## Bayesian Multilevel Compositional Data Analysis: Introduction, Evaluation, and Application

Flora Le ${ }^{1}$<br>Tyman E. Stanford ${ }^{2}$<br>Dorothea Dumuid ${ }^{2}$<br>Joshua F. Wiley ${ }^{1}$<br>${ }^{1}$ School of Psychological Sciences and Turner Institute for Brain and Mental Health, Monash University<br>${ }^{2}$ Alliance for Research in Exercise, Nutrition and Activity, Allied Health and Human Performance, University of South Australia


#### Abstract

Multilevel compositional data commonly occur in various fields, particularly in intensive, longitudinal studies using ecological momentary assessments. Examples include data repeatedly measured over time that are non-negative and sum to a constant value, such as sleep-wake movement behaviours in a 24 -hour day. This article presents a novel methodology for analysing multilevel compositional data using a Bayesian inference approach. This method can be used to investigate how reallocation of time between sleep-wake movement behaviours may be associated with other phenomena (e.g., emotions, cognitions) at a daily level. We explain the theoretical details of the data and the models, and outline the steps necessary to implement this method. We introduce the R package multilevelcoda to facilitate the application of this method and illustrate using a real data example. An extensive parameter recovery simulation study verified the robust performance of the method. Across all simulation conditions investigated in the simulation study, the model had minimal convergence issues (convergence rate $>99 \%$ ) and achieved excellent quality of parameter estimates and inference, with an average bias of 0.00 (range $-0.09,0.05$ ) and coverage of 0.95 (range $0.93,0.97$ ). We conclude the article with recommendations on the use of the Bayesian compositional multilevel modelling approach, and hope to promote wider application of this method to answer robust questions using the increasingly available data from intensive, longitudinal studies.


## Translational Abstract

In intensive, longitudinal studies, researchers often seek to understand how behaviours, emotion, and cognition interact in everyday life. When movement behaviours, such as sleep, physical activity, and sedentary behaviour, are repeatedly measured over time, they have a complex multilevel compositional structure. Therefore, analyses must address their unique properties to produce analytically and practically meaningful insights. In this article, we present a novel methodology to analyse multilevel compositional data using Bayesian inference. We describe the data structure and outline the steps necessary to implement this method. To facilitate its practical application, we introduce an R package multilevelcoda and demonstrate how to conduct this analysis and interpret the results in a real data application. We also verify the performance of this method through a simulation study. Across the tested conditions, the model had minimal convergence issues and produced unbiased parameter estimates and valid inferences. This method is recommended for analysing data with a multilevel compositional structure, such as daily movement behaviours. With the growing data availability from wearable devices and EMAs in psychological
research, we encourage the research community to apply this method in their own work to shed novel light on how reallocation of movement behaviours are associated with emotional experiences and cognitive processes in real life.

Keywords: multilevel modeling, compositional data analysis, isotemporal substitution model, Bayesian inference, intensive longitudinal data

## Bayesian Multilevel Compositional Data Analysis: <br> Introduction, Evaluation, and Application

Across the fields of clinical and health science research, the rise of ecological momentary assessments (EMAs), wearable devices, daily-intensive longitudinal study designs, has enabled researchers to capture real-time phenomena, such as health behaviours, cognition and emotion, and how they interact in everyday life. In many contexts, data that are repeatedly measured have a complex multilevel compositional structure. Some examples of multilevel compositional data include 24-hour sleep-wake movement behaviours (e.g., time spent in behaviours like sleep, physical activity, and sedentary behaviour, during the 24 -hour day), dietary behaviour (e.g., proportions of total caloric intake of macronutrients like proteins, fats and carbohydrates). These data are compositional, as they can be expressed as percentages (or proportions) or units that sum to a total constant value. They also have a multilevel structure, as they are usually measured across time and nested within clusters (e.g., daily measures of behaviours nested within people). Further, multilevel compositional data often contain two sources of variability: between-person (i.e., differences between individuals) and within-person (i.e., changes within individuals, such as the deviation from the average of an individual). These two unique processes open up an avenue to investigate beyond the average changes over time in a person and towards how the individual might fluctuate substantially around that mean, frequently using a multilevel modelling (MLM) approach.

Despite the growing evidence from EMA studies supporting the independent associations between daily movement behaviours and emotional experiences and cognitive processes (Hartson et al., 2023), there is uncertainty about how daily reallocation of time across behaviours are associated with these phenomena. Specifically, existing analyses often focus on one of the behaviour components and neglect the compositional and/or multilevel nature of movement behaviours. Due to the fixed 24 hours in a day, a person who increases time spent in one behaviour must compensate by decreasing time spent in one or more other behaviours. For example, they can only increase time spent in physical activity by spending less time in other behaviours (e.g., sleep, sedentary behaviour). This is illustrated in Figure 1. In epidemiological research, the compositional approach, particularly the compositional isotemporal substitution model (Dumuid et al., 2018, 2019), has been increasingly employed to investigate how reallocation of one behaviour to another, while keeping the total time fixed, are associated with physical and mental health outcomes (Janssen et al., 2020; Groves et al., 2023; Grgic et al., 2018; Miatke et al.,
2023). Existing epidemiological studies are, however, often interested in outcomes that are stable over time (i.e., cross-sectional), such as incidence of diseases. In contrast, longitudinal studies using EMA designs often seek to identify the between- and within-person processes in the variable trajectories as they emerge across time. With the growing data availability from EMAs in psychological research, emerging advanced approaches that accommodate the theoretical properties of multilevel compositional data could, therefore, facilitate more conceptually and analytically meaningful analyses, leading to new health insights.

