## Isospin violation effect and three-body decays of the $T_{cc}^+$ state

Zhi-Feng Sun<sup>1,2,3,4</sup>,\* Ning Li<sup>5</sup>,<sup>†</sup> and Xiang Liu<sup>1,2,3,4</sup>‡

<sup>1</sup>School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China

<sup>2</sup>Lanzhou Center for Theoretical Physics, Key Laboratory of Theoretical Physics

of Gansu Province, Lanzhou University, Lanzhou, Gansu 730000, China

<sup>3</sup>MoE Frontiers Science Center for Rare Isotopes, Lanzhou University, Lanzhou, Gansu 730000, China

<sup>3</sup>Research Center for Hadron and CSR Physics, Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China

<sup>5</sup>School of Physics, Sun Yat-Sen University, Guangzhou 510275, China

(Dated: May 2, 2024)

In this work, we make a study of  $T_{cc}^+$  state observed by the LHCb collaboration in 2021. In obtaining the effective potentials using the One-Boson-Exchange Potential Model we use an exponential form factor, and find that in the short and medium range, the contributions of the  $\pi$ ,  $\rho$  and  $\omega$  exchanges are comparable while in the long range the pion-exchange contribution is dominant. Based on the assumption that  $T_{cc}^+$  is a loosely bound state of  $D^*D$ , we focus on its three-body decay using the meson-exchange method. Considering that the difference between the thresholds of  $D^{*+}D^0$  and  $D^{*0}D^+$  is even larger than the binding energy of  $T_{cc}^+$ , the isospin-breaking effect is amplified by the small binding energy of  $T_{cc}^+$ . Explicitly including such an isospin-breaking effect we obtain, by solving the Schrödinger equation, that the probability of the isoscalar component is about 91% while that of the isovector component is around 9% for  $T_{cc}^+$ . Using the experimental value of the mass of  $T_{cc}^+$  as an input, we obtain the wave function of  $T_{cc}^+$  and further obtain its width via the three-body hadronic as well as the radiative decays. The total width we obtain is in agreement with the experimental value of the LHCb measurement with a unitarised Breit-Wigner profile. Conversely, the current results support the conclusion that  $T_{cc}^+$  is a hadronic molecule of  $D^*D$ .

#### I. INTRODUCTION

The search for the exotic hadrons and the exploration of their nature have been important goals of the particle physics [1–7]. The doubly heavy multiquark is particularly important, as its study can deepen our understanding of the nonperturbative strong interaction in the low-energy region and enrich our identification of the hadronic spectrum. In 2021, the LHCb Collaboration announced the observation of the doubly charmed tetraquark  $T_{cc}^+$  in the  $D^0D^0\pi^+$  mass distribution [8, 9]. This discovery is a turning point in hadron physics, providing the first evidence for a clearly exotic meson state with two charm quarks. Using a relativistic P-wave two-body Breit-Wigner function with a Blatt-Weisskopf form factor as the natural resonance profile, the location of the peak relative to the  $D^{*+}D^0$  mass threshold,  $\delta m_{BW}$ , and the width,  $\Gamma_{BW}$ , are determined to be [8]

$$\delta m_{BW} = -273 \pm 61 \pm 5^{+11}_{-14} \text{ keV},$$
  

$$\Gamma_{BW} = 410 \pm 165 \pm 43^{+18}_{-38} \text{ keV}.$$

In order to assess the fundamental properties of nearthreshold resonances, the LHCb Collaboration uses advanced parametrisations, i.e., a unitarised Breit-Wigner profile. The extracted mass relative to the  $D^{*+}D^0$  threshold and the width are given respectively by [9]

$$\delta m_U = -360 \pm 40^{+4}_{-0} \text{ keV},$$
  

$$\Gamma_U = 48 \pm 2^{+0}_{-14} \text{ keV}.$$

Before this discovery, there were many works searching for the structures with two heavy and two light quarks [10– 27, 31–61]. The striking feature that  $T_{cc}^+$  is extremely close to the threshold.  $T_{cc}^+$  has motivated much work investigating this state within the picture of the hadronic molecular state [62–100], tetraquark state [91–93, 101–109], and many other explanations [91, 110–116].

In Ref. [27] we investigated the possible tetraquarks with double charm. We found that the interaction between  $D^*$  and D is attractive for the system  $D^*D$  with quantum number  $I(J^P) = 0(1^+)$ , and it can form a loosely bound state. In the present work, we still consider  $T_{cc}^+$  as the  $D^{*+}D^0/D^{*0}D^+$  molecular state. Since the mass difference of  $D^{*+}D^0$  and  $D^{*0}D^+$  is so large compared to the small binding energy of  $T_{cc}^+$ , the isospin violation effect should not be neglected, and in the present work we explicitly take this effect into account.

In the compact tetraquark picture, the heavy diquarkantiquark symmetry contributes a strong attractive force for the doubly heavy systems, so that this scheme can generate a deep bound state relative to the corresponding two-meson threshold. In order to clarify whether the hadronic molecular picture is correct, in this work we develop a method to calculate the three-body decay of  $T_{cc}^+$  based on  $T_{cc}^+$  being a hadronic molecule of  $D^*D$ . Note that in Refs. [65, 71, 97], the authors have already studied the three-body decay of  $T_{cc}^+$ . Unlike these works, we study the three-body decay of  $T_{cc}^+$  in the framework of the  $D^{*+}D^0/D^{*0}D^+$  molecular state based on the one-bosonexchange model, where its molecular structure is represented by the wave functions obtained by solving the Schrödinger equation.

In this work, we also make an improvement to the oneboson-exchange model. In the past, to calculate the effective potentials a monopole form factor was introduced at each vertex to suppress the high-momentum (or short-range) contribution. However, when the mass of the exchanged particle is

<sup>\*</sup>sunzf@lzu.edu.cn

<sup>&</sup>lt;sup>†</sup>lining59@mail.sysu.edu.cn

<sup>&</sup>lt;sup>‡</sup>xiangliu@lzu.edu.cn

large, the monopole form factor does not work very well due to the suppression by its numerator. We therefore choose an exponentially parameterized form factor in the derivation of the effective potential to regulate the short-range interaction.

We also extend the effective Lagrangians from  $[SU(3)_{L} \otimes$  $SU(3)_R]_{global} \otimes [SU(3)_V]_{local}$  to  $[U(3)_L \otimes U(3)_R]_{global} \otimes [U(3)_V]_{local}$ , in which  $\pi$ , K,  $\eta^{(\prime)}$  are identified as the pseudoscalar Goldstone bosons, while  $\rho$ ,  $\omega$ ,  $K^*$  and  $\phi$  are taken as the hidden local symmetry gauge bosons.

The paper is organized as follows. In Sec. II, we will present the effective potentials with an exponentially parameterized form factor. The formalism for the three-body decay of  $T_{cc}^+$  will be discussed explicitly in Sec. III. The numerical results are then presented and analyzed in Sec. IV. In the final section V we will summarize our results and draw conclusions.

## **II. EFFECTIVE POTENTIAL**

In this work, we consider the Lagrangians by extending the hidden local symmetry  $[SU(3)_L \otimes SU(3)_R]_{global} \otimes [SU(3)_V]_{local}$ to  $[U(3)_L \otimes U(3)_R]_{global} \otimes [U(3)_V]_{local}$ . In such case, the exchanged light pseudoscalar and vector mesons can be grouped into compact matrices, respectively, which are as follows

$$M = \begin{pmatrix} \frac{\pi^{0}}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}} & \pi^{+} & K^{+} \\ \pi^{-} & -\frac{\pi^{0}}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}} & K^{0} \\ K^{-} & \bar{K}^{0} & -\frac{2}{\sqrt{6}}\eta + \frac{\eta'}{\sqrt{3}} \end{pmatrix}, (1)$$
$$\hat{\rho}^{\mu} = \begin{pmatrix} \frac{\rho^{0}}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^{+} & K^{*+} \\ \rho^{-} & -\frac{\rho^{0}}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}^{\mu}.$$
(2)

Taking into account also the heavy quark spin symmetry, the Lagrangian of the interaction between the light meson and the heavy meson containing a charm or bottom quark is constructed as [28, 29]

$$\mathcal{L} = ig \operatorname{Tr} \left[ H_b^{(Q)} \gamma_{\mu} \gamma_5 A_{ba}^{\mu} \bar{H}_a^{(Q)} \right] + i\beta \operatorname{Tr} \left[ H_b^{(Q)} v_{\mu} (V_{ba}^{\mu} - \rho_{ba}^{\mu}) \bar{H}_a^{(Q)} \right] + i\lambda \operatorname{Tr} \left[ H_b^{(Q)} \sigma_{\mu\nu} F_{ba}^{\mu\nu} \bar{H}_a^{(Q)} \right],$$
(3)

where

$$H_{a}^{(Q)} = \frac{1+\psi}{2} \left[ P_{a}^{*\mu} \gamma_{\mu} - P_{a} \gamma_{5} \right], \tag{4}$$

$$\bar{H}_{a}^{(Q)} = \gamma_{0} H_{a}^{(Q)\dagger} \gamma_{0} = \left[ P_{a}^{*^{\dagger}\mu} \gamma_{\mu} + P_{a}^{\dagger} \gamma_{5} \right] \frac{1+\psi}{2}, \qquad (5)$$

$$A^{\mu} = \frac{1}{2} \left( \xi^{\dagger} \partial^{\mu} \xi - \xi \partial^{\mu} \xi^{\dagger} \right), \tag{6}$$

$$V^{\mu} = \frac{1}{2} \left( \xi^{\dagger} \partial^{\mu} \xi + \xi \partial^{\mu} \xi^{\dagger} \right),$$

$$F_{\mu\nu} = \partial_{\mu} \rho_{\nu} - \partial_{\nu} \rho_{\mu} - \left[ \rho_{\mu}, \rho_{\nu} \right],$$
(7)
(8)

$$F_{\mu\nu} = \partial_{\mu}\rho_{\nu} - \partial_{\nu}\rho_{\mu} - \left[\rho_{\mu}, \rho_{\nu}\right], \qquad ($$

