tag{11}$$

where a_0 is the Bohr radius. To move into momentum space we Fourier transform in 3D:

$$\psi(\vec{p}) = \int d^3x e^{i\vec{p}\cdot\vec{x}} \psi(\vec{x}) = \int d\cos\theta \, d\phi \, dr r^2 e^{i\vec{p}\cdot\vec{x}} \psi(\vec{x}). \tag{12}$$

Choosing coordinates for the $\vec{x}$ integral such that $\vec{p}$ points along $\theta = 0$,

$$\psi(\vec{p}) = 2\pi \int d\cos\theta \, dr r^2 e^{ipr\cos\theta} \psi(r) \tag{13}$$

$$= \frac{8}{\sqrt{\pi}} \frac{Z^{3/2}}{a_0^{3/2}} \frac{a_0^3 Z}{\left(a_0^2 p^2 + Z^2\right)^2} \equiv \mathcal{A} \left(1 + \frac{p_i^2}{\left(Z/a_0\right)^2}\right)^{-2}. \tag{14}$$

We have collected several constants into a normalization factor $\mathcal{A}$ that we can restore later if needed. To obtain ϕ in terms of ψ, consider that each initial momentum magnitude p_i maps to a unique final momentum magnitude p_f. Thus their wave function amplitudes should also map, as

$$\phi(p_f) = f[\psi(p_i[p_f]), p_f]. \tag{15}$$

Getting the precise shape for this final state wave function depends on whether we consider the 1D or 3D problem, but in order to satisfy our present purposes it suffices to use an example wave function with the correct mean and width. We will thus use our trusty workhorse, the Gaussian wave packet,

$$\phi(p_f) = \mathcal{B} \exp\left[-\frac{\left(p - p_{rel}\right)^2}{4\sigma_{rel}^2}\right], \tag{16}$$

where p_{rel} and σ_{rel} represent the relative momentum and width, respectively. We estimate p_{rel} and σ_{rel} by noting that peak of Eq. 14 occurs at $p_i^{mean} = 0$ with its half width at half maximum (HWHM) at $(p_i^{HWHM})^2 = \left(\sqrt{2} - 1\right)(Z/a_0)^2$. This suggests we might as a reasonable estimate take ϕ to have peak and HWHM at the kinematically related points at the corresponding neutrino momenta (with an extra factor of $\frac{1}{2}$ to account for the fact the kinematically related peak is one sided),

$$p_{rel} = M, \qquad \sigma_{rel} \sim \frac{1}{2\sqrt{2\ln 2}} \left[M + \frac{p_i^2}{2m_e}\right]_0^{p_i^{HWHM}} = \frac{\zeta Z^2}{2a_0^2 m_e}. \tag{17}$$

With $\zeta = \frac{\sqrt{2}-1}{2\sqrt{2\ln 2}} \sim 0.18$. To find the quantity representing neutrino wave-packet width using the methodology of Ref. [1], we need to calculate the off-diagonal width of the reduced neutrino density matrix. This full-system density matrix is defined as

$$\rho = |\Phi_f\rangle\langle\Phi_f|. \tag{18}$$

The neutrino reduced density matrix, from which all oscillation phenomenology can be calculated, is

$$\rho_\nu = \int dp_N \langle p_N|\rho|p_N\rangle. \tag{19}$$

Writing this out with all its ingredients, where subscripts 1,2 refer to the variables that appear on the left and right of the density matrix, and $|\nu(p)\rangle$ is a neutrino state vector with momentum p,

$$\rho_\nu = \int d^3P_1 \tilde{\Psi}(\vec{P}_1)\left[\int d^3p_1\, \tilde{\psi_f}(\vec{p}_1)\right]\int d^3P_2 \tilde{\Psi}(\vec{P}_2)\left[\int d^3p_2\, \tilde{\psi_f}(\vec{p}_2)\right]\delta(p_{N1}-p_{N2})\,|\nu(\vec{p}_{\nu 1})\rangle\langle\nu(\vec{p}_{\nu 2})| \tag{20}$$

We will carry all the Gaussian normalization factors forward in an arbitrary constant $\mathcal{N}$, which will cancel when we calculate the quantities of interest. We can easily take care of the p_N delta function,

$$= \mathcal{N}\int d^3p_{\nu 1}d^3p_{\nu 2}d^3p_N \exp\left[-\frac{(p_{\nu 1}+p_N)^2}{4\sigma_P^2} - \frac{(p_{\nu 2}+p_N)^2}{4\sigma_P^2} - \frac{(p_{\nu 1}-p_N-p_{rel})^2}{4\sigma_{rel}^2} - \frac{(p_{\nu 2}-p_N-p_{rel})^2}{4\sigma_{rel}^2}\right]|\nu(\vec{p}_{\nu 1})\rangle\langle\nu(\vec{p}_{\nu 2})|. \tag{21}$$