## Figure 1

An example composition of time spent in movement behaviours of an individual is shown in Panel A. Due to the fixed 24-hour day, the individual can reallocate time across behaviours differently, but they must keep the total time fixed. They can increase an hour of moderate-to-vigorous physical activity at the expense of sleep (Panel B). Alternatively, they can increase an hour of moderate-to-vigorous physical activity at the expense of sedentary behaviour (Panel C).
A


B

C

Minutes in $\square$ Moderate-to-vigorous Physical Activity $\square$ Sleep $\square$ Awake in Bed Light Physical Activity $\square$ Sedentary Behaviour

Bayesian MLMs offer great flexibility in modelling statistical phenomena that exist in different levels. Although model estimation by Bayesian and frequentist approaches can include both population-level and group-level effects (also referred to as fixed and random effects), Bayesian MLMs have been increasingly employed due to their flexible model estimation and straightforward interpretation of results. Importantly, a fundamental aspect of Bayesian framework is quantifying uncertainty parameters and models using probability theory, which is not provided by frequentist approaches (Gelman et al., 2013; Wagenmakers et al., 2016). Using Bayes's theorem, the posterior distribution of any parameters can be
derived to reflect the knowledge of the model (i.e., prior) and the observed data (i.e., likelihood). This allows for making probabilistic inferences about parameters (or their functions), enabling any post-hoc analyses involving calculated quantities to be directly and intuitively estimated. With the rapid development of software for Bayesian posterior sampling, including the probabilistic programming language Stan (Carpenter et al., 2017), easily accessible front-end R package brms (Bürkner, 2017) for model fitting with similar syntax to frequentist MLMs, (i.e., lme4, Bates et al., 2015), MLMs with complex data structure, such as multilevel composition, can be conceptually simple and computationally tractable in Bayesian framework.

In this paper, we present a novel methodology to model multilevel compositional data using Bayesian inference. This method can be used to address the interdependence of day-to-day movement behaviour composition and investigate how reallocation of movement behaviours may be associated with other phenomena at a daily level. We start by describing the structure of multilevel compositional data and the model specification, with a focus on models with compositional variables as predictors at both between- and within-person levels. To facilitate the implementation of this approach in a robust and principled workflow, we introduce the R package multilevelcoda (Le and Wiley, 2023; Le et al., 2024) and present its applications on a data set with daily repeated measures. We then use the results from the real data application as a starting point of a simulation study to assess the accuracy and precision of parameter estimates. We conclude the paper with a discussion on the use of this approach and recommendations on its practical applications.

## Multilevel Compositional Data

Detailed structure of single-level compositional data and the relevant transformations have been described previously (Van den Boogaart and Tolosana-Delgado, 2013; Dumuid et al., 2018; Smithson and Broomell, 2022). Here, we focus on the compositional data with multilevel structure, considering both the between (i.e., cluster-specific mean) and within (i.e., mean-centered deviate) levels, and two-level data hierarchy (i.e., daily observations nested within people).

For $d=1, \ldots, D$ part composition of $j=1, \ldots, J$ individuals across $i=1, \ldots, I$ time points, a multilevel composition is defined as a vector of $D$ positive components that sum to a constant $\kappa$, observed at the $i^{\text {th }}$ time point for the $j^{\text {th }}$ person. We denote the multilevel composition and its between- and
within-person components as

$$
\begin{aligned}
x_{i j} & =\left(x_{1 i j}, x_{2 i j}, \ldots, x_{D i j}\right), \text { where } \sum_{i=1}^{D} x_{i}=\kappa \\
& =\mathscr{C}\left(x_{1 \cdot j}^{(b)} \cdot x_{1 i j}^{(w)}, x_{2 \cdot j}^{(b)} \cdot x_{2 i j}^{(w)}, \ldots, x_{D \cdot j}^{(b)} \cdot x_{D i j}^{(w)}\right) \\
& =x_{\cdot j}^{(b)} \oplus x_{i j}^{(w)}
\end{aligned}
$$

in which superscripts $(b)$ and $(w)$ denote the between and within components of the composition, $\oplus$ is the perturbation operation on the simplex (closure operation applied to the element-wise product), and $\mathscr{C}\left(x_{i j}\right)=\kappa^{x}$ is the closure operation that ensures the compositional parts of the vector $x_{i j}$ sum to the constant $\kappa$ (Aitchison, 1986). The between- and within-person subcompositions are essentially compositions themselves as

$$
\begin{aligned}
& x_{\cdot j}^{(b)}=\mathscr{C}\left(x_{1 \cdot j}^{(b)}, x_{2 \cdot j}^{(b)}, \ldots, x_{D \cdot j}^{(b)}\right) \text { and } \\
& x_{i j}^{(w)}=\mathscr{C}\left(x_{1 i j}^{(w)}, x_{2 i j}^{(w)}, \ldots, x_{D i j}^{(w)}\right) .
\end{aligned}
$$

Compositions are elements of the $D-\operatorname{simplex}\left(\mathscr{S}^{D}\right)$ whose properties are incompatible with standard mathematical operations (e.g., addition, multiplication) and statistical models (e.g., linear regression) (for detailed discussion on the properties of compositional data and their consequences, see Aitchison, 1994, 1986).

Compositional data analysis (CoDA; Aitchison, 1986; Pawlowsky-Glahn and Buccianti, 2011) is a log-ratio analysis paradigm that utilises the relative information contained in compositional data. Although several transformations exist, isometric log-ratio (ilr) transformation (Egozcue et al., 2003) preserves the metric properties of the composition and accounts for the dependencies between its parts, so that standard statistical methods can be applied to the transformed data. This method involves transforming the $D$-part composition in the simplex $\left(\mathscr{S}^{D}\right)$ to a set of $(D-1)$-dimension ilr coordinates in the Euclidean space $\left(\mathbb{R}^{D-1}\right)$ isometrically (i.e., preserving angles and distances). This isometry is constructed using the sequential binary partition (SBP), a $D \times(D-1)$ matrix that maps the $D$ compositional parts and their membership in the $(D-1)$ ilr coordinates (Egozcue and Pawlowsky-Glahn, 2005). A SBP is obtained by first partitioning the compositional parts into two non-empty sets, where one set corresponds to the first ilr coordinate's numerator (coded as +1 ) and the other set corresponds to the the first ilr coordinate's denominator (coded as -1 ), and where applicable, compositional part uninvolved in the ilr are coded as 0 .