After expanding the Lagrangians in Eq. (3), we have the terms we need for our calculation

(1, 0, 0, 0).

$$\mathcal{L}_{P^{(*)}P^{(*)}M} = -i\frac{2g}{f_{\pi}}\epsilon_{\alpha\mu\nu\lambda}v^{\alpha}P^{*\mu}\partial^{\nu}MP^{*\lambda^{\dagger}}$$

$$-\frac{2g}{f_{\pi}}(P\partial^{\lambda}MP^{*\dagger}_{\lambda} + P^{*}_{\lambda}\partial^{\lambda}MP^{\dagger}), \qquad (9)$$

$$\mathcal{L}_{P^{(*)}P^{(*)}V} = -\sqrt{2}\beta g_{V}P(v\cdot\hat{\rho})P^{\dagger}$$

$$-2\sqrt{2}\lambda g_{V}\epsilon_{\lambda\mu\alpha\beta}v^{\lambda}(P\partial^{\alpha}\hat{\rho}^{\beta}P^{*\mu^{\dagger}} + P^{*\mu}\partial^{\alpha}\hat{\rho}^{\beta}P^{\dagger})$$

$$+\sqrt{2}\beta g_{V}P^{\mu*}(v\cdot\hat{\rho})P^{*\dagger}_{\mu}$$

$$-i2\sqrt{2}\lambda g_V P^{*\mu}(\partial_\mu \hat{\rho}_\nu - \partial_\nu \hat{\rho}_\mu) P^{*\nu\dagger}.$$
 (10)

With the Lagrangians above, we derive the effective potentials, which are related to the scattering amplitudes by

$$V(q) = -\frac{\mathcal{M}(q)}{4\sqrt{m_1 m_2 m_3 m_4}},$$
 (11)

where  $m_i$  (i = 1, 2, 3, 4) is the mass of the heavy meson in the initial or final state. By performing the Fourier transformation, we get the effective potentials in coordinate space

$$V(r) = \frac{1}{(2\pi)^3} \int d^3 q e^{i q \cdot r} V(q) F^2(q).$$
(12)

Here, F(q) is the form factor, which suppresses the contribution of high momenta, i.e., small distance. And the presence of such a form factor is dictated by the extended (quark) structure of the hadrons [117]. In this work, we adopt the exponentially parameterized form factor

$$F(q) = e^{(q_0^2 - q^2)/\Lambda^2},$$
 (13)

where  $q_0$  is the zero-th component of the four momentum of exchanged meson, and  $\Lambda$  is the cutoff. In the past, a monople form factor is usually used,

$$F_M(q) = \frac{\Lambda^2 - m_{ex}^2}{\Lambda^2 - q_0^2 + q^2}.$$
 (14)

However, in the case of the heavier  $\rho$ ,  $\omega$  or  $\phi$  exchanges,  $F_M(q)$  is suppressed by the numerator  $\Lambda^2 - m_{ex}^2$ , which may affect the final results. In this work, we will compare the effective potentials and the probability distributions obtained with these two form factors.

The expressions of the effective potentials are listed in Ta-

TABLE I: The effective potentials.

	$D^{*0}D^{+}$	$D^{*+}D^0$
$D^{*0}D^{+}$	$-V^D_{\rho^0} + V^D_{\omega} + V^C_{\rho^-} + V^C_{\pi^-}$	$2V^D_{\rho^-} - \tfrac{1}{2}V^C_{\rho^0} + \tfrac{1}{2}V^C_{\omega} - \tfrac{1}{2}\tilde{V}^C_{\pi^0} + \tfrac{1}{6}V^C_{\eta} + \tfrac{1}{3}V^C_{\eta'}$
$D^{*+}D^{0}$	$2V^{D}_{\rho^{-}} - \frac{1}{2}V^{C}_{\rho^{0}} + \frac{1}{2}V^{C}_{\omega} - \frac{1}{2}\tilde{V}^{C}_{\pi^{0}} + \frac{1}{6}V^{C}_{\eta} + \frac{1}{3}V^{C}_{\eta'}$	$-V^D_{\rho^0} + V^D_\omega + V^C_{\rho^+} + V^C_{\pi^+}$

ble I with the definition of the following functions

$$V_V^D = \frac{1}{4}\beta^2 g_V^2(\boldsymbol{\epsilon}_1 \cdot \boldsymbol{\epsilon}_3^{\dagger}) Y(\Lambda, m_V, q_0, r), \qquad (15)$$

$$V_{V}^{C} = 2\lambda^{2}g_{V}^{2} \left[ \frac{2}{3} \boldsymbol{\epsilon}_{1} \cdot \boldsymbol{\epsilon}_{4}^{\dagger} \nabla^{2} Y(\Lambda, \tilde{m}_{V}, q_{0}, r) - \frac{1}{3}S(\hat{\boldsymbol{r}}, \boldsymbol{\epsilon}_{1}, \boldsymbol{\epsilon}_{4}^{\dagger})r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} Y(\Lambda, \tilde{m}_{V}, q_{0}, r) \right], \quad (16)$$

$$V_{p}^{C} = \frac{g^{2}}{f_{\pi}^{2}} \left[ \frac{1}{3} \boldsymbol{\epsilon}_{1} \cdot \boldsymbol{\epsilon}_{4}^{\dagger} \nabla^{2} Y(\Lambda, \tilde{m}_{p}, q_{0}, r) + \frac{1}{3} S(\hat{\boldsymbol{r}}, \boldsymbol{\epsilon}_{1}, \boldsymbol{\epsilon}_{4}^{\dagger}) r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} Y(\Lambda, \tilde{m}_{p}, q_{0}, r) \right], \quad (17)$$

$$\tilde{V}_{p}^{C} = \frac{g^{2}}{f_{\pi}^{2}} \left[ \frac{1}{3} \boldsymbol{\epsilon}_{1} \cdot \boldsymbol{\epsilon}_{4}^{\dagger} \nabla^{2} U(\Lambda, \tilde{m}_{p}^{\prime}, q_{0}, r) + \frac{1}{3} S(\hat{\boldsymbol{r}}, \boldsymbol{\epsilon}_{1}, \boldsymbol{\epsilon}_{4}^{\dagger}) r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} U(\Lambda, \tilde{m}_{p}^{\prime}, q_{0}, r) \right].$$
(18)

In the above,  $\tilde{m}_V^2 = m_V^2 - q_0^2$ ,  $\tilde{m}_p^2 = m_p^2 - q_0^2$ ,  $\tilde{m}_p'^2 = q_0^2 - m_p^2$ and  $q_0 = \frac{m_2^2 - m_1^2 + m_3^2 - m_4^2}{2(m_3 + m_4)}$ . The functions *Y* and *U* are defined as follows

$$Y(\Lambda,\mu,q_0,r) = \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot \mathbf{r}} \frac{1}{\mathbf{q}^2 + \mu^2 - i\epsilon} e^{2(q_0^2 - \mathbf{q}^2)/\Lambda^2}$$
$$= -\frac{e^{2q_0^2/\Lambda^2}}{(2\pi)^2 r} \frac{\partial}{\partial r} \left\{ \frac{\pi}{2\mu} \left[ e^{-\mu r} + e^{\mu r} + e^{-\mu r} \right] \right\}$$
$$\times \operatorname{erf}\left( \frac{r\Lambda}{2\sqrt{2}} - \frac{\sqrt{2\mu}}{\Lambda} \right) - e^{\mu r} \operatorname{erf}\left( \frac{r\Lambda}{2\sqrt{2}} + \frac{\sqrt{2\mu}}{\Lambda} \right) \right] e^{2\mu^2/\Lambda^2} \right\}, \tag{19}$$

$$U(\Lambda,\mu,q_{0},r) = \int \frac{d^{3}q}{(2\pi)^{3}} e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \frac{1}{\boldsymbol{q}^{2}-\mu^{2}-i\epsilon} e^{2(q_{0}^{2}-\boldsymbol{q}^{2})/\Lambda^{2}}$$
$$= \frac{e^{2q_{0}^{2}/\Lambda^{2}}}{(2\pi)^{2}r} \frac{\partial}{\partial r} \left\{ \pi \left[ -\frac{1}{2i\mu} \left( e^{-i\mu r} \operatorname{erf} \left( \frac{r\Lambda}{2\sqrt{2}} - \frac{\sqrt{2}i\mu}{\Lambda} \right) - e^{i\mu r} \operatorname{erf} \left( \frac{r\Lambda}{2\sqrt{2}} + \frac{\sqrt{2}i\mu}{\Lambda} \right) \right) - \frac{i}{\mu} \cos(\mu r) \right] e^{-2\mu^{2}/\Lambda^{2}} \right\}.$$
(20)

The derivation of Eqs. (19)-(20) can be found in the Appendix of Ref. [118].

