Can the QCD axion feed a dark energy component?

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A pseudo Nambu-Goldstone boson (PNGB) coupled to a confining gauge group via an anomalous term is characterised, during the confining phase transition, by a temperature dependent mass $m^2(T) \propto T^{-n}$. For n > 2, a non-relativistic population of such particles dominating the cosmological energy density would act as dark energy (DE), accelerating the expansion. We study the possibility that a PNGB φ_b coupled to a hidden gauge group that is presently undergoing confinement could realise this scenario. To obtain the observed amount of DE, the number density of φ_b must be boosted by some mechanism. Assuming that the QCD axion φ_a constitutes the dark matter (DM), a non-adiabatic level crossing between φ_a and φ_b shortly before matter-DE equality can convert a small fraction of DM into DE, providing such mechanism and explaining the coincidence puzzle.

Introduction. Unveiling the true nature of dark energy (DE) and dark matter (DM) undoubtedly stands as one of the most formidable endeavors in contemporary fundamental physics. We propose a framework in which DE and DM emerge as intricately connected phenomena. They are both explained in terms of elementary particles, albeit characterised by a rather peculiar intertwined dynamics. As a first assumption, we posit that the QCD axion embodies the entirety of DM [1– 3]. The QCD axion (see [4] for a review) is a hypothetical particle whose existence is implied by the most elegant solution to the strong CP problem [5, 6], the socalled Peccei-Quinn (PQ) mechanism [7-10]. The axion arises as a Nambu-Goldstone boson (NGB) of a spontaneously broken global Abelian symmetry endowed with a mixed anomaly with the color gauge group $SU(3)_c$. The anomaly represents an explicit breaking of the global U(1) that provides the axion with a tiny mass m_a . However, at temperatures well above the QCD confining temperature T_c , the axion is massless because free color charges in the plasma screen the non-perturbative effects responsible for generating its mass. As T decreases towards T_c , color charges get confined into color singlets, and a mass is generated, which at $T \lesssim T_c$ reaches its final value $m_a \sim \Lambda_{QCD}^2 / F$, where $\Lambda_{QCD} \sim O(100 \,\text{MeV})$ is the QCD scale, and $F \gtrsim 10^9 \,\text{GeV}$ is the axion decay constant. The dependence of the axion mass on temperature is generally written as $m_a^2(T) \sim T^{-n}$. In the dilute instanton gas approximation (DIGA) [11, 12] at lowest order one has $n = \beta_0 + n_f - 4$ where n_f is the number of light quarks and $\beta_0 = \frac{11}{3}N - \frac{1}{3}n_sT_s - \frac{4}{3}n_fT_f$ is the one loop coefficient of the β -function, N is the degree of the confining gauge group, n_s the number of light scalars and $T_{s,f}$ the index of the corresponding representations $(T_{s,f} = 1/2 \text{ for the fundamental})$. In QCD with $n_f = 3, (0)$ one obtains n = 8, (7). However, the exponent n is not a constant but it has a rather intricate temperature dependence, and lattice simulations (see Ref. [13] for a review) have found that while at temperatures well above the transition, values of n are generally close to the DIGA results, at temperatures relevant for the onset of oscillations of the axion field the dependence on T is milder. This is intriguing because for values around $n \approx 6$ (see e.g. Refs. [14, 15]) a population of cold axions would contribute an approximately constant energy density component $\rho_a = m_a n_a$. More generally, within a sufficiently small time/temperature window such that $n \approx \text{const.}$, one obtains from the conservation law $d(\rho_a a^3) = -p_a da^3$ (with a the cosmological scale factor and p_a the pressure) an effective equation of state $p = w\rho$ with $w = -\frac{n}{6}$. Thus, already for n > 2 a component of such particles dominating the cosmological energy density would accelerate the expansion, behaving as quintessence for n < 6 [16], as a cosmological constant for n = 6, and for n > 6 as phantom DE [17]. Moreover, a variation of n with the temperature would be reflected in w(z) varying with the redshift.¹