Using this principle, each of the previously constructed sets are recursively partitioned into two non-empty sets until no further partitions of the subcompositional parts are possible (after $D-1$ steps). ilr coordinates can be interpreted as log ratio of the subcomposition in the numerator in relation to the subcomposition in the denominator. Although the order of parts in composition might be mathematically arbitrary, the SBP can be reconstructed to be intuitive and interpretable depending on application.

For a $D$-part composition $x_{i j} \in \mathscr{S}^{D}$, the corresponding set of $D-1$ ilr coordinates is $\left(z_{1 i j}, z_{2 i j}, \ldots, z_{(D-1) i j}\right)=z_{i j} \in \mathbb{R}^{D-1}$. The individual $k^{t h}(k=1,2, \ldots, D-1)$ ilr coordinate observed at time point $i$ for individual $j, z_{k i j}$, can be expressed as

$$
z_{k i j}=\sqrt{\frac{r_{k i j} s_{k i j}}{r_{k i j}+s_{k i j}}} \ln \left(\frac{\tilde{x}_{R_{k i j}}}{\tilde{x}_{S_{k i j}}}\right), \quad k=1,2, \ldots, D-1
$$

where $\tilde{x}_{R_{k i j}}$ and $\tilde{x}_{S_{k i j}}$ are the geometric mean of a subcomposition in the numerator $\left(R_{k i j}\right)$ and the denominator $\left(S_{k i j}\right)$, respectively, with $r_{k i j}$ and $s_{k i j}$ being the size of the sets $R_{k i j}$ and $S_{k i j}$, respectively, and $\sqrt{\frac{r_{k i j} s_{k i j}}{r_{k i j}+s_{k i j}}}$ being a normalising constant.

As the ilr coordinates exist in the Euclidean space $\mathbb{R}^{D-1}$, the decomposition of the $(D-1)$-dimension ilr coordinates $z_{i j}$ can be equivalently be decomposed into its between- and within-person components using the usual addition operation, that is

$$
\begin{aligned}
z_{i j} & =\left(z_{1 \cdot j}^{(b)}+z_{1 i j}^{(w)}, z_{2 \cdot j}^{(b)}+z_{2 i j}^{(w)}, \ldots, z_{(D-1) \cdot j}^{(b)}+z_{(D-1) i j}^{(w)}\right) \\
& =z_{\cdot j}^{(b)}+z_{i j}^{(w)}
\end{aligned}
$$

in which superscript ${ }^{(b)}$ and ${ }^{(w)}$ also denote the between and within components of the ilr coordinates.
The ilr coordinates are linearly independent multivariate real values. Therefore, once the multilevel composition has been re-expressed as a set of corresponding ilr coordinates, they can be entered into standard statistical models (Mateu-Figueras et al., 2011), such as MLMs. Importantly, the ilr transformation is invertible, meaning that the ilr coordinates can be back-transformed via their $1-1$ relationship to the original composition for further investigation, if required (Egozcue and Pawlowsky-Glahn, 2005).

## Bayesian Compositional Multilevel Model

Our exposition of Bayesian inference will be kept to a minimum, given the rich and growing literature that offers methodological guidance on Bayesian analyses, including both introductions (Kruschke, 2014; McElreath, 2018) and advanced topics (Gelman et al., 2013). While several perspectives on Bayesian inference exist (for a review, see Levy and McNeish, 2023), our method adopts Bayesian approach due to its computational advantages when estimating complex models and the exchangeability assumption when building MLMs. We now explain the MLMs with compositional predictors and its associated post-hoc substitution models.

## Bayesian Compositional Multilevel Model

We consider a continuous, normally distributed outcome variable observed at time point $i$ for individual $j$ as $y_{i j}$. within-person effects of a $D$-part composition (expressed as a set of $(D-1)$-dimension ilr coordinates). A linear MLM of $y_{i j}$ with a varying intercept (also referred to as random intercept) can be written as

$$
y_{i j}=\beta_{0 j}+\overbrace{\sum_{k=1}^{D-1} \beta_{k} z_{k \cdot j}^{(b)}}^{\text {between }}+\underbrace{\sum_{k=1}^{D-1} \beta_{(k+D-1)} z_{k i j}^{(w)}}_{\text {within }}+\varepsilon_{i j}
$$

where

$$
\begin{aligned}
\beta_{0 j} & =\gamma_{0}+u_{0 j} \\
u_{0 j} & \sim \operatorname{Normal}\left(0, \sigma_{u}^{2}\right) \\
\varepsilon_{i j} & \sim \operatorname{Normal}\left(0, \sigma_{\varepsilon}^{2}\right)
\end{aligned}
$$

The between- and within-person components of the composition (expressed as a set of ilr coordinates) are $z^{(b)}$ and $z^{(w)}$, with the subscripts denoting that the between component is unique to individual $j$ and the within component is unique to time $i$ for individual $j$. In this example model, all $z^{(b)}$ and $z^{(w)}$ are included as population-level effects, however, the $z^{(w)}$ can be allowed to vary if necessary. The between- and within-person effects of the ilr coordinates are $\beta_{k}$ and $\beta_{k+D-1}$. Because each ilr coordinate is decomposed into its between- and within-person components, for $D-1 \operatorname{ilr}$ coordinates, the corresponding $\beta$ for them in the model is $2(D-1)$.