Using the Gaussian expansion method (GEM) [119], we solve the coupled-channel Schrödinger equation to find the bound state solutions,

$$\left(\hat{K} + \hat{M} + \hat{V}\right)\Psi = E\Psi.$$
(21)

Here,  $\hat{K} = \text{diag}(-\frac{\tilde{\Delta}}{2\mu_1}, -\frac{\tilde{\Delta}}{2\mu_2}, \cdots)$ ,  $\hat{M} = \text{diag}(0, M_2 - M_1, M_3 - M_1, \cdots)$ . For the central-force field problem, the full wave function can be decoupled into the radial part and the angular components. After integrating out the angular part, one obtains a one-dimensional radial Schödinger equation where the double derivative operator  $\tilde{\Delta} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r}\right) - \frac{l(l+1)}{r^2}$ . For the function  $U(\Lambda, \mu, q_0, r)$ , which is complex, we use the real part in order to solve the radial part of the stationary Schrödinger equation, which is real.

## III. THREE BODY DECAY OF $T_{cc}^+$

The main task of the current work is to derive a practical general formula for the calculation of the strong and radiative three-body decay of  $T_{cc}^+$ . Here, the  $T_{cc}^+$  state is considered as the  $D^{*+}D^0/D^{*0}D^+$  molecular state. The decays occur first with the t-channel exchange and then sequentially the vector charmed meson  $D^*$  decays into  $\pi D$  or  $\gamma D$ . The widths of the strong and radiative decay are

$$\Gamma_{T_{cc}^{+} \to \pi^{+,0} D^{0,+} D^{0}} = \frac{1}{(2\pi)^{3}} \frac{1}{32m_{T_{cc}^{+}}^{3}} \int dm_{34}^{2} dm_{45}^{2} \\ \times \frac{1}{3} \sum_{S_{c}^{T}} |\mathcal{M}_{T_{cc}^{+} \to \pi^{+,0} D^{0,+} D^{0}}|^{2} \frac{1}{S}, \quad (22)$$

and

$$\Gamma_{T_{cc}^{+} \to \gamma D^{+} D^{0}} = \frac{1}{(2\pi)^{3}} \frac{1}{32m_{T_{cc}^{+}}^{3}} \int dm_{34}^{2} dm_{45}^{2} \\ \times \frac{1}{3} \sum_{S_{z}^{T}, S_{z}^{\gamma}} |\mathcal{M}_{T_{cc}^{+} \to \gamma D^{+} D^{0}}|^{2}, \qquad (23)$$

respectively, where  $S_z^T$  and  $S_z^\gamma$  are the spin magnetic quantum numbers of  $T_{cc}^+$  and the photon respectively. 1/S is the symmetry factor, i.e., 1/2! for  $T_{cc}^+ \to \pi^+ D^0 D^0$ , 1 for  $T_{cc}^+ \to \pi^0 D^+ D^0$  and 1 for  $T_{cc}^+ \to \gamma D^+ D^0$ .  $\mathcal{M}_{T_{cc}^+ \to \pi^+ D^0 D^0}$ ,  $\mathcal{M}_{T_{cc}^+ \to \pi^0 D^+ D^0}$ and  $\mathcal{M}_{T_{cc}^+ \to \gamma D^+ D^0}$ .  $\mathcal{M}_{T_{cc}^+ \to \pi^+ D^0 D^0}$ ,  $\mathcal{M}_{T_{cc}^+ \to \pi^0 D^+ D^0}$ and  $\mathcal{M}_{T_{cc}^+ \to \gamma D^+ D^0}$  are the amplitudes corresponding to the Feynman diagrams in Fig. 1. In fact, there should be the same number of diagrams with pseudoscalar(vector) charmed meson associated with the vector(pseudoscalar) charmed meson in the t-channel process. However, after specifically calculating such diagrams, we found that their contributions are close to zero. Therefore, we do not show such diagrams in Fig. 1. The Feynmann diagrams Fig. 1 (a)-(d) are calculated using the Lagrangians in Eqs. (9)-(10) to obtain the strong decay amplitudes. To calculate the radiative decay amplitude, however, we need two more vertices describing the interactions of



FIG. 1: The diagrams of the strong and radiative decays of  $T_{cc}^+$ . The exchanged mesons are  $\pi$ ,  $\eta$ ,  $\eta'$ ,  $\rho$  and  $\omega$ . The same number of diagrams with charmed pseudoscalar (vector) meson connected to vector (pseudoscalar) meson are not shown because their contributions are close to zero, see the main text.

$$T_{cc}^+$$
 with  $\gamma D^{*0} D^0$  and  $\gamma D^{*+} D^+$ , i.e.,

$$\mathcal{L} = e g_{D^0 D^{*0} \gamma} \epsilon^{\mu \nu \alpha \beta} \partial_{\mu} A_{\nu} \nu_{\alpha} D_{\beta}^{*0} D^{0\dagger}, \qquad (24)$$

$$\mathcal{L} = e g_{D^* D^{*+} \gamma} \epsilon^{\mu \nu \alpha \beta} \partial_{\mu} A_{\nu} v_{\alpha} D_{\beta}^{*+} D^{+\dagger}.$$
(25)

The elementary charge e = 0.303. To determine the value of the couplings  $g_{D^0D^{*0}\gamma}$  and  $g_{D^+D^{*+}\gamma}$ , we use the decay width of  $D^*$  in Ref. [71]. And we get  $g_{D^0D^{*0}\gamma} = 1.911 \text{ GeV}^{-1}$  and  $g_{D^+D^{*+}\gamma} = 0.478 \text{ GeV}^{-1}$ .

Before calculating the amplitudes corresponding to the diagrams in Fig. 1, we apply a power counting to estimate the contribution of each diagram to simplify the calculation. The order of a quantity X is defined as  $O(X) = -\log_{10} X$ . Neglecting the higher-order contributions, we obtain the following decay amplitudes

$$\mathcal{M}_{T_{cc}^{+} \to \pi^{+} D^{0} D^{0}} = -\sqrt{\frac{2\pi m_{T_{cc}^{+}}}{E_{D^{*0}} E_{D^{+}}}} \int_{0}^{\infty} drr j_{0}(k_{5}r) u_{S}^{D^{*0} D^{+}}(r) \mathcal{A}_{\rho^{-}}^{(a)}$$
$$-\sqrt{\frac{2\pi m_{T_{cc}^{+}}}{E_{D^{*0}} E_{D^{+}}}} \int_{0}^{\infty} drr j_{0}(k_{4}r) u_{S}^{D^{*0} D^{+}}(r) \mathcal{A}_{\rho^{-}}^{(b)},$$
(26)

$$\mathcal{M}_{T_{cc}^{+} \to \pi^{0} D^{+} D^{0}} = -\sqrt{\frac{2\pi m_{T_{cc}^{+}}}{E_{D^{*0}} E_{D^{+}}}} \int_{0}^{\infty} drr j_{0}(k_{5}r) u_{S}^{D^{*0} D^{+}}(r) \mathcal{A}_{\rho^{-}}^{(c)} -\sqrt{\frac{2\pi m_{T_{cc}^{+}}}{E_{D^{*+}} E_{D^{0}}}} \int_{0}^{\infty} drr j_{0}(k_{4}r) u_{S}^{D^{*+} D^{0}}(r) \mathcal{A}_{\rho^{+}}^{(d)},$$
(27)

$$\mathcal{M}_{T_{cc}^{+} \to \gamma D^{+} D^{0}} = -\sqrt{\frac{2\pi m_{T_{cc}^{+}}}{E_{D^{*0}} E_{D^{+}}}} \int_{0}^{\infty} drr j_{0}(k_{5}r) u_{S}^{D^{*0}D^{+}}(r) \mathcal{A}_{\rho^{-}}^{(e)}$$
$$-\sqrt{\frac{2\pi m_{T_{cc}^{+}}}{E_{D^{*+}} E_{D^{0}}}} \int_{0}^{\infty} drr j_{0}(k_{4}r) u_{S}^{D^{*+}D^{0}}(r) \mathcal{A}_{\rho^{+}}^{(f)}.$$
(28)