For our present purposes it suffices to consider the 1D problem, to avoid needing to do all these integrals in 3D, so we proceed as

$$\rho_\nu^{1D} = \mathcal{N}\int dp_{\nu 1}dp_{\nu 2}dp_N \exp\left[-\frac{(p_{\nu 1}+p_N)^2+(p_{\nu 2}+p_N)^2}{4\sigma_P^2} - \frac{(p_{\nu 1}-p_N-p_{rel})^2+(p_{\nu 2}-p_N-p_{rel})^2}{4\sigma_{rel}^2}\right]|\nu(\vec{p}_{\nu 1})\rangle\langle\nu(\vec{p}_{\nu 2})|. \tag{22}$$

We can take care of the second nuclear momentum integral at this point,

$$= \mathcal{N}\int dp_{\nu 1}dp_{\nu 2} \exp\left[-\frac{1}{8}\left(\frac{1}{\sigma_P^2}+\frac{1}{\sigma_{rel}^2}\right)(p_{\nu 1}-p_{\nu 2})^2 - \frac{(p_{\nu 1}+p_{\nu 2}-p_{rel})^2}{2\left(\sigma_P^2+\sigma_{rel}^2\right)}\right]|\nu(\vec{p}_{\nu 1})\rangle\langle\nu(\vec{p}_{\nu 2})|. \tag{23}$$

To find the position-space density matrix, project Eq. 23 onto position basis states left and right,

$$\rho_\nu(y_1,y_2) = \mathcal{N}\int dp_{\nu 1}dp_{\nu 2} \exp\left[-\frac{1}{8}\left(\frac{1}{\sigma_P^2}+\frac{1}{\sigma_{rel}^2}\right)(p_{\nu 1}-p_{\nu 2})^2 - \frac{(p_{\nu 1}+p_{\nu 2}-p_{rel})^2}{2\left(\sigma_P^2+\sigma_{rel}^2\right)} + ip_{\nu 1}y_1 - p_{\nu 2}y_2\right]. \tag{24}$$

Switching coordinates to $p_\pm = p_{\nu 1} \pm p_{\nu 2}$ and $y_\pm = y_1 \pm y_2$ (including a factor of $\frac{1}{2}$ for the Jacobian of the integral transformation),

$$\rho_\nu(y_1,y_2) = \mathcal{N}\frac{1}{2}\int dp_- dp_+ \exp\left[-\frac{1}{8}\left(\frac{1}{\sigma_P^2}+\frac{1}{\sigma_{rel}^2}\right)p_-^2 - \frac{(p_+-p_{rel})^2}{2\left(\sigma_P^2+\sigma_{rel}^2\right)} + i\left(p_+y_- + p_-y_+\right)\right]. \tag{25}$$

The two p integrals are now just Gaussian Fourier transforms,

$$\rho_\nu(y_1,y_2) = \mathcal{N}\frac{1}{2}\exp\left[-\frac{1}{2\Delta_D^2}(y_1+y_2)^2 - \frac{1}{8\Delta_{OD}^2}(y_1-y_2)^2 - ip_{rel}(y_1-y_2)\right], \tag{26}$$

written here in a form to be comparable with Eq. 16 of Ref. [1]. From this expression we can read off the diagonal and off-diagonal widths in position space,

$$\Delta_D^2 = \frac{1}{4}\left(\frac{1}{\sigma_P^2}+\frac{1}{\sigma_{rel}^2}\right), \quad \Delta_{OD}^2 = \frac{1}{4(\sigma_P^2+\sigma_{rel}^2)}. \tag{27}$$

The neutrino wave packet width in this formalism is associated with the value of Δ_{OD}.

$$\sigma_{\nu,x} \sim \sqrt{\Delta_{OD}} = \frac{1}{2\sqrt{\sigma_P^2+\sigma_{rel}^2}}. \tag{28}$$

We can consider the behavior of this expression in two limits. If the atom is very spatially localized, uncertainty on the center of mass coordinate will dominate and $\sigma_{\nu,x} \sim \sigma_X = 1/2\sigma_P$. If the atom is more delocalized than this, the neutrino wave function's size is dictated by the relative momentum scale, and is

$$\sigma_{\nu,x} \sim \frac{a_0^2 m_e}{\zeta Z^2} \sim 2.7\,\mathrm{nm}. \tag{29}$$

Since the solid lattice localizes the center of mass of the atom to scales smaller than an atomic diameter, we expect that the uncertainty associated with σ_{rel} will be the dominant one in the electron capture neutrino production process at BeEST.

III. EXTRACTING CONSTRAINTS ON THE NEUTRINO SPATIAL WIDTH FROM BEEST DATA

Along with predicting the outgoing neutrino width following an electron capture decay, a connected issue is the method by which sensitivity to the width can be derived based on measuring the nuclear recoil energy spectrum.

The authors of Ref. [2] aim to contrast the approaches advocated by different schools of thought on this matter. We note that the first approach, assigning equal energy width to neutrino and recoil, is one we too would subscribe to in the case where the energy width were limited by intrinsic line width of the decay. However, since the electron capture lifetime is 53 days, this would imply an energy width of 10^{-22} eV. It is well understood that it is not this width that is encoded in measurement of the recoil system in electron capture. Instead, this quantity is dictated by homogeneous broadening due to widths associated with de-excitation processes in the daughter atom [13], as well as inhomogeneous broadening associated with solid state effects in materials [14]. The latter is understood to be the dominant contribution in the BeEST case [15].