## Bayesian Compositional Substitution Multilevel Model

Substitution analysis examines the expected difference in an outcome when a fixed unit $t$ of the composition is reallocated from one compositional component to another, while the other components remain fixed (Dumuid et al., 2019). Given the different sources of variability in the composition, we can investigate the changes in an outcome associated with the reallocation of compositional parts at between-person (i.e., differences in composition between individuals) and within-person (i.e., changes in composition within an individual across time points) levels

Table 1 outlines the steps for Bayesian compositional substitution MLM. A common reference composition is the compositional mean, thus, we briefly provide the notations for this scenario. When considering the compositional mean as the reference composition, there is no within-person variance at the compositional mean. Thus, the within-person component of the composition, $x_{0}^{(w)}$, becomes the neutral element of the simplex, $1_{D}=\mathscr{C}(1,1, \ldots, 1)=(\kappa / D, \kappa / D, \ldots, \kappa / D)$. The reference composition and its corresponding ilr transformation can be simplified to

$$
\begin{aligned}
& x_{0}=x_{0}^{(b)} \oplus 1_{D}=x_{0}^{(b)} \\
& z_{0}=z_{0}^{(b)}+0=z_{0}^{(b)}
\end{aligned}
$$

The predicted outcome by the complete compositional predictor at the compositional mean, $\hat{y}_{0}$, become

$$
\begin{aligned}
\hat{y}_{0} & =\hat{\beta}_{0 j}+\sum_{k=1}^{D-1} \hat{\beta}_{k} z_{k 0}^{(b)}+\sum_{k=1}^{D-1} \hat{\beta}_{(k+D-1)} z_{k 0}^{(w)} \\
& =\hat{\beta}_{0 j}+\sum_{k=1}^{D-1} \hat{\beta}_{k} z_{k 0}^{(b)}+0 \\
& =\hat{\beta}_{0 j}+\sum_{k=1}^{D-1} \hat{\beta}_{k} z_{k 0}^{(b)}
\end{aligned}
$$

We follow with steps 5-8 as outlined in Table 1 .

## Software Implementation

We implemented this method in a free, open-source, easy-to use R package multilevelcoda (Le and Wiley, 2023; Le et al., 2024). multilevelcoda is built on brms and Stan, which are easily accessible to lay users. The focus of multilevlecoda is on a streamlined and efficient workflow from dealing with raw multilevel compositional data, performing ilr transformations, estimating Bayesian compositional MLMs

Table 1
Steps to Perform Bayesian Compositional Substitution Multilevel Model.


#### Abstract

Step Notation


1. Select a reference composition
2. Decompose into its between and within levels
3. Re-express composition as ilr coordinates
4. Estimate the outcome by the complete composition at the reference composition

$$
\begin{gathered}
x_{0} \\
x_{0}^{(b)} \text { and } x_{0}^{(w)} \\
z_{k 0}^{(b)} \text { and } z_{k 0}^{(w)} \\
\hat{y}_{0}=\hat{\beta}_{0 j}+\sum_{k=1}^{D-1} \hat{\beta}_{k} z_{k 0}^{(b)}+\sum_{k=1}^{D-1} \hat{\beta}_{(k+D-1)} z_{k 0}^{(w)}
\end{gathered}
$$

## A. Between substitution

5A. Calculate the new composition for the reallocation at the between-person level

$$
\begin{gathered}
x_{0}^{(b)^{\prime}} \\
z_{0}^{(b)^{\prime}} \text { and } z_{0}^{(w)^{\prime}}
\end{gathered}
$$

7A. Estimate the outcome at the between-person reallocation $\quad \hat{y}_{0}^{(b)^{\prime}}=\hat{\beta}_{0 j}+\sum_{k=1}^{D-1} \hat{\beta}_{k} z_{k 0}^{(b)^{\prime}}+\sum_{k=1}^{D-1} \hat{\beta}_{(k+D-1)} z_{k 0}^{(w)}$
8A. Estimate the difference in outcome between
the between-person reallocation and the reference

$$
\Delta \hat{y}^{(b)}=\hat{y}_{0}^{(b)^{\prime}}-\hat{y}_{0}
$$

## B. Within substitution

5B. Calculate the new composition for the reallocation at the within-person level

$$
\begin{gathered}
x_{0}^{(w)^{\prime}} \\
z_{0}^{(b)^{\prime}} \text { and } z_{0}^{(w)^{\prime}}
\end{gathered}
$$

6B. Re-express the new composition as ilr coordinates
7B. Estimate the outcome for the within-person reallocation
8B. Estimate the difference in outcome between the within-person reallocation and the reference
and the associated subsitution models, and visualising final results (Figure 2).

## Illustrative Real Data Study

## Aims

We demonstrated our approach in a real data application. The objectives of this study are to 1) examine the relationship between the 24 -hour movement behaviours and sleepiness, and 2 ) investigate the changes in sleepiness associated with the change in movement behaviours at both between- and within-person levels.

Figure 2
Estimation Procedure using package multilevelcoda.


## Method

## Data

The data come from three studies with similar daily intensive designs and repeated measures:
Activity, Coping, Emotions, Stress, and Sleep (ACES. $N=187$ ); Diet, Exercise, Stress, Emotions, Speech, and Sleep (DESTRESS, $N=78$ ); and Stress and Health Study (SHS, $N=96$ ). Study materials are available on the Open Science framework for ACES (https://doi.org/10.17605/OSF.IO/H5497), DESTRESS (https://doi.org/10.17605/OSF.IO/QM63W), and SHS (https://doi.org/10.17605/OSF.IO/TZ48Y). Details of the data collection have been described previously (Le et al., 2022; Yap et al.,,2020). This data set had the data structure found in typical applications of multilevel analysis in psychological research (i.e., daily observations of movement behaviours are nested within individuals). For the purposes of this illustration, we used the data of 345 healthy adults, from whom we have repeated measurements of sleepiness and five movement behaviours of total sleep time, time awake in bed, moderate-to-vigorous physical activity (MVPA), light physical activity (LPA), and sedentary behaviour (SB). Sleepiness was a single item and self-reported 3-4 times daily, which was averaged to obtain the average daily level of sleepiness. The five behaviours were recorded via an actigraph for 7-12 days and scored using the GGIR R package (van Hees et al., 2023, 2014, 2015, 2018, Migueles et al., 2019). These data are available from the corresponding author upon request.