Note here that we neglect the D-wave contributions which are much smaller than those of the S-wave. In Eqs. (26)-(28), the specific expressions of  $\mathcal{R}_{\rho^-}^{(a)}, \ldots$  are

$$\mathcal{A}_{\rho^{-}}^{(a)} = i \frac{4g\beta^2 g_V^2}{f_{\pi}} m_{D^0} m_{D^{*+}} \sqrt{m_{D^{*0}}} m_{D^+} \\ \times \frac{Y(\Lambda, \tilde{M}_1, \tilde{q}_1, r)}{k^2 - m_{D^{*+}}^2 + im_{D^{*+}} \Gamma_{D^{*+}}} \epsilon_1 \cdot k_3,$$
(29)

$$\mathcal{A}_{\rho^{-}}^{(b)} = i \frac{4g\beta^2 g_V^2}{f_{\pi}} m_{D^0} m_{D^{*+}} \sqrt{m_{D^{*0}} m_{D^+}} \\ \times \frac{Y(\Lambda, \tilde{M}_1, \tilde{q}_1, r)}{k'^2 - m_{D^{*+}}^2 + im_{D^{*+}} \Gamma_{D^{*+}}} \epsilon_1 \cdot k_3,$$
(30)

$$\mathcal{A}_{\rho^{-}}^{(c)} = -i \frac{2 \sqrt{2}g\beta^{2}g_{V}^{2}}{f_{\pi}} m_{D^{+}} m_{D^{*+}} \sqrt{m_{D^{*0}}m_{D^{0}}} \times \frac{Y(\Lambda, \tilde{M}_{1}, \tilde{q}_{1}, r)}{\Phi_{1} \cdot k_{2}} \epsilon_{1} \cdot k_{2}, \qquad (31)$$

 $\mathcal{A}^{(d)}_{o^+}$ 

$$= i \frac{2\sqrt{2}g\beta^2 g_V^2}{f_{\pi}} m_{D^0} m_{D^{*0}} \sqrt{m_{D^{*+}} m_{D^+}}$$

$$\times \frac{Y(\Lambda, \tilde{M}_{2}, \tilde{q}_{2}, r)}{k'^{2} - m_{D^{*0}}^{2} + im_{D^{*0}}\Gamma_{D^{*0}}} \epsilon_{1} \cdot k_{3}, \qquad (32)$$

$$\mathcal{A}_{\rho^{-}}^{(e)} = -i2eg_{D^{+}D^{*+}\gamma}\beta^{2}g_{V}^{2}m_{D^{+}}m_{D^{*+}}\sqrt{m_{D^{*0}}m_{D^{0}}} \\ \times \frac{Y(\Lambda,\tilde{M}_{1},\tilde{q}_{1},r)[\epsilon_{1}\cdot(\boldsymbol{k}_{3}\times\epsilon_{3}^{\dagger})]}{k^{2}-m_{D^{*+}}^{2}+im_{D^{*+}}\Gamma_{D^{*+}}},$$
(33)

$$\mathcal{A}_{\rho^{+}}^{(f)} = -i2eg_{D^{0}D^{*0}\gamma}\beta^{2}g_{V}^{2}m_{D^{0}}m_{D^{*0}}\sqrt{m_{D^{*+}}m_{D^{+}}} \\ \times \frac{Y(\Lambda,\tilde{M}_{2},\tilde{q}_{2},r)[\boldsymbol{\epsilon}_{1}\cdot(\boldsymbol{k}_{3}\times\boldsymbol{\epsilon}_{3}^{\dagger})]}{k^{\prime2}-m_{D^{*0}}^{2}+im_{D^{*0}}\Gamma_{D^{*0}}}.$$
(34)

Here,  $j_0(x) = \sin x/x$  is the spherical Bessel function of order 0.  $\tilde{q}_1 = \frac{m_{D^+}^2 - m_{D^{*0}}^2 + m_{D^{*+}}^2 - m_{D^0}^2}{2(m_{D^{*+}} + m_{D^0})}$ ,  $\tilde{q}_2 = \frac{m_{D^0}^2 - m_{D^{*+}}^2 + m_{D^{*0}}^2 - m_{D^+}^2}{2(m_{D^{*0}} + m_{D^+})}$ ,  $\tilde{M}_1 = \sqrt{m_{\rho^-}^2 - \tilde{q}_1^2}$ , and  $\tilde{M}_2 = \sqrt{m_{\rho^+}^2 - \tilde{q}_2^2}$ .  $u_0^{D^{*0}D^+}/r$  and  $u_0^{D^{*+}D^0}/r$  are the S-wave radial wave functions of  $D^{*0}D^+$  and  $D^{*+}D^0$  channels, respectively.  $k^{\mu} = k_3^{\mu} + k_4^{\mu}$  is the four-momentum of the intermediate  $D^*$  in Fig. 1 (a), (c) and (e), while  $k'^{\mu} = k_3^{\mu} + k_5^{\mu}$  is the four-momentum of the intermediate  $D^*$  meson in Fig. 1 (b), (d) and (f).

Note that the amplitudes in Eqs. (26)-(28) have the same tensor structure as those in Refs. [65, 71], i.e.,

$$\tilde{\mathcal{M}}_{T_{cc}^{+} \to \pi^{+} D^{0} D^{0}} \sim k_{\pi^{+} \mu} \frac{-g^{\mu \nu} + k^{\mu} k^{\nu} / m_{D^{*}}^{2}}{k^{2} - m_{D^{*}}^{2} + i m_{D^{*}} \Gamma_{D^{*}}} \epsilon_{\nu} \\
\sim k_{\pi^{+}} \cdot \epsilon,$$
(35)

$$\tilde{\mathcal{M}}_{T_{cc}^{+} \to \pi^{0} D^{+} D^{0}} \sim k_{\pi^{0} \mu} \frac{-g^{\mu \nu} + k^{\mu} k^{\nu} / m_{D^{*}}^{2}}{k^{2} - m_{D^{*}}^{2} + i m_{D^{*}} \Gamma_{D^{*}}} \epsilon_{\nu}$$
$$\sim k_{\pi^{0}} \cdot \epsilon, \qquad (36)$$

$$\tilde{\mathcal{M}}_{T_{cc}^{+} \to \gamma D^{+} D^{0}} \sim \frac{-g^{\mu \nu} + k^{\mu} k^{\nu} / m_{D^{*}}^{2}}{k^{2} - m_{D^{*}}^{2} + i m_{D^{*}} \Gamma_{D^{*}}} \epsilon^{\mu \nu \alpha \beta} k_{\gamma \mu} \epsilon_{\gamma \nu} k_{\alpha} \epsilon_{\nu}$$
$$\sim \epsilon \cdot (\mathbf{k}_{\gamma} \times \epsilon_{\gamma}). \tag{37}$$

### **IV. NUMERICAL RESULTS**

With the analytic results given before, we solve the Schrödinger equation to obtain the radial wave function, and then we calculate the decay width based on the radial wave function. In this section we will present the numerical results, perform the discussion and draw the conclusion.

# A. Comparison of the effective potentials with different form factors

As mentioned above, we use the exponential form factor in our calculation. For comparison, we take the  $D^{*+}D^0 \rightarrow$  $D^{*+}D^0$  process as an example, and plot the effective potentials with the exponential form factor (upper panel) and the monopole form factor (lower panel) in Fig. 2. It can clearly be seen that for the exponential form factor the pionexchange contribution and the vector-meson-exchange contribution are comparable in the short and medium interaction range whereas in the long range the pion-exchange contribution is dominant (see Fig. 2 (a) and (b)). In the case of the monopole form factor, the vector exchange contribution is relatively small compared to the pion exchange for the whole interaction range. This is because the masses of the vector mesons are larger than that of pion, and their contributions are suppressed by the numerator of the monopole form factor  $F_M(\mathbf{q}) = \frac{\Lambda^2 - m_{ex}^2}{\Lambda^2 - q_e^2 + \mathbf{q}^2}$ , where the cutoff  $\Lambda$  is around 1000 MeV.

### B. Isospin Symmetry Violation

The threshold difference betwen  $D^{*0}D^+$  and  $D^{*+}D^0$  is about 1.34 MeV, which is much larger than the the small binding energy of  $T_{cc}^+$ . Therefore, the isospin symmetry does not fully hold. Here, we explicitly consider the isospin symmetry violation effect for  $T_{cc}^+$ .