In the case of the QCD axion, the dependence of the mass on the temperature has no effects on the cosmological expansion, since this occurs in the radiation dominated era. By the time of matter/radiation equality, m_a has long reached its zero temperature constant value, behaving as cold DM. It is, however, natural to ask if a mass-varying mechanism could be at work in a dark sector (DS) containing an axion coupled to a confining gauge group G_b that is undergoing a phase transition at present time. Energetic considerations suggest that in its simplest form this scenario is not viable, because the amount of energy that the axion potential can provide is bounded by the G_b confining scale $\rho_b < \Lambda_b^4$. In turn, to ensure an ongoing DS phase transition, Λ_b must be below

¹ Recently the DESI collaboration has reported results from precise baryon acoustic oscillation measurements in galaxy, quasar and Lyman- α forest tracers [18]. Combined with CMB data or type Ia supernovae, their results indicate a preference for DE time-variance at a late epoch over the Λ CDM model that, depending on the combined data set, can exceed the 3σ level.

the present DS temperature T_{DS} . Therefore, similarly to the QCD case, during the mass-varying era the DS remains dominated by radiation from the dark plasma.

In this paper we propose a mechanism that allows to circumvent this argument, and that can yield $\rho_b \gg \Lambda_b^4$. We assume that at a certain temperature $T_{\rm LC}$ somewhat above the temperature of matter/DE equality $T_{\rm DE}$, the T-dependent dark axion mass $m_b(T)$ hits a level crossing (LC) with the mass of the QCD axion $m_b(T_{\rm LC}) \simeq m_a$. Then a fraction of DM gets converted into a DE component, that will come to dominate the energy density at $T = T_{\rm DE}$. Due to the different scaling $\rho_b/\rho_a \sim$ $(T_{\rm LC}/T)^{n/2}$, only a small fraction of DM needs to be converted, which implies a strongly non-adiabatic LC.² In summary, the particle physics interpretation of the DE phenomenon that we are suggesting neatly accounts for the DE/DM coincidence puzzle, it is consistent with variations of the effective DE equation of state, and it predicts that in the far future, once T will fall below Λ_b , DE will end up contributing to DM. Thus, even phantom DE (n > 6) would not constitute a cosmological problem, because it would remain a transitory regime [28].

Two coupled axions system. Consider two confining gauge gropus G_a , G_b with confining scales $\Lambda_a > \Lambda_b$, and two set of fermions transforming under $G_a \otimes G_b$ as $\psi_{L,R} \sim$ $[(d^a_{\psi}, T^a_{\psi}), (d^b_{\psi}, T^b_{\psi})], \chi_{L,R} \sim [(d^a_{\chi}, T^a_{\chi}), (d^b_{\chi}, T^b_{\chi})]$, where $d^{a,b}$ and $T^{a,b}$ denote respectively the dimension and index of the representation. Consider the Yukawa Lagrangian

$$\mathcal{L}_Y = \overline{\psi}_L \psi_R \Phi_1 + \overline{\chi}_L \chi_R \Phi_2 \,, \tag{1}$$

where $\Phi_{1,2}$ are two gauge singlet scalars, acquiring VEVs $v_{1,2}$. There are in total six fields carrying six overall phases. Two conditions for rephasing invariance are fixed by Eq. (1). The remaining four global symmetries are two independent baryon numbers B_{ψ} , B_{χ} , and two global Peccei-Quinn (PQ) symmetries $U(1)_{q,p}$ under which the scalar fields carry charges $\Phi_1 \sim (q_1, 0)$ and $\Phi_2 \sim (0, p_2)$. The Yukawa terms impose the conditions $q_{\psi_L} - q_{\psi_R} = q_1$, $p_{\chi_L} - p_{\chi_R} = p_2$ and, without loss of generality, we can normalize $q_1 = p_2 = 1$. Then the mixed $U(1)_{q,p} - G_{a,b}$ anomaly coefficients are given by:

$$n_1 = 2d_{\psi}^b T_{\psi}^a, \ m_1 = 2d_{\psi}^a T_{\psi}^b, \ n_2 = 2d_{\chi}^b T_{\chi}^a, \ m_2 = 2d_{\chi}^a T_{\chi}^b, \ (2)$$

where the factors of 2 ensure that the coefficients are integers. After $U(1)_{q,p}$ spontaneous breaking the Yukawa terms in Eq. (1) can be rewritten as

$$\mathcal{L}_{Y}^{\text{eff}} = \overline{\psi}_{L} \psi_{R} v_{1} e^{i\frac{a_{1}}{v_{1}}} + \overline{\chi}_{L} \chi_{R} v_{2} e^{i\frac{a_{2}}{v_{2}}} . \tag{3}$$