## Analytical Approach

The movement behaviours make up of a 5-part composition $(D=5)$, which corresponds to a 4-dimensional set of ilr coordinates. Individuals with missing data and zero values of any behaviours were excluded, as missing data and zeros hamper the analysis of compositional data (as the ilr transformation cannot compute 0 s ). The two sets of between- and within-person ilr coordinates were constructed using a SBP that represents the relative information of compositional parts as follows:

$$
\begin{aligned}
z_{1 \cdot j}^{(b)} & =\sqrt{\frac{6}{5}} \ln \frac{\left(\text { Sleep }^{(b)} \cdot{\text { Awake in } \left.\mathrm{Bed}^{(b)}\right)^{1 / 2}}_{\left(\mathrm{MVPA}^{(b)} \cdot \mathrm{LPA}^{(b)} \cdot \mathrm{SB}^{(b)}\right)^{1 / 3}}\right.}{z_{2 \cdot j}^{(b)}}=\sqrt{\frac{1}{2}} \ln \frac{\mathrm{Sleep}^{(b)}}{{\text { Awake in } \mathrm{Bed}^{(b)}}^{(b)}} \\
z_{3 \cdot j}^{(b)} & =\sqrt{\frac{2}{3}} \ln \frac{\mathrm{MVPA}^{(b)}}{\left(\mathrm{LPA}^{(b)} \cdot \mathrm{SB}^{(b)}\right)^{1 / 2}} \\
z_{4 \cdot j}^{(b)} & =\sqrt{\frac{1}{2}} \ln \frac{\mathrm{LPA}^{(b)}}{\mathrm{SB}^{(b)}}
\end{aligned}
$$

and

$$
\begin{aligned}
z_{1 i j}^{(w)} & =\sqrt{\frac{6}{5}} \ln \frac{\left(\mathrm{Sleep}^{(w)} \cdot{\text { Awake in } \left.\mathrm{Bed}^{(w)}\right)^{1 / 2}}_{\left(\mathrm{MVPA}^{(w)} \cdot \mathrm{LPA}^{(w)} \cdot \mathrm{SB}^{(w)}\right)^{1 / 3}}\right.}{z_{2 i j}^{(w)}}=\sqrt{\frac{1}{2}} \ln \frac{\mathrm{Sleep}^{(w)}}{{\text { Awake in } \mathrm{Bed}^{(w)}}^{(w)}} \\
z_{3 i j}^{(w)} & =\sqrt{\frac{2}{3}} \ln \frac{\mathrm{MVPA}^{(w)}}{\left(\mathrm{LPA}^{(w)} \cdot \mathrm{SB}^{(w)}\right)^{1 / 2}} \\
z_{4 i j}^{(w)} & =\sqrt{\frac{1}{2}} \ln \frac{\mathrm{LPA}^{(w)}}{\mathrm{SB}^{(w)}} .
\end{aligned}
$$

The ilr coordinates represent the relative effects of behaviours (increasing some while at decreasing others), accounting for the constrained nature between behaviours within the 24 -hour day. Specifically, across the between- and within-person levels, they represent the effects of (1) increasing sleep and time awake in bed while proportionally decreasing MVPA, LPA, and SB, (2) increasing sleep while proportionally decreasing time awake in bed, (3) increasing MVPA while proportionally decreasing LPA and SB , and (4) increasing LPA while proportionally decreasing SB.

We considered a Bayesian compositional MLM with a varying-intercept. The predictors are the 5-part movement composition, expressed as a total of 8 between- and within-person ilr coordinates. The outcome of the model is next-day sleepiness. A varying-intercept by participants was included to account
for non-independence. The model was fitted with weakly informative priors, 4 chains, and 4 cores, with 3000 iterations including 500 warmups (total of 10000 post-warmup draws), using CmdStanR (Stan Development Team, 2022) as back-end. We used weakly-informative priors, which play a minimal role in the computation of the posterior distribution, and maximise the influence of the data. For the population-level effects, student's $t$ distribution were used for the constant (i.e., fixed) intercept, and flat priors (improper priors over the reals) were used for the constant parameters of the predictors. Group-level effects also have their standard deviation parameters (i.e., varying intercept and residual), which were specified using student's $t$ distribution. The priors for the standard deviation parameters are restricted to be non-negative and have a half student-t prior with 3 degrees of freedom and a scale parameter that depends on the standard deviation of the outcome. These priors are only weakly informative, but provide some regularisation to improve convergence and sampling efficiency (Bürkner, 2024). Prior information is given in Table 2

Table 2
Priors Setting for Bayesian Compositional Multilevel Models.

|  | Parameter | Prior |
| :--- | :---: | ---: |
| Population-level |  |  |
| Intercept | $\gamma_{0}$ | student_t $(3,2.6,3.1)$ |
| $1^{\text {st }}$ between ilr | $\beta z_{1 \cdot j}^{(b)}$ | flat |
| $2^{\text {nd }}$ between $i l r$ | $\beta z_{2 \cdot j}^{(b)}$ | flat |
| $3^{\text {rd }}$ between $i l r$ | $\beta z_{3 . j}^{(b)}$ | flat |
| $4^{\text {th }}$ between $i l r$ | $\beta z_{4 \cdot j}^{(b)}$ | flat |
| $1^{\text {st }}$ within $i l r$ | $\beta z_{1 i j}^{(w)}$ | flat |
| $2^{\text {nd }}$ within $i l r$ | $\beta z_{2 i j}^{(w)}$ | flat |
| $3^{\text {rd }}$ within $i l r$ | $\beta z_{3 i j}^{(w)}$ | flat |
| $4^{\text {th }}$ within $i l r$ | $\beta z_{4 i j}^{(w)}$ | flat |
| Group-level |  |  |
| Intercept | $\sigma_{u}$ | student_t $(3,0,3.1)$ |
| Residual | $\sigma_{\varepsilon}$ | student_t $(3,0,3.1)$ |

The Bayesian compositional substitution MLM was then conducted for both between- and within-person levels, examining the predicted change in sleepiness associated with the pairwise reallocation from 1 to 30 minutes between the composition of 24-hour movement behaviours.