In the following, we apply two types of bases, one is for the different channels (labelled by 1) and the other is for the different isospin states (labelled by 2). The wave functions of  $T_{cc}^+$  in these two bases are

$$\psi_{T_{cc}^{+}} = \begin{pmatrix} \frac{u_{S}^{p^{*0}D^{+}}}{r} | {}^{3}S_{1} \rangle \\ \frac{u_{D}^{p^{*0}D^{+}}}{r} | {}^{3}D_{1} \rangle \\ \frac{u_{S}^{p^{*0}D^{+}}}{r} | {}^{3}S_{1} \rangle \\ \frac{u_{D}^{p^{*1}D^{0}}}{r} | {}^{3}D_{1} \rangle \end{pmatrix}, \ \psi_{T_{cc}^{+}}^{+} = \begin{pmatrix} -\frac{u_{S}^{p^{*0}D^{+}} + u_{S}^{p^{*1}D^{0}}}{\sqrt{2}r} | {}^{3}S_{1} \rangle \\ -\frac{u_{D}^{p^{*0}D^{+}} + u_{D}^{p^{*1}D^{0}}}{\sqrt{2}r} | {}^{3}D_{1} \rangle \\ \frac{u_{D}^{p^{*0}D^{+}} - u_{S}^{p^{*1}D^{0}}}{\sqrt{2}r} | {}^{3}D_{1} \rangle \end{pmatrix},$$

Λ	Ε	R <sub>rms</sub>	$P_{D^{*0}D^+({}^3S_1)}$	$P_{D^{*0}D^+({}^3D_1)}$	$P_{D^{*+}D^0({}^3S_1)}$	$P_{D^{*+}D^0(^3D_1)}$
782	200.3	6.7	23.9	0.1	75.7	0.2
786	236.2	6.2	25.3	0.1	74.3	0.2
790	274.5	5.7	26.6	0.1	73.0	0.2
794	315.3	5.4	27.8	0.1	71.9	0.2
798	358.6	5.1	28.9	0.1	70.7	0.2

respectively. These two wave functions are related by each other via the relation  $\psi'_{T_{cc}^+} = K \psi_{T_{cc}^+}$  with the transformation matrix

$$K = \begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0\\ 0 & -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0\\ 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{pmatrix}.$$
 (38)

With the effective potentials shown in Sec. II, we solve the Schrödinger equation using the basis 1. The numerical results are given in Table II. When the cutoff  $\Lambda$  is fixed at 782-798 MeV, we obtain a loosely bound state of  $D^*D$  with a binding energy 274 – 359 keV, which is in agreement with the experimental value from LHCb. The radial probability distributions of the different channels are shown in Fig. 3, in which the cutoffs are fixed at 798 MeV and 1187 MeV for the respective exponential and monopole form factors, so that the binding energies of these two cases are both around 360 keV. It can clearly be seen that the probability distributions of the S-wave are almost the same. Since the S-wave contributions are dominant, replacing the monopole form factor with the exponential form factor does not change the results too much.

Considering the basis 2, the probability of the isovector component within  $T_{cc}^+$  is

$$\rho_{10} = \int dr \frac{\left[u_{S}^{D^{*0}D^{+}} + u_{S}^{D^{*+}D^{0}}\right]^{2} + \left[u_{D}^{D^{*0}D^{+}} + u_{D}^{D^{*+}D^{0}}\right]^{2}}{2},(39)$$

while that of the isoscalar component is

$$\rho_{00} = \int dr \frac{\left[u_{S}^{D^{*0}D^{+}} - u_{S}^{D^{*+}D^{0}}\right]^{2} + \left[u_{D}^{D^{*0}D^{+}} - u_{D}^{D^{*+}D^{0}}\right]^{2}}{2}.$$
(40)

Numerically, if the cutoff is chosen as 790 - 798 MeV, the probabilities of  $D^{*+}D^0$  and  $D^{*0}D^+$  are 73.3%-71.0% and 26.7%-29.0% respectively. The probability of the isoscalar component is 90.2%-92.2% while that of the isovector component is 9.8%-7.8%, revealing a large isospin symmetry violation effect for the loosely bound state of  $T_{cc}^{+}$ .



FIG. 2: The effective potentials for the  $D^{*+}D^0 \rightarrow D^{*+}D^0$  process. (a) and (b) correspond to the exponential form factor with different interaction range, and (c) and (d) the monopole form factor with different interaction range. In all cases, the cutoff  $\Lambda$  is chosen to be 1000 MeV.

## C. The Three-Body Decay of $T_{cc}^+$

In Ref. [8], a relativistic P-wave two-body Breit-Wigner function with a Blatt-Weisskopf form factor is used as the natural resonance profile, while in Ref. [9], a unitarised Breit-Wigner profile is used in the analysis. The experimental widths are  $410 \pm 165 \pm 43^{+18}_{-38}$  keV and  $48 \pm 2^{+0}_{-14}$  keV, respectively. In this subsection, we will calculate the three-body decay width to investigate which result is reasonable. Using the analytic formulae given in Sec. III and the radial wave function of  $T^+_{cc}$ , we calculate the strong and radiative three-body decay of  $T^+_{cc}$  and list the numerical results in Table III.

For strong decays of  $T_{cc}^+$ , there are two possible channels, i.e.,  $T_{cc}^+ \rightarrow D^0 D^0 \pi^+$  and  $T_{cc}^+ \rightarrow D^0 D^+ \pi^0$ . Note that the decay  $T_{cc}^+ \rightarrow D^+ D^+ \pi^0$  is kinetically forbidden. If we fix the cutoff  $\Lambda$  at 782 – 798 MeV, the total width of these two channels is 17.3 - 22.4 keV. The partial decay widths of  $T_{cc}^+ \rightarrow D^0 D^0 \pi^+$ and  $T_{cc}^+ \rightarrow D^0 D^+ \pi^0$  are 14.8 keV and 7.6 keV respectively for the cutoff  $\Lambda = 798$  MeV. Note that if the isospin symmetry is exact, in which case the masses of  $D^{(*)+}$  and  $D^{(*)0}$  are equal, then these two partial widths should be exactly the same. This means that the difference between the partial widths is caused by the isospin symmetry violation effect.

For radiative decays, the only possible channel is  $T_{cc}^+ \rightarrow D^0 D^+ \gamma$ . If the cutoff is fixed as 782 – 798 MeV, the width of the radiative decay is 0.7 – 1.0 keV.

Consequently, the total decay width of  $T_{cc}^+$  is 23.4 keV with  $\Lambda = 798$  MeV, which is close to the lower limit of the experimental value  $48 \pm 2^{+0}_{-14}$  keV using a unitarised Breit-Wigner profile. This result therefore supports the conclusion that  $T_{cc}^+$  is a hadronic molecule composed of  $D^{*+}D^0/D^{*0}D^+$ .

TABLE III: The numerical results of the three-body decay widths as a function of the cutoff.  $\Gamma_R$ ,  $\Gamma_S$  and  $\Gamma_{tot}$  are the radiative, strong and total decay widths.

$\Lambda(MeV)$	782	786	790	794	798
$\Gamma_R$ (keV)	0.7	0.8	0.9	0.9	1.0
$\Gamma_S$ (keV)	17.3	18.6	19.9	21.2	22.4
$\Gamma_{tot}$ (keV)	18.0	19.4	20.7	22.1	23.4

### V. SUMMARY

In the present work, we revisit the tetraquark  $T_{cc}^+$  observed by the LHCb collaboration in 2021. We consider  $T_{cc}^+$  as a  $D^{*+}D^0/D^{*0}D^+$  molecular state with the isospin symmetry violation effect explicitly included using the one-boson-exchange potential model. Based on this result, we focus on the threebody strong and radiative decays of  $T_{cc}^+$  which can give us much more information about the structure of  $T_{cc}^+$ .

In order to calculate the effective potentials, we use the exponential form factor  $F(q) = e^{(q_0^2 - q^2)/\Lambda^2}$  instead of the monopole form factor  $F_M = \frac{\Lambda^2 - m_{e_x}^2}{\Lambda^2 - q_0^2 + q^2}$  which is frequently used in the one-boson-exchange potential model. We find that for the monopole form factor the contribution of  $\rho$  and  $\omega$  exchanges is much smaller than that of the pion-exchange over the whole interaction range. This is due to the suppression of the numerator of the monopole form factor. However, if we use the exponential form factor, we find a different behavior of the potentials, i.e., in the short and medium interaction range



FIG. 3: The radial probability distributions for different channels. (a) and (b) are the results using the exponential form factor with  $\Lambda = 798$  MeV, while (c) and (d) are the results using the monopole form factor with  $\Lambda = 1187$  MeV.

(around 0 - 0.3 fm and 0.3 - 2 fm, respectively), the vectormeson-exchange contributions are comparable to those of the pion-exchange, while in the long range the pion-exchange potential is dominant.

In this work, we use the exponential form factor in our calculation. After solving the Schrödinger equation, we obtain a loosely bound state of  $D^{*+}D^0/D^{*0}D^+$  with binding energy of 358.6 keV if the cutoff is fixed at 798 MeV. On the other hand, if we choose a cutoff 790 MeV, the binding energy is 274.5 keV. LHCb shows that the binding energy of the  $T_{cc}^+$ is  $-273 \pm 61 \pm 5_{-14}^{+11}$  keV and  $-360 \pm 40_{-0}^{+4}$  keV, when using a relativistic P-wave two-body Breit-Wigner function with a Blatt-Weisskopf form factor as the natural resonance profile and a unitarised Breit-Wigner profile, respectively. Thus, from a spectrum perspective, our results support that  $T_{cc}^+$  can be explained as a  $D^{*+}D^0/D^{*0}D^+$  molecular state.

Since the threshold difference between  $D^{*+}D^0$  and  $D^{*0}D^+$  is large compared to the small binding energy of  $T_{cc}^+$ , the isospin symmetry is violated for the  $D^{*+}D^0/D^{*0}D^+$  system. After including an isospin breaking effect, we find that the probability of the isoscalar component is about 91% and that of the isovector component is about 9%.