Removing the complex phases via chiral rotations of the fermion fields generates the anomalous terms $\frac{C_i}{16\pi^2} F_i \cdot \tilde{F}_i$

SU(2):	$2\left(1 ight)$	3 (4)	4(10) 5(20)	6(35)	7(56)
SU(3):	3 (1)	6(5)	$8(6) \ 10(15)$	15 (20)	21(35)
SU(4):	4(1)	6(2)	$10(6) \ 15(8)$	20(13)	20 ′ (16)
SU(5):	5(1)	10 (3)	15(7) 24(10)	35 (28)	40 (22)

TABLE I. SU(N) (N = 2, 3, 4, 5) representations of lowest dimension (in bold face) with twice the value of the index given in parenthesis.

(i = a, b) with coefficients:

$$C_a = n_1 \frac{a_1}{v_1} + n_2 \frac{a_2}{v_2}, \qquad C_b = m_1 \frac{a_1}{v_1} + m_2 \frac{a_2}{v_2}.$$
 (4)

We see that if the dimension/index of the representations satisfy the condition $(d_{\psi}^a d_{\chi}^b)/(d_{\psi}^b d_{\chi}^a) = (T_{\psi}^a T_{\chi}^b)/(T_{\psi}^b T_{\chi}^a)$ (i.e. $n_1/n_2 = m_1/m_2$) then $C_a \propto C_b$. The field combination orthogonal to $C_{a,b}$ would then decouple from the symmetry breaking effects generated by the anomalies, maintaining a flat potential. If instead the relation holds only approximately, then the flat direction would get lifted only slightly by anomaly effects, realising the Kim-Nilles-Peloso (KNP) mechanism [29]. While enforcing an exact equality is straightforward, enforcing it to a given level of approximation requires a careful choice of groups and representations. This can be understood by looking at the dimension and index values of SU(N)representations in Table I.

Let us now assume $\Lambda_a \gg \Lambda_b$, so that at a temperature $T \sim \Lambda_a$ the combination of NGB $\varphi_a/F \sim C_a$ acquires a mass while the orthogonal combination $\varphi_b/f \sim$ $-n_2a_1/v_2 + n_1a_2/v_1$ remains massless The mass eigenstates (φ_a, φ_b) are related to (a_1, a_2) by an orthogonal transformation with angle $\vartheta = \arctan(\frac{n_2v_1}{n_1v_2})$. Following Ref. [30], we can obtain the respective decay constants from the variations of the fields $\delta a_i = v_i \alpha_i$ (i = 1, 2):

$$\delta\varphi_a \equiv F\eta = c_\vartheta \delta a_1 + s_\vartheta \delta a_2 = \frac{v_1 v_2}{\sqrt{}} (n_1 \alpha_1 + n_2 \alpha_2) \quad (5)$$

where $c_{\vartheta}(s_{\vartheta}) = \cos \vartheta(\sin \vartheta)$, $\sqrt{} = \sqrt{n_1^2 v_2^2 + n_2^2 v_1^2}$, and $\eta = n_1 \alpha_1 + n_2 \alpha_2 \neq 0$ is an arbitrary non-vanishing shift. Therefore $F = v_1 v_2 / \sqrt{}$. A variation in the orthogonal direction is identified by the condition $n_1 \hat{\alpha}_1 + n_2 \hat{\alpha}_2 = 0$, that is $\hat{\alpha}_1 = -n_2 \eta'$, $\hat{\alpha}_2 = n_1 \eta'$ for some arbitrary shift η' . We then have

$$\delta\varphi_b = -s_\vartheta \delta \hat{a}_1 + c_\vartheta \delta \hat{a}_2 = \sqrt{-\cdot \eta'} \,. \tag{6}$$

Finally, by expressing $a_{1,2}$ in C_b in Eq. (4) in terms of φ_a, φ_b one can easily obtain

$$\mathcal{C}_b = \frac{\varphi_a}{F'} + \frac{\varphi_b}{f} \,, \tag{7}$$

with

² Two axions LC, tipycally in the adiabatic regime, has been previously harnessed in various contexts unrelated to DE [19–27].