Significance of individual parameters was assessed using the Bayesian $95 \%$ posterior credible interval, with $95 \%$ credible intervals (CIs) not containing 0 providing evidence for the probability that the true estimate would lie within the interval. All analyses were performed in R v4.3.1 ( R Core Team, 2022), using package multilevelcoda v1.1.0 (model estimation, workflow outlined in Figure 2), brms (Bürkner, 2017), future (parallel processing, Bengtsson, 2021), and ggplot2 (results visualisation, Wickham, 2016).

All analysis code is available at: https://github.com/florale/multilevelcoda-sim.

## Results

## Bayesian Compositional Multilevel Model

Results from the MLM predicting next-day sleepiness from a 5-part composition are presented in Table 3, supporting the effects of all within-person ilr coordinates (indicated by 95\% CIs not containing 0s), but not any between-person ilr coordinates (indicated by $95 \%$ CIs containing 0 s ). This demonstrated the relationships between movement behaviours and next-day sleepiness occurred only at within-person level, but not between-person level. Overall, the $1^{s t}$ within-person ilr coordinate (longer time spent on sleep behaviours than usual [sleep and time awake in bed], relative to wake behaviours [MVPA, LPA, and SB]) predicted -0.59 [95\% CI -0.70, -0.49] lower next-day sleepiness. The $2^{\text {nd }}$ within-person ilr coordinate (longer sleep than usual, relative to spending time staying awake in bed), also predicted lower -0.44 [95\% CI -0.55, -0.34] next-day sleepiness. Similarly, the $3^{r d}$ within-person ilr coordinate (higher-than-usual MVPA, relative to LPA and SB ) and the $4^{\text {th }}$ within ilr coordinate (higher-than-usual LPA relative to SB ), predicted lower sleepiness $(-0.27[95 \%$ CI $-0.39,-0.16]$ and $-0.20[95 \%$ CI $-0.35,-0.06]$, respectively).

## Bayesian Compositional Substitution Multilevel Model

Consistent with the main MLM, reallocation of time between movement behaviours predicted changes in sleepiness at the within-person level, but not the between-person level. Individuals slept longer-than-usual at the expense of any behaviours, except MVPA, at within level, experienced lower level of next-day sleepiness. However, when individuals sacrificed their sleep on a given day is for any other behaviours (i.e., including MVPA), they experienced a higher level of sleepiness. Additionally, individuals who spent longer time in LPA at the expense of time awake in bed on a given day, also experienced a higher level of sleepiness the next day, and vice versa. Results of the substitution model for 30-minute reallocation are in Table 4. For brevity, we presented only the significant results for the reallocation from 1

Table 3
Associations of the 24-hour Sleep-Wake Movement Behaviours and Sleepiness.

| Parameter | Interpretation | Posterior mean <br> and 95\% credible intervals |
| :--- | :--- | :---: |
| Between-person level |  |  |
| $z_{1}^{(b)}$ | Longer sleep and awake in bed, <br> relative to MVPA, LPA, and SB | 0.16 |
| $z_{2}^{(b)}$ | Longer sleep, <br> relative to awake in bed | $[-0.15,0.46]$ |
| $z_{3}^{(b)}$ | Longer MVPA, <br> relative to LPA and SB | -0.01 |
| $z_{4}^{(b)}$ | Longer LPA, <br> relative to SB | $[-0.27,0.25]$ |

Within-person level

| $z_{1}^{(w)}$ | Longer-than-usual sleep and awake in bed, <br> relative to MVPA, LPA, and SB on a given day | $-0.59^{*}$ |
| :--- | :--- | :---: |
| $z_{2}^{(w)}$ | Longer-than-usual sleep, <br> relative to awake in bed on a given day | $[-0.69,-0.49]$ |
| $z_{3}^{(w)}$ | Longer-than-usual MVPA, <br> relative to LPA and SB on a given day | $\left[-0.54^{*}\right.$ |
| $z_{4}^{(w)}$ | Longer-than-usual LPA, <br> relative to SB within level on a given day | $[-0.34]$ |

Notes. ${ }^{*} 95 \%$ credible intervals not containing 0.
to 30 minutes of Sleep and Awake in bed, respectively, in Figure 3

## Simulation Study

## Aims

In a series of simulation studies, we investigated the performance of the Bayesian compositional MLM and Bayesian compositional substitution MLM in parameter recovery. Our simulation study were based on the real data study, where the objective was to examine the relationship between 24-hour movement behaviour composition and sleepiness.