Since LHCb adopted two methods to analyze the experimental data of the  $T_{cc}^+$ , there are two different results of its mass and width. In order to check which one is more reasonable, we study the three-body decay of  $T_{cc}^+$  and develop for the first time a general method to calculate the three-body decay width. We find that if we take the experimental value of the binding energy, around 360 keV, as an input the total width is 23.4 keV, which is close to the experimental value using a unitarised Breit-Wigner profile. Therefore, our results for the three-body decay width of  $T_{cc}^+$  also support that the  $T_{cc}^+$  is a hadronic molecule of  $D^{*+}D^0/D^{*0}D^+$ .

### Acknowledgments

We would like to thank Prof. Shi-Lin Zhu for suggestive discussion. This project is supported by the Fundamental Research Funds for the Central Universities under Grant No. lzujbky-2022-sp02, the National Natural Science Foundation of China (NSFC) under Grant Nos. 11705069, 12335001, 11965016 and 12247101, the project for topnotch innovative talents of Gansu province, the National Key Research and Development Program of China under Contract No. 2020YFA0406400, and the Guangdong Basic and Applied Basic Research Foundation under Grant No. 2023A1515011704.

 N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo and C. Z. Yuan, "The XYZ states: experimental and theoretical status and perspectives,"

Phys. Rept. 873, 1-154 (2020).

- [2] H. X. Chen, W. Chen, X. Liu and S. L. Zhu, "The hiddencharm pentaquark and tetraquark states," Phys. Rept. 639, 1-121 (2016).
- [3] F. K. Guo, C. Hanhart, U. G. Meißner, Q. Wang, Q. Zhao and B. S. Zou, "Hadronic molecules," Rev. Mod. Phys. 90, no.1, 015004 (2018) [erratum: Rev. Mod. Phys. 94, no.2, 029901 (2022)].
- [4] Y. R. Liu, H. X. Chen, W. Chen, X. Liu and S. L. Zhu, "Pentaquark and Tetraquark states," Prog. Part. Nucl. Phys. 107, 237-320 (2019).
- [5] L. Meng, B. Wang, G. J. Wang and S. L. Zhu, "Chiral perturbation theory for heavy hadrons and chiral effective field theory for heavy hadronic molecules," Phys. Rept. **1019**, 1-149 (2023).
- [6] H. X. Chen, W. Chen, X. Liu, Y. R. Liu and S. L. Zhu, "An updated review of the new hadron states," Rept. Prog. Phys. 86, no.2, 026201 (2023).
- [7] M. Z. Liu, Y. W. Pan, Z. W. Liu, T. W. Wu, J. X. Lu and L. S. Geng, "Three ways to decipher the nature of exotic hadrons: multiplets, three-body hadronic molecules, and correlation functions," [arXiv:2404.06399 [hep-ph]].
- [8] R. Aaij *et al.* [LHCb], "Observation of an exotic narrow doubly charmed tetraquark," Nature Phys. 18, no.7, 751-754 (2022).
- [9] R. Aaij et al. [LHCb], "Study of the doubly charmed tetraquark T<sup>+</sup><sub>cc</sub>," Nature Commun. 13, no.1, 3351 (2022).
- [10] J. I. Ballot and J. M. Richard, "FOUR QUARK STATES IN ADDITIVE POTENTIALS," Phys. Lett. B 123, 449-451 (1983).
- [11] S. Zouzou, B. Silvestre-Brac, C. Gignoux and J. M. Richard, "FOUR QUARK BOUND STATES," Z. Phys. C 30, 457 (1986).
- [12] L. Heller and J. A. Tjon, "On the Existence of Stable Dimesons," Phys. Rev. D 35, 969 (1987).
- [13] J. Carlson, L. Heller and J. A. Tjon, "Stability of Dimesons," Phys. Rev. D 37, 744 (1988).
- [14] B. Silvestre-Brac, "Systematics of  $Q^2$  ( $\overline{Q}^2$ ) systems with a chromomagnetic interaction," Phys. Rev. D **46**, 2179-2189 (1992).
- [15] A. V. Manohar and M. B. Wise, "Exotic *QQqq̄* states in QCD," Nucl. Phys. B **399**, 17-33 (1993).
- [16] D. M. Brink and F. Stancu, "Tetraquarks with heavy flavors," Phys. Rev. D 57, 6778-6787 (1998).
- [17] M. S. Cook and H. R. Fiebig, "A Lattice study of interaction mechanisms in a heavy light meson meson system," [arXiv:hep-lat/0210054 [hep-lat]].
- [18] B. A. Gelman and S. Nussinov, "Does a narrow tetraquark  $cc\bar{u}d$  state exist?," Phys. Lett. B **551**, 296-304 (2003).
- [19] D. Janc and M. Rosina, "The  $T_{cc} = DD^*$  molecular state," Few Body Syst. **35**, 175-196 (2004).
- [20] J. Vijande, A. Valcarce and K. Tsushima, "Dynamical study of QQ-ūd mesons," Phys. Rev. D 74, 054018 (2006).
- [21] D. Ebert, R. N. Faustov, V. O. Galkin and W. Lucha, "Masses of tetraquarks with two heavy quarks in the relativistic quark model," Phys. Rev. D 76, 114015 (2007).
- [22] J. Vijande, A. Valcarce and N. Barnea, "Exotic meson-meson molecules and compact four-quark states," Phys. Rev. D 79, 074010 (2009).
- [23] Y. Yang, C. Deng, J. Ping and T. Goldman, "S-wave  $QQ\bar{q}\bar{q}$  state in the constituent quark model," Phys. Rev. D **80**, 114023 (2009).
- [24] J. M. Dias, S. Narison, F. S. Navarra, M. Nielsen and J. M. Richard, "Relation between  $T_{cc,bb}$  and  $X_{c,b}$  from QCD,"

Phys. Lett. B 703, 274-280 (2011).

- [25] S. Ohkoda, Y. Yamaguchi, S. Yasui, K. Sudoh and A. Hosaka, "Exotic mesons with double charm and bottom flavor," Phys. Rev. D 86, 034019 (2012).
- [26] M. L. Du, W. Chen, X. L. Chen and S. L. Zhu, "Exotic QQqq, QQqs and QQss states," Phys. Rev. D 87, no.1, 014003 (2013).
- [27] N. Li, Z. F. Sun, X. Liu and S. L. Zhu, "Coupled-channel analysis of the possible  $D^{(*)}D^{(*)}, \overline{B}^{(*)}\overline{B}^{(*)}$  and  $D^{(*)}\overline{B}^{(*)}$  molecular states," Phys. Rev. D **88**, no.11, 114008 (2013).
- [28] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, "Phenomenology of heavy meson chiral Lagrangians," Phys. Rept. 281, 145-238 (1997).
- [29] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, "Light vector resonances in the effective chiral Lagrangian for heavy mesons," Phys. Lett. B 292, 371-376 (1992).
- [30] G. J. Ding, "Are Y(4260) and Z<sup>+</sup><sub>2</sub> are D<sub>1</sub>D or D<sub>0</sub>D<sup>\*</sup> Hadronic Molecules?," Phys. Rev. D 79, 014001 (2009).
- [31] Y. Ikeda, B. Charron, S. Aoki, T. Doi, T. Hatsuda, T. Inoue, N. Ishii, K. Murano, H. Nemura and K. Sasaki, "Charmed tetraquarks T<sub>cc</sub> and T<sub>cs</sub> from dynamical lattice QCD simulations," Phys. Lett. B **729**, 85-90 (2014).
- [32] S. Q. Luo, K. Chen, X. Liu, Y. R. Liu and S. L. Zhu, "Exotic tetraquark states with the  $qq\bar{Q}\bar{Q}$  configuration," Eur. Phys. J. C **77**, no.10, 709 (2017).
- [33] T. Mehen, "Implications of Heavy Quark-Diquark Symmetry for Excited Doubly Heavy Baryons and Tetraquarks," Phys. Rev. D 96, no.9, 094028 (2017).
- [34] C. E. Fontoura, G. Krein, A. Valcarce and J. Vijande, "Production of exotic tetraquarks  $QQ\bar{q}\bar{q}$  in heavy-ion collisions at the LHC," Phys. Rev. D **99**, no.9, 094037 (2019).
- [35] H. Xu, B. Wang, Z. W. Liu and X. Liu, "DD\* potentials in chiral perturbation theory and possible molecular states," Phys. Rev. D 99, no.1, 014027 (2019) [erratum: Phys. Rev. D 104, no.11, 119903 (2021)].
- [36] A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, "Lattice Prediction for Deeply Bound Doubly Heavy Tetraquarks," Phys. Rev. Lett. 118, no.14, 142001 (2017).
- [37] A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, "Evidence for charm-bottom tetraquarks and the mass dependence of heavy-light tetraquark states from lattice QCD," Phys. Rev. D 99, no.5, 054505 (2019).
- [38] S. S. Agaev, K. Azizi and H. Sundu, "Strong decays of doublecharmed pseudoscalar and scalar *ccūd* tetraquarks," Phys. Rev. D 99, no.11, 114016 (2019).
- [39] Y. Tan, W. Lu and J. Ping, "Systematics of  $QQ\bar{q}\bar{q}$  in a chiral constituent quark model," Eur. Phys. J. Plus **135**, no.9, 716 (2020).
- [40] G. Yang, J. Ping and J. Segovia, "Doubly-heavy tetraquarks," Phys. Rev. D 101, no.1, 014001 (2020).
- [41] J. B. Cheng, S. Y. Li, Y. R. Liu, Z. G. Si and T. Yao, "Doubleheavy tetraquark states with heavy diquark-antiquark symmetry," Chin. Phys. C 45, no.4, 043102 (2021).
- [42] H. J. Lipkin, "A MODEL INDEPENDENT APPROACH TO MULTI - QUARK BOUND STATES," Phys. Lett. B 172, 242-247 (1986).
- [43] F. S. Navarra, M. Nielsen and S. H. Lee, "QCD sum rules study of  $QQ \bar{u}\bar{d}$  mesons," Phys. Lett. B 649, 166-172 (2007).
- [44] C. Semay and B. Silvestre-Brac, "Diquonia and potential models," Z. Phys. C **61**, 271-275 (1994).
- [45] J. Vijande, E. Weissman, A. Valcarce and N. Barnea, "Are there compact heavy four-quark bound states?," Phys. Rev. D

76, 094027 (2007).