Equations of motion. The $G_a \times G_b$ strong dynamics generates the following potential:

$$V = \Lambda_a^4 \left[1 - \cos\left(\frac{\varphi_a}{F}\right) \right] + \Lambda_b^4 \left[1 - \cos\left(\frac{\varphi_a}{F'} + \frac{\varphi_b}{f}\right) \right],\tag{9}$$

from which one can obtain the equations of motion. In the limit of small oscillations, they can be written as:

$$A + 3HA + \mathcal{M}^2 A = 0, \qquad (10)$$

with

$$A = \begin{pmatrix} \varphi_a \\ \varphi_b \end{pmatrix}, \qquad \mathcal{M}^2 = m_a^2 \begin{pmatrix} 1 + \epsilon^2 r & \epsilon r \\ \epsilon r & r \end{pmatrix}, \quad (11)$$

and we have defined $m_a = \Lambda_a^2/F$, $r = r(T) = m_b^2(T)/m_a^2$, $\epsilon = f/F'$. Let us now assume that at zero temperature $m_b = \Lambda_b^2/f > m_a$. Since $\Lambda_a \gg \Lambda_b$ this implies the condition $f/F \ll 1$. A class of models satisfying this condition is easily obtained by assuming $v_1/v_2 \ll 1$ and by choosing ψ to be a singlet of G_a . Then $n_1 = 0$ and we have $f/F = (n_2/m_1)(v_1/v_2)$ as well as $\epsilon = f/F' =$ $(m_2/m_1)(v_1/v_2) \ll 1$ Under these conditions, it is clear that at temperatures around $T \sim \Lambda_b$, when $m_b(T)$ is still evolving, while m_a has long reached its constant value, a level crossing (LC) will be encountered. Neglecting for simplicity the highly suppressed ϵ^2 term in $(\mathcal{M}^2)_{11}$, the LC is defined by the condition $m_b(T_{\rm LC}) = m_a$. Let us now introduce the time variable $x = t/t_{\rm LC}$, and let us define the instantaneous mass basis A_m as

$$A = R(x) A_m, \qquad M^2 = R^{\dagger}(x) \mathcal{M}^2 R(x) , \qquad (12)$$

where $M^2 = \text{diag}(M^2_+, M^2_-)$ and R(x) is an orthogonal matrix defined in terms of an angle $\beta(r(x))$. We have:

$$M_{\pm}^2 = \frac{m_a^2}{2}(1+r\pm\Delta), \quad \tan\beta = \frac{2\epsilon r}{1-r+\Delta} \qquad (13)$$

with $\Delta = \sqrt{(1-r)^2 + 4\epsilon^2 r^2}$. At LC $\tan \beta \to 1$ and R(x=1) describes maximal mixing. At $r = (1+\frac{3}{2}\epsilon)^{-1}$ $\tan \beta = \frac{1}{2}$ so that the width of the resonance is $\Delta r \sim 3\epsilon$. In the instantaneous mass basis Eq. (11) becomes

$$\ddot{A}_m + 3\mathsf{H}\dot{A}_m + \mathsf{M}^2 A_m = 0, \qquad (14)$$

where

$$\mathsf{H} = H + \frac{2}{3} R^\dagger \dot{R}, \qquad \mathsf{M}^2 = M^2 + R^\dagger \ddot{R} + 3 H R^\dagger \dot{R} \,. \label{eq:H}$$

and

$$R^{\dagger}\dot{R} = i\sigma_2 \cdot \frac{\epsilon}{\Delta^2}\dot{r},$$

$$R^{\dagger}\ddot{R} = i\sigma_2 \cdot \left[\frac{2(1-r-4r\epsilon^2)}{\Delta^2}\dot{r}^2 + \ddot{r}\right]\frac{\epsilon}{\Delta^2} - \sigma_0 \cdot \frac{\epsilon^2}{\Delta^4}\dot{r}^2$$