Table 4
Estimated Difference in Sleepiness Associated with Reallocation of 30 minutes Across Sleep-Wake Movement Behaviours.

|  | $\downarrow$ Sleep | $\downarrow$ Awake in bed | $\downarrow$ MVPA | $\downarrow$ LPA | $\downarrow$ SB |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Between-person level |  |  |  |  |  |
| $\uparrow$ Sleep | - | -0.05 | -0.05 | 0.04 | 0.01 |
|  |  | $[-0.15,0.05]$ | $[-0.22,0.12]$ | $[-0.08,0.16]$ | $[-0.02,0.04]$ |
| $\uparrow$ Awake in bed | 0.03 |  | -0.02 | 0.07 | 0.04 |
|  | $[-0.04,0.09]$ | - | $[-0.20,0.17]$ | $[-0.06,0.20]$ | $[-0.02,0.10]$ |
| $\uparrow$ MVPA | 0.02 | -0.03 |  | 0.07 | 0.04 |
|  | $[-0.08,0.13]$ | $[-0.17,0.11]$ | - | $[-0.14,0.27]$ | $[-0.06,0.14]$ |
| $\uparrow$ LPA | -0.03 | -0.08 | -0.08 | - | -0.02 |
|  | $[-0.13,0.06]$ | $[-0.21,0.05]$ | $[-0.32,0.17]$ |  | $[-0.11,0.07]$ |
| $\uparrow$ SB | -0.01 | -0.06 | -0.06 | 0.03 | - |
| Within-person level | $[-0.04,0.02]$ | $[-0.16,0.03]$ | $[-0.22,0.11]$ | $[-0.09,0.15]$ |  |
| $\uparrow$ Sleep | - |  |  |  |  |
|  |  | -0.04 | -0.04 | $-0.11^{*}$ | $-0.06^{*}$ |
| $\uparrow$ Awake in bed | $0.04^{*}$ | $[-0.08,0.00]$ | $[-0.10,0.01]$ | $[-0.15,-0.06]$ | $[-0.08,-0.05]$ |
|  | $[0.02,0.7]$ | - | 0.00 | $-0.07^{*}$ | -0.02 |
| $\uparrow$ MVPA | $0.05^{*}$ | 0.00 | $[-0.06,0.06]$ | $[-0.12,-0.02]$ | $[-0.04,0.00]$ |
|  | $[0.01,0.08]$ | $[-0.04,0.05]$ | - | -0.06 | -0.02 |
|  | $0.10^{*}$ | $0.05^{*}$ | 0.05 | $[-0.14,0.01]$ | $[-0.05,0.01]$ |
| $\uparrow$ LPA | $[0.06,0.13]$ | $[0.01,0.10]$ | $[-0.03,0.13]$ | - | 0.03 |
|  | $0.07^{*}$ | 0.03 | 0.02 | -0.04 | $[-0.01,0.06]$ |
| $\uparrow$ SB | $[0.05,0.08]$ | $[-0.01,0.06]$ | $[-0.03,0.08]$ | $[-0.09,0.00]$ | - |

Notes. Values presented are posterior means and $95 \%$ credible intervals. ${ }^{*} 95 \%$ credible intervals not containing 0 .

## Method

## Simulation Conditions

We created a range of simulation conditions including different values of the number of clusters $(J)$, cluster size $(I)$, the number of compositional parts $(D)$, and the magnitude of sample variability (assessed by the varying-intercept variance $\sigma_{u}^{2}$ and residual variance $\sigma_{\varepsilon}^{2}$ ). The values for the number of clusters and cluster sizes were informed by our review of a systematic review and meta-analyses on daily

Figure 3
Estimated changes in Sleepiness for Reallocation of 24-hour Sleep-Wake Movement Behaviours.


Notes. LPA = Light Physical Activity, MVPA = Moderate-to-Vigorous Physical Activity, and SB= Sedentary Behavior.
sleep and physical activity (Atoui et al., 2021). Given the different number of compositional parts used in existing studies, we constructed models with different numbers of possible compositional parts and assessed their performances using different sets of ground truth values. Finally, we examined the influences of sample variability, including varying-intercept variance $\left(\sigma_{u}^{2}\right)$ and residual variance $\left(\sigma_{\varepsilon}^{2}\right)$ on the estimation of our models. Table 5]summarises the factors and their levels considered in this simulation study. The combination of these factors resulted in a total of 240 scenarios. For each cell of the simulation design, 2000 replications were generated $\left(n_{\text {sim }}=2000\right)$, resulting in $4[I] \times 4[J] \times 3[D] \times 5[\sigma] \times 2000=480$ 000 data sets to be analysed.

Table 5
Factors and Their Levels in the Simulation Study.

| Factor | Notation | Levels |
| :--- | :---: | :--- |
| Number of clusters | J | $3,5,7,14$ |
| Cluster size | I | $30,50,360,1200$ |
| Number of | D | $3,4,5$ |
| compositional parts |  | $\sigma_{u}^{2}=1$ and $\sigma_{\varepsilon}^{2}=1$, |
|  |  | $\sigma_{u}^{2}=1.5$ and $\sigma_{\varepsilon}^{2}=0.5$, |
| Variance | $\sigma_{u}^{2}$, | $\sigma_{u}^{2}=0.5$ and $\sigma_{\varepsilon}^{2}=1.5$, |
|  | $\sigma_{\varepsilon}^{2}$ | $\sigma_{u}^{2}=1$ and $\sigma_{\varepsilon}^{2}=0.5$, |
|  |  | $\sigma_{u}^{2}=1$ and $\sigma_{\varepsilon}^{2}=1.5$ |

Notes. $\sigma_{u}^{2}=$ varying-intercept variance, $\sigma_{\varepsilon}^{2}=$ residual variance.

## Data Generation

In the following, we described the simulation procedure to generate data sets resembling the data structure used in real data study. The varying-intercept $u_{0 j}$ was generated from $\operatorname{Normal}\left(0, \sigma_{u}^{2}\right)$. The design matrices of the predictors, the between-person ilr $\left(z^{(b)}\right)$ and within-person ilr $\left(z^{(w)}\right)$ corresponding to 5-part composition (total sleep time, time in bed awake, MVPA, LPA, and SB) were generated, respectively, as follows:

$$
z^{(b)} \sim \operatorname{MVNormal}\left(\mu^{z^{(b)}}, \Sigma^{z^{(b)}}\right)
$$

and

$$
z^{(w)} \sim \operatorname{MVNormal}\left(\mu^{z^{(w)}}, \Sigma^{z^{(w)}}\right)
$$

with values of the means and covariances informed by the data set used in the real data study.
Compositional data were then generated by inverse-transforming the 4-dimension ilr coordinates. At this step, when necessary, the 4-part and 3-part compositions were created by collapsing variables. The 4-part composition was obtained by collapsing total sleep time and wake in bed to a single variable named sleep. The 3-part composition was created by collapsing MVPA and LPA to a single variable named physical activity. These compositions were transformed again to ilr coordinates for model estimation.