- [46] S. H. Lee and S. Yasui, "Stable multiquark states with heavy quarks in a diquark model," Eur. Phys. J. C 64, 283-295 (2009).
- [47] M. Karliner and J. L. Rosner, "Discovery of doubly-charmed  $\Xi_{cc}$  baryon implies a stable ( $bb\bar{u}\bar{d}$ ) tetraquark," Phys. Rev. Lett. **119**, no.20, 202001 (2017).
- [48] E. J. Eichten and C. Quigg, "Heavy-quark symmetry implies stable heavy tetraquark mesons  $Q_i Q_j \bar{q}_k \bar{q}_l$ ," Phys. Rev. Lett. **119**, no.20, 202002 (2017).
- [49] Z. G. Wang, "Analysis of the axialvector doubly heavy tetraquark states with QCD sum rules," Acta Phys. Polon. B 49, 1781 (2018).
- [50] P. Junnarkar, N. Mathur and M. Padmanath, "Study of doubly heavy tetraquarks in Lattice QCD," Phys. Rev. D 99, no.3, 034507 (2019).
- [51] M. Z. Liu, T. W. Wu, M. Pavon Valderrama, J. J. Xie and L. S. Geng, "Heavy-quark spin and flavor symmetry partners of the X(3872) revisited: What can we learn from the one boson exchange model?," Phys. Rev. D 99, no.9, 094018 (2019).
- [52] Z. M. Ding, H. Y. Jiang and J. He, "Molecular states from  $D^{(*)}\bar{D}^{(*)}/B^{(*)}\bar{B}^{(*)}$  and  $D^{(*)}D^{(*)}/\bar{B}^{(*)}\bar{B}^{(*)}$  interactions," Eur. Phys. J. C **80**, no.12, 1179 (2020).
- [53] R. Molina, T. Branz and E. Oset, "A new interpretation for the  $D_{s2}^*(2573)$  and the prediction of novel exotic charmed mesons," Phys. Rev. D 82, 014010 (2010).
- [54] S. Pepin, F. Stancu, M. Genovese and J. M. Richard, "Tetraquarks with color blind forces in chiral quark models," Phys. Lett. B 393, 119-123 (1997).
- [55] J. Vijande, F. Fernandez, A. Valcarce and B. Silvestre-Brac, "Tetraquarks in a chiral constituent quark model," Eur. Phys. J. A 19, 383 (2004).
- [56] G. Q. Feng, X. H. Guo and B. S. Zou, "QQ'ūd bound state in the Bethe-Salpeter equation approach," [arXiv:1309.7813 [hep-ph]].
- [57] C. Deng, H. Chen and J. Ping, "Systematical investigation on the stability of doubly heavy tetraquark states," Eur. Phys. J. A 56, no.1, 9 (2020).
- [58] M. Z. Liu, J. J. Xie and L. S. Geng, " $X_0(2866)$  as a  $D^*\bar{K}^*$  molecular state," Phys. Rev. D **102**, no.9, 091502 (2020).
- [59] W. Park, S. Noh and S. H. Lee, "Masses of the doubly heavy tetraquarks in a constituent quark model," Nucl. Phys. A 983, 1-19 (2019).
- [60] Q. F. Lü, D. Y. Chen and Y. B. Dong, "Masses of doubly heavy tetraquarks T<sub>QQ</sub>' in a relativized quark model," Phys. Rev. D 102, no.3, 034012 (2020).
- [61] E. Braaten, L. P. He and A. Mohapatra, "Masses of doubly heavy tetraquarks with error bars," Phys. Rev. D 103, no.1, 016001 (2021).
- [62] R. Chen, Q. Huang, X. Liu and S. L. Zhu, "Predicting another doubly charmed molecular resonance T<sup>'+</sup><sub>cc</sub> (3876)," Phys. Rev. D 104, no.11, 114042 (2021).
- [63] A. Feijoo, W. H. Liang and E. Oset, " $D^0D^0\pi^+$  mass distribution in the production of the  $T_{cc}$  exotic state," Phys. Rev. D **104**, no.11, 114015 (2021).
- [64] S. Fleming, R. Hodges and T. Mehen, " $T_{cc}^+$  decays: Differential spectra and two-body final states," Phys. Rev. D 104, no.11, 116010 (2021).
- [65] L. Meng, G. J. Wang, B. Wang and S. L. Zhu, "Probing the long-range structure of the T<sup>+</sup><sub>cc</sub> with the strong and electromagnetic decays," Phys. Rev. D 104, no.5, 051502 (2021).
- [66] M. Albaladejo, " $T_{cc}^+$  coupled channel analysis and predictions," Phys. Lett. B **829**, 137052 (2022).
- [67] L. R. Dai, R. Molina and E. Oset, "Prediction of new  $T_{cc}$

states of *D*\**D*\* and *D*<sub>s</sub>\**D*\* molecular nature," Phys. Rev. D **105**, no.1, 016029 (2022) [erratum: Phys. Rev. D **106**, no.9, 099902 (2022)].

- [68] C. Deng and S. L. Zhu, "T<sup>+</sup><sub>cc</sub> and its partners," Phys. Rev. D 105, no.5, 054015 (2022).
- [69] M. L. Du, V. Baru, X. K. Dong, A. Filin, F. K. Guo, C. Hanhart, A. Nefediev, J. Nieves and Q. Wang, "Coupled-channel approach to T<sup>+</sup><sub>cc</sub> including three-body effects," Phys. Rev. D 105, no.1, 014024 (2022).
- [70] H. W. Ke, X. H. Liu and X. Q. Li, "Possible molecular states of  $D^{(*)}D^{(*)}$  and  $B^{(*)}B^{(*)}$  within the Bethe–Salpeter framework," Eur. Phys. J. C **82**, no.2, 144 (2022).
- [71] X. Z. Ling, M. Z. Liu, L. S. Geng, E. Wang and J. J. Xie, "Can we understand the decay width of the  $T_{cc}^+$  state?," Phys. Lett. B **826**, 136897 (2022).
- [72] Y. Liu, M. A. Nowak and I. Zahed, "Holographic tetraquarks and the newly observed  $T_{cc}^+$  at LHCb," Phys. Rev. D **105**, no.5, 054021 (2022).
- [73] K. Chen, R. Chen, L. Meng, B. Wang and S. L. Zhu, "Systematics of the heavy flavor hadronic molecules," Eur. Phys. J. C 82, no.7, 581 (2022).
- [74] X. K. Dong, F. K. Guo and B. S. Zou, "A survey of heavy-heavy hadronic molecules," Commun. Theor. Phys. 73, no.12, 125201 (2021).
- [75] Q. Xin and Z. G. Wang, "Analysis of the doubly-charmed tetraquark molecular states with the QCD sum rules," Eur. Phys. J. A 58, no.6, 110 (2022).
- [76] Y. Huang, H. Q. Zhu, L. S. Geng and R. Wang, "Production of  $T_{cc}^+$  exotic state in the  $\gamma p \rightarrow D^+ \bar{T}_{cc}^- \Lambda_c^+$  reaction," Phys. Rev. D **104**, no.11, 116008 (2021).
- [77] H. Ren, F. Wu and R. Zhu, "Hadronic Molecule Interpretation of T<sup>+</sup><sub>cc</sub> and Its Beauty Partners," Adv. High Energy Phys. 2022, 9103031 (2022).
- [78] M. J. Zhao, Z. Y. Wang, C. Wang and X. H. Guo, "Investigation of the possible DD
  <sup>\*</sup>/BB
  <sup>\*</sup> and DD<sup>\*</sup>/BB
  <sup>\*</sup> bound states," Phys. Rev. D 105, no.9, 096016 (2022).
- [79] S. S. Agaev, K. Azizi and H. Sundu, "Hadronic molecule model for the doubly charmed state T<sup>+</sup><sub>cc</sub>," JHEP 06, 057 (2022).
- [80] J. He and X. Liu, "The quasi-fission phenomenon of double charm  $T_{cc}^+$  induced by nucleon," Eur. Phys. J. C **82**, no.4, 387 (2022).
- [81] M. Padmanath and S. Prelovsek, "Signature of a Doubly Charm Tetraquark Pole in *DD*<sup>\*</sup> Scattering on the Lattice," Phys. Rev. Lett. **129**, no.3, 032002 (2022).
- [82] M. Albaladejo and J. Nieves, "Compositeness of S-wave weakly-bound states from next-to-leading order Weinberg's relations," Eur. Phys. J. C 82, no.8, 724 (2022).
- [83] J. B. Cheng, Z. Y. Lin and S. L. Zhu, "Double-charm tetraquark under the complex scaling method," Phys. Rev. D 106, no.1, 016012 (2022).
- [84] L. M. Abreu, "A note on the possible bound  $D^{(*)}D^{(*)}, \bar{B}^{(*)}\bar{B}^{(*)}$  and  $D^{(*)}\bar{B}^{(*)}$  states," Nucl. Phys. B **985**, 115994 (2022).
- [85] S. Chen, C. Shi, Y. Chen, M. Gong, Z. Liu, W. Sun and R. Zhang, " $T_{cc}^+$ (3875) relevant  $DD^*$  scattering from  $N_f = 2$  lattice QCD," Phys. Lett. B **833**, 137391 (2022).
- [86] Z. S. Jia, M. J. Yan, Z. H. Zhang, P. P. Shi, G. Li and F. K. Guo, "Hadronic decays of the heavy-quark-spin molecular partner of *T*<sup>+</sup><sub>cc</sub>," Phys. Rev. D **107**, no.7, 074029 (2023).
- [87] B. Wang and L. Meng, "Revisiting the  $DD^*$  chiral interactions with the local momentum-space regularization up to the third order and the nature of  $T_{cc}^+$ ," Phys. Rev. D **107**, no.9, 094002 (2023).
- [88] L. Dai, S. Fleming, R. Hodges and T. Mehen, "Strong decays of T<sup>+</sup><sub>cc</sub> at NLO in an effective field theory," Phys. Rev. D 107,