Both the effects of homogeneous broadening due to finite lifetime and inhomogeneous broadening due to the variable environment of the emitter can be incorporated simply into the present formalism. The energy broadening would appear as a widening of the δ function in Eq. 5, as was done in Ref. [1]. Due to the very small width expected, this is not understood to limit the neutrino coherent width in this system. Accounting for the inhomogeneous broadening due to variability of the initial state amounts to taking a weighted sum of reduced density matrices for neutrinos produced from emitters in each possible kind of vacancy, as given in Ref. [14]. Since this represents an additional source of classical uncertainty about the initial state, which is in any case traced out of the final state density matrix incoherently, it will not impact the prediction for the off-diagonal width that dictates neutrino spatial coherence via $\sigma_{\nu,x}$. While neither of these effects will modify the predictions of this calculation concerning the predicted spatially coherent width of the neutrino wave packet, they do broaden the electron capture energy peak, implying that it must be a lower limit that is extracted from BeEST data. Ref. [2] recognizes this clearly, and appropriately derives a lower limit on the neutrino width.

The approach we advocate for in assigning a neutrino spatial width limit is to compare the expected off-diagonal width of the neutrino position-space density matrix to the on-diagonal width of the recoil momentum-space density matrix. The latter quantity is what is encoded in the energy spread measurement that is accessed experimentally by BeEST. Running through an argument similar to that in the previous section, the momentum space reduced density matrix for the recoil is

$$\rho_{N'}(p_1,p_2) = \mathcal{N}\exp\left[-\frac{1}{2(2\Delta_{OD})^{-2}}(p_1+p_2)^2 - \frac{1}{2(2\Delta_D)^{-2}}(p_1-p_2)^2\right], \tag{30}$$

from which we can extract that

$$\sigma_{N,p'} = \sigma_{\nu,p} = \frac{1}{2\sigma_{N',x}} = \frac{1}{2\sigma_{\nu,x}}. \tag{31}$$

This contradicts Eq. 2 in the BeEST paper, which cites Ref. [1] for its origin,

$$\sigma_{\nu,x} \stackrel{?}{=} \frac{2m_N^2}{m_N^2 - m_{N'}^2}\left(1+\frac{p_N c}{\sqrt{m_N^2c^4+p_N^2c^2}}\right)^{-1}\sigma_{N,x}. \tag{32}$$

The apparent conflict is resolved by noting that Eq. 32 relates the neutrino wave packet width $\sigma_{\nu,x}$ to the delocalization scale in an initial state system $\sigma_{N,x}$ in a two body decay scenario. In the BeEST analysis, this formula appears to have been mis-applied, with $\sigma_{N',x}$ on the right instead of $\sigma_{N,x}$. Even with $\sigma_{N,x}$ on the right, this formula would not be strictly applicable under the notation we are using in this paper. According to the present notation, $\sigma_{N,x}$ would denote the width of the initial state nucleus alone, not the full initial state system.

As such, we recommend the following prescription to extract a lower limit on the coherent width of the neutrino wave packet. First, note from Eq. 31 that $\sigma_{\nu,p} = \sigma_{N,p'}$. Intuitively this can be understood by considering that the two particles recoil against each other in approximately the center of mass with equal momenta. Then, taking the measured energy spectrum width of the recoil of $\sigma_{N',E}$ =2.9 eV and applying the mapping to momentum width as described in Ref. [2],

$$\sigma_{\nu,p} = \sigma_{N,p'} = \sqrt{m/2E}\,\sigma_{N',E}, \tag{33}$$

Finally, applying the Heisenberg uncertainty principle for the neutrino wave packet, we find

$$\sigma_{\nu,x} = \sigma_{N',x} \geq \frac{\hbar}{2\sigma_{\nu,p}} = 6.2\,\mathrm{pm}, \tag{34}$$

with equality in the case where the wave packet is Gaussian. This is the limit on the spatial neutrino wave packet width from BeEST data to which our prediction above, $\sigma_{\nu,x} \sim 2.7\,\mathrm{nm}$, should be consistently compared.

IV. CONCLUSION

We have used a density matrix formalism and careful consideration of the relevant distance scales in the electron capture decay of ^{7}Be ($e^- + {}^7\text{Be} \to {}^7\text{Li} + \nu_\text{e}$) to predict the width of the emerging neutrino wavepacket. We find that the uncertainty associated with the relative momentum between the initial state electron and parent nucleus, rather than the two-body initial state center of mass coordinate, sets the localization scale relevant for the neutrino width. Our resulting prediction, $\sigma_{\nu,x} \sim 2.7$ nm, can be compared to recent experimental results from the BeEST Collaboration ($\sigma_{\nu,x} > 6.2$ pm).

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