with σ_2 the Pauli matrix and σ_0 the identity. The nature of the LC is characterised by the ratio between the

splitting of the two levels, and the off-diagonal entries in M^2 that mix these levels, evaluated at LC:

$$\gamma = \left| \frac{\operatorname{Tr}\left(\sigma_{3}\mathsf{M}^{2}\right)}{\operatorname{Tr}\left(\frac{i}{2}\sigma_{2}\mathsf{M}^{2}\right)} \right|_{\mathrm{LC}} = \left| \frac{8m_{a}^{2}\epsilon^{2}}{3H\dot{r} - 2\dot{r}^{2} + \ddot{r}} \right|_{\mathrm{LC}} = \frac{36(\epsilon t_{\mathrm{LC}}m_{a})^{2}}{n(2n-3)}.$$
(15)

If, in the resonance region, the splitting is much larger than the variation of r ($\gamma \gg 1$), the heavier state (φ_a at $t \ll t_{\rm LC}$) remains the heavier, and emerges as φ_b at $t \gg t_{\rm LC}$, that is, in crossing the resonance region, the two axions swap their "flavor" identities. This defines the adiabatic regime, which is then realised when $\epsilon \omega \equiv$ $\epsilon t_{\rm LC} m_a \gg 1$. Since m_a corresponds to the oscillation frequency at LC, and $\epsilon t_{\rm LC}$ to the width of the resonant region, $\gamma \gg 1$ can also be interpreted as the requirement that several oscillations occur within this region.

The adiabatic LC phenomenon is well known in condensed matter physics, and analytic treatments valid under certain assumptions were formulated long ago, most notably by Landau [31] and Zener [32]. In particle physics, the phenomenon is realised in the Mikheyev-Smirnov-Wolfenstein (MSW) enhancement of in-matter $\nu_e \rightarrow \nu_{\mu}$ conversion of solar neutrinos [33, 34]. However, our two-axion LC differs from the MSW effect in that besides the mass splitting between the two "flavour" states, also the off diagonal entries in \mathcal{M}^2 are time-dependent. It bears a closer resemblance to a variant MSW realization [35] where the off-diagonal entries stem not from vacuum mixing angles, but from (hypothetical) $\nu_e - \nu_{\mu}$ flavor changing interactions with electrons and nucleons, so that they also vary as a function of the matter density.

We are interested in a LC occurring during matter domination $(a(t) \sim t^{2/3}, H = 2/(3t))$. Let us consider the evolution around $x \sim 1$. $m_b^2(T) = m_a^2(T_{\rm LC}/T)^n$ gives $r(x) = x^{\frac{2n}{3}}$. Eq. (10) can then be written as:

$$\ddot{\varphi}_a + \frac{2}{x}\dot{\varphi}_a + \omega^2\varphi_a + \epsilon\omega^2 x^{\frac{2n}{3}}\varphi_b = 0, \qquad (16)$$

$$\ddot{\varphi}_b + \frac{2}{x}\dot{\varphi}_b + \omega^2 x^{\frac{2n}{3}}\varphi_b + \epsilon \omega^2 x^{\frac{2n}{3}}\varphi_a = 0, \qquad (17)$$

where $\omega = m_a t_{\rm LC}$ and the dots represent derivatives with respect to x.

Dark energy from the QCD axion. Let us now study whether the LC mechanism can be harnessed to generate a density of mass-varying axions sufficient to drive a cosmological acceleration. Assuming that both φ_a and φ_b are dark sector particles would ease the construction. However, taking $G_a = SU(3)_{\rm QCD}$ and φ_a as a QCD axion supplying the entirety of DM is more economical, it provides a compelling connection between DM and DE, and it also yields testable (at least in principle) predictions. A sketch of the evolution of the DM and DE components, neglecting for simplicity the baryon contribution, is depicted in figure 1 (not to scale). The DM energy density ρ_{φ_a} dominates at early times (EDM), dropping as T^3 .