no.7, 076001 (2023).

- [89] Y. Li, Y. B. He, X. H. Liu, B. Chen and H. W. Ke, "Searching for doubly charmed tetraquark candidates  $T_{cc}$  and  $T_{cc\bar{s}}$  in  $B_c$  decays," Eur. Phys. J. C **83**, no.3, 258 (2023).
- [90] Y. Hu, J. Liao, E. Wang, Q. Wang, H. Xing and H. Zhang, "Production of doubly charmed exotic hadrons in heavy ion collisions," Phys. Rev. D 104, no.11, L111502 (2021).
- [91] X. Chen and Y. Yang, "Doubly-heavy tetraquark states ccūd and bbūd," Chin. Phys. C 46, no.5, 054103 (2022).
- [92] K. Azizi and U. Özdem, "Magnetic dipole moments of the  $T_{cc}^+$  and  $Z_V^{++}$  tetraquark states," Phys. Rev. D **104**, no.11, 114002 (2021).
- [93] L. M. Abreu, H. P. L. Vieira and F. S. Navarra, "Multiplicity of the doubly charmed state T<sup>+</sup><sub>cc</sub> in heavy-ion collisions," Phys. Rev. D 105, no.11, 116029 (2022).
- [94] I. Vidana, A. Feijoo, M. Albaladejo, J. Nieves and E. Oset, "Femtoscopic correlation function for the T<sub>cc</sub>(3875)<sup>+</sup> state," Phys. Lett. B 846, 138201 (2023).
- [95] L. R. Dai, L. M. Abreu, A. Feijoo and E. Oset, "The isospin and compositeness of the *T<sub>cc</sub>*(3875) state," Eur. Phys. J. C 83, no.10, 983 (2023).
- [96] B. R. He, M. Harada and B. S. Zou, "Quark model with hidden local symmetry and its application to  $T_{cc}$ ," Phys. Rev. D **108**, no.5, 054025 (2023).
- [97] Z. S. Jia, Z. H. Zhang, G. Li and F. K. Guo, "Radiative decays of the heavy-quark-spin molecular partner of T<sup>+</sup><sub>cc</sub>," Phys. Rev. D 108, no.9, 094038 (2023).
- [98] Y. D. Lei and H. S. Li, "Electromagnetic properties of the T<sup>+</sup><sub>cc</sub> molecular states," Phys. Rev. D 109, no.7, 076014 (2024).
- [99] M. Sakai and Y. Yamaguchi, "Analysis of  $T_{cc}$  and  $T_{bb}$  based on the hadronic molecular model and their spin multiplets," Phys. Rev. D **109**, no.5, 054016 (2024).
- [100] V. Montesinos, M. Albaladejo, J. Nieves and L. Tolos, "Properties of the  $T_{cc}(3875)^+$  and  $T_{c\bar{c}}(3875)^-$  and their heavy-quark spin partners in nuclear matter," Phys. Rev. C **108**, no.3, 035205 (2023).
- [101] S. S. Agaev, K. Azizi and H. Sundu, "Newly observed exotic doubly charmed meson  $T_{cc}^+$ ," Nucl. Phys. B **975**, 115650 (2022).
- [102] Y. Jin, S. Y. Li, Y. R. Liu, Q. Qin, Z. G. Si and F. S. Yu, "Color and baryon number fluctuation of preconfinement system in production process and T<sub>cc</sub> structure," Phys. Rev. D 104, no.11, 114009 (2021).
- [103] T. W. Wu and Y. L. Ma, "Doubly heavy tetraquark multiplets as heavy antiquark-diquark symmetry partners of heavy baryons," Phys. Rev. D 107, no.7, L071501 (2023).
- [104] X. Z. Weng, W. Z. Deng and S. L. Zhu, "Doubly heavy

tetraquarks in an extended chromomagnetic model \*," Chin. Phys. C **46**, no.1, 013102 (2022).

- [105] L. M. Abreu, F. S. Navarra, M. Nielsen and H. P. L. Vieira, "Interactions of the doubly charmed state T<sup>+</sup><sub>cc</sub> with a hadronic medium," Eur. Phys. J. C 82, no.4, 296 (2022).
- [106] Y. Kim, M. Oka and K. Suzuki, "Doubly heavy tetraquarks in a chiral-diquark picture," Phys. Rev. D 105, no.7, 074021 (2022).
- [107] S. Noh and W. Park, "Nonrelativistic quark model analysis of  $T_{cc}$ ," Phys. Rev. D **108**, no.1, 014004 (2023).
- [108] X. Y. Liu, W. X. Zhang and D. Jia, "Doubly heavy tetraquarks: Heavy quark bindings and chromomagnetically mixings," Phys. Rev. D 108, no.5, 054019 (2023).
- [109] D. Wang, K. R. Song, W. L. Wang and F. Huang, "Spectrum of S- and P-wave  $cc\bar{q}\bar{q}'(\bar{q},\bar{q}'=\bar{u},\bar{d},\bar{s})$  systems in a chiral SU(3) quark model," Phys. Rev. D **109**, no.7, 074026 (2024).
- [110] M. J. Yan and M. P. Valderrama, "Subleading contributions to the decay width of the T<sup>+</sup><sub>cc</sub> tetraquark," Phys. Rev. D 105, no.1, 014007 (2022).
- [111] C. R. Deng and S. L. Zhu, "Decoding the double heavy tetraquark state T<sup>+</sup><sub>cc</sub>," Sci. Bull. 67, 1522 (2022).
- [112] J. Z. Wang, Z. Y. Lin and S. L. Zhu, "Cut structures and an observable singularity in the three-body threshold dynamics: The  $T_{cc}^+$  case," Phys. Rev. D **109**, no.7, L071505 (2024).
- [113] J. Shi, E. Wang and Q. Wang, "Investigating the isospin property of T<sup>+</sup><sub>cc</sub> from its Dalitz plot distribution," Phys. Rev. D 106, no.9, 096012 (2022).
- [114] T. Kinugawa and T. Hyodo, "Compositeness of T<sub>cc</sub> and X(3872) by considering decay and coupled-channels effects," Phys. Rev. C 109, no.4, 045205 (2024).
- [115] L. R. Dai, J. Song and E. Oset, "Evolution of genuine states to molecular ones: The  $T_{cc}(3875)$  case," Phys. Lett. B **846**, 138200 (2023).
- [116] Y. Ma, L. Meng, Y. K. Chen and S. L. Zhu, "Doubly heavy tetraquark states in the constituent quark model using diffusion Monte Carlo method," Phys. Rev. D 109, no.7, 074001 (2024).
- [117] R. Machleidt, K. Holinde and C. Elster, "The Bonn Meson Exchange Model for the Nucleon Nucleon Interaction," Phys. Rept. 149, 1-89 (1987).
- [118] W. He, D. S. Zhang and Z. F. Sun, "The  $Z_b$  states as the mixture of the molecular and diquark-anti-diquark components within the effective field theory," [arXiv:2403.02099 [hep-ph]].
- [119] E. Hiyama, Y. Kino and M. Kamimura, "Gaussian expansion method for few-body systems," Prog. Part. Nucl. Phys. 51, 223-307 (2003).