FIG. 1. Sketch of the evolution of the cosmological energy density, neglecting the baryon contribution. The initial Early DM (EDM) phase evolves at $t > t_{\rm LC}$ into a mixed DM+DE phase. After G_b confinement ($t > t_b$) DE is converted into a Late DM (LDM) component reinstating matter domination.

At $t_{\rm LC}$ a tiny fraction of QCD axions is converted into φ_b whose mass is growing as $\sim T^{-n/2}$ providing a DE component ($\rho_{\varphi_b} \approx \text{const.}$ for $n \approx 6$). After $\rho_{\varphi_b} = \rho_{DM}$ is reached at $t_{\rm DE}$, the DE component starts dominating, eventually matching the present time (t_0) values of $\Omega_{\rm DE}$ and $\Omega_{\rm DM}$. In the far future, after G_b confinement is completed $(t > t_b)$, $m_b = \text{const.}$ and φ_b will end up contributing a Late DM (LDM) component, reinstating matter domination. Figure 2 gives an example of the dynamics of LC conversion. We have fixed n = 6 for the evolution of $m_b^2(T)$, $\epsilon \omega = 1$ for the adiabaticity parameter, and $\omega = 50$ that, although unrealistically small, allows to easily integrate numerically the system of coupled equations (16)-(17). At $t \ll t_{\rm LC}$ the initial amplitude of φ_b is vanishingly small. The amplitude of φ_a is relatively large, and decreases with time because of the cosmological expansion. Around $t \simeq t_{\rm LC}$ the frequency of φ_b approaches that of φ_a , and a partial conversion takes place. In the picture the amplitude of φ_b is multiplied by ten, so that the efficiency of the conversion at the level of amplitudes is about 10%. After LC both amplitudes decrease as 1/t. However, while the frequency of φ_a remains constant, that of φ_b keeps increasing as t^2 .

There are severe constraints on our scenario. We are interested in the evolution of $m_b(T)$ around $T_{\rm LC}$:

$$m_b(T_{\rm LC}) \sim \frac{\Lambda_b^2}{f} \left(\frac{T_b}{T_{\rm LC}}\right)^3 = m_a = \frac{\Lambda_a^2}{F}$$
 (18)

where $T_b \approx \Lambda_b$ is the G_b confinement temperature, and $\Lambda_a = \Lambda_{QCD}$. We take for simplicity the same temperature for the QCD and G_b sectors (we will comment on the consequences of dropping this assumption in the next section.) In viable models we will generally find



FIG. 2. An example of the LC mechanism for n = 6, $\epsilon \omega = 1$ and $\omega = 50$. The φ_b amplitude is increased by a factor of 10.

 $F' \sim F \propto v_2 \ (f \propto v_1)$, so we can write

$$\epsilon \simeq \frac{f}{F} = \frac{\Lambda_b^2}{\Lambda_a^2} \frac{T_b^3}{T_{\rm LC}^3} \lesssim 10^{-25} \left(\frac{\Lambda_b}{10^{-4} {\rm eV}} \frac{160 {\rm MeV}}{\Lambda_a}\right)^2 \quad (19)$$

where for the inequality we have used $T_b/T_{\rm LC} < T_0/T_{\rm DE}$ as is implied by $T_{\rm LC} > T_{DE}$ and by the requirement that m_b is still evolving $(T_b < T_0)$, together with $T_0/T_{\rm DE} = (1 + z_{DE})^{-1}$ with $z_{DE} \approx 0.5$ the redshift at which DE starts dominating. An additional condition is implied by the requirement that the PQ symmetry is broken at a temperature sufficiently above $T_{\rm LC}$, say $f \gtrsim 10^{-2} \,\mathrm{eV}$. Together with Eq. (19) this yields $F \gtrsim 10^{14} \,\mathrm{GeV} \ (m_a \lesssim 6 \cdot 10^{-8} \,\mathrm{eV})$ which implies a preinflationary QCD axion scenario, with a moderate tuning of the initial misalignment angle ($\theta_a \leq 6\%$) in order to reproduce Ω_{DM} . Another consequence of Eq. (19) is that, assuming as a benchmark $t_{\rm LC} = 10^9 \, {\rm yr} \, (T_{\rm LC} \sim$ $1.3 \cdot 10^{-3} \,\mathrm{eV}, z_{\mathrm{LC}} \sim 4.6$) we obtain for the adiabaticity parameter in Eq. (15) $\gamma \leq 1$, that is the LC occurs in the deep non-adiabatic regime, so that only a tiny fraction of QCD axions is converted into φ_b . Notably, in order to reach DE-matter equality at $z_{\rm DE} \sim 0.5$, one needs at LC $\frac{\rho_{\rm DE}}{\rho_m}\Big|_{\rm LC} = \left(\frac{1+z_{\rm DE}}{1+z_{\rm LC}}\right)^3 \sim 1\% - 2\%, \text{ consistently with a non$ adiabatic conversion. Let us now specify some minimal possibilities for G_h and for the fermions representations. $G_b = SU(2)$, with $\psi \sim (1,2)$ and $\chi \sim (3,2)$. We have $F = v_2/2$, $F' = v_2/3$ and $f = v_1$. In this case the exponent for the evolution of $m_b^2(T)$ in the DIGA approximation is $n = \frac{11}{3}$ which gives for the equation of state $w \simeq -0.61$, so that DE behaves as quintessence. $G_b = SU(3)$, with $\psi \sim (1,3)$ and $\chi \sim (3,3)$. We have

 $G_b = SU(3)$, with $\psi \sim (1, 3)$ and $\chi \sim (3, 3)$. We have $F = F' = v_2/3$ and $f = v_1$. This model encompasses a wider range of possibilities. In the DIGA approximation $n = \frac{22}{3}$ which gives $w \simeq -1.2$. This opens up the possibility that during the cosmological evolution DE could behave initially as phantom DE, evolving at later times into a cosmological constant and eventually into quintessence.

Discussion. A precise study of the dynamics of the LC in the physical regime is an extremely difficult task. Analytic tools as the Landau-Zener (LZ) approximation [31, 32] cannot be employed because they rely on assumptions that are not respected in our case (a mass splitting linear function of time and time independent off-diagonal mixing terms). Moreover, it is known that the LZ formula cannot be extrapolated to the strong nonadiabatic regime [36-38]. On the other hand, attempts at numerical integration of Eqs. (16)-(17) encounter daunting obstacles, related to the well known problem of following the dynamics of fast oscillating fields at late cosmological times. In our case this is represented by the huge value of the dimensionless parameter ω that, for the values of m_a and $t_{\rm LC}$ mentioned above, is $\omega \sim 10^{24}$. For manageable values of ω the numerical results confirm the qualitative behaviour depicted in figure 2. However, pursuing numerical solutions of Eqs. (16)-(17) in the physical regime certainly deserves further efforts.

The G_a and G_b sectors undergo thermal decoupling since the very early times of the cosmological evolution. This is because the only portal is represented by the χ fermions that transform under both gauge groups, and can mediate via loop box diagrams gluon scattering $g_a g_a \leftrightarrow g_b g_b$. However, the large mass $m_{\chi} \sim v_2 \sim$ $10^{14} \,\mathrm{GeV}$ ensures that this process is unable to keep thermal equilibrium (it also ensures that the χ 's, which could potentially constitute cosmologically dangerous strongly interacting stable relics [39], are effectively inflated away.) G_a - G_b thermal decoupling can imply that our benchmark values for Λ_b and f have to be modified by a factor of a few. It is, however, a welcome feature of the construction, in that for $T_b < T_a$ the contribution of dark radiation to the effective number of neutrino species $N_{\rm eff}^{\nu}$ can be reduced. In fact, given that the visible sector gets reheated by annihilation of all the standard model (SM) degrees of freedom (except photons and neutrinos), it turns out that the $G_b = SU(2)$ model remains compatible at 90% c.l. with the recent DESI result on N_{eff}^{ν} [18]. For $G_b = SU(3)$ some further entropy injection in the visible sector would be needed. The additional degrees of freedom of the minimal supersymmetric SM would sufficie

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