The outcome vector $y$ was generated from a normal distribution:

$$
y \sim \operatorname{Normal}\left(\gamma_{0}+u_{0 j}+\sum_{k=1}^{D-1} \beta_{k} z_{k \cdot j}^{(b)}+\sum_{k=1}^{D-1} \beta_{(k+D-1)} z_{k i j}^{(w)}, \quad \sigma_{\varepsilon}^{2}\right)
$$

with the values for the constant parameters set to be close to those found in the real data study.

## Estimands

The primary estimands of the simulation study are the parameters of the Bayesian MLM models, including the constant parameter estimates: the intercept $\left(\gamma_{0}\right)$, the between-person and within-person ilr coordinates ( $\beta \mathrm{s}$ ), and the varying parameters: varying-intercept $\left(\sigma_{u}\right)$ and residual error $\left(\sigma_{\varepsilon}\right)$. For the Bayesian compositional substitution MLM, estimation of predicted change in outcome at between-level $\left(\Delta \hat{y}_{i j}^{(b)}\right)$ and within-level ( $\Delta \hat{y}_{i j}^{(w)}$ ) were evaluated for all possible pairwise substitution between compositional parts, totalling to $2 \times D \times(D-1)$ parameters.

## Evaluation Criteria

Model performance of 2000 replications across 240 conditions was evaluated using the following criteria.

1. Quality of the MCMC-based sampling procedure of the Bayesian compositional MLM. We considered the proportion of replication that sufficiently converged ( $\hat{R}<1.05$, Vehtari et al., 2021) and had no divergent transition. Effective sample size (ESS) was investigated both at the bulk of the distribution (e.g., for the mean or median) and in the tails (e.g., for posterior interval estimates and inferences about extreme quatiles). Any parameters with ESS $>400$ indicated sampling inefficiency and required further diagnostics (Vehtari et al., 2021).
2. Quality of model performance was evaluated in terms of accuracy in parameter estimates and inference, using three performance measures: bias, coverage, and bias-eliminated coverage (Morris et al., 2019). Monte Carlo standard errors were used to calculate $95 \%$ confidence interval.

## Analysis of Simulated Data

Using package multilevelcoda, each simulated data set was fitted in Bayesian MLM with a varying-intercept to predict next-day sleepiness from the $D$-part behaviour composition, expressed as a total of $2(D-1)$ between- and within-person ilr coordinates. The Bayesian substitution MLM was then
conducted, and the model performance in estimating the predicted change in outcome for 30-minute reallocation was evaluated. The simulation study results were summarised using package rsimsum (Gasparini, 2018) and visualised using package ggplot2 (Wickham, 2016). Reproducible material for this study is available at: https://github.com/florale/multilevelcoda-sim.

## Simulation Results

We found minimal effects of certain simulation conditions on model estimation. Therefore, for brevity, the descriptive statistics of the simulation results of the Bayesian compositional MLM and its associated substitution models were collapsed across 240 conditions and summarised in Table 6 .

Table 6
Descriptives Statistics of the Simulation Study.

|  | Bayesian <br> Compositional <br> MLM | Bayesian <br> Compositional <br> Substitution <br> MLM |
| :--- | :---: | ---: |
| Number of |  |  |
| divergent | 0.01 | - |
| transitions | $(0,134)$ | - |
| $\hat{R}$ | 1.00 | - |
| Bulk-ESS | $(1.00,1.07)$ | - |
| Tail-ESS | 6193.83 | - |
| Bias | $(52.06,27047.59)$ | $(-0.03,0.04)$ |
| Coverage | 5600.04 | 0.95 |
| Bias-Eliminated | $(107.91,9465.94)$ | $(0.93,0.97)$ |
| Coverage | $(-0.09,0.05)$ | 0.95 |

Notes. Values are mean and range. MLM = multilevel model.

## Quality of Estimation Procedure

Divergences were observed in 1312 replications ( $0.00 \%$ ), predominantly with small number of clusters $(73.6 \% \mathrm{~J}: 30)$, small cluster size ( $90.5 \% \mathrm{I}: 3$ ), and large residual variation ( $97.6 \% \sigma_{\varepsilon}^{2}: 1.5$ ). An additional $17(0.00 \%)$ replications had $\hat{R}>1.05$, demonstrating convergence issues. These replications were excluded for the evaluation of parameter estimates and inference.

In contrast, low bulk ESS were observed as sample size increases with large between-person heterogeneity and small within-person heterogeneity. Particularly, 27651 replications ( $0.06 \%$ ) had bulk ESS $<400$ for some parameters, predominantly with large number of clusters ( $51.1 \% \mathrm{~J}: 1200$ ), large cluster size $(70.1 \% \mathrm{I}: 14)$, and small residual variation $\left(95.6 \% \sigma_{\varepsilon}^{2}: 0.5\right)$. The low ESS values under these conditions represent a technical difficulty posed by the MCMC sampling methods, wherein small variation in the sample (i.e., $\sigma_{\varepsilon}^{2}$ ) cause the sampler to produce higher within-chain correlation (Betancourt and Girolami, 2015). Additionally, the default non-centered parameterisation (i.e., separation of population parameters and hyperparameters in the prior, Papaspiliopoulos et al., 2007) used in our model estimation procedure can be less efficient for large data sets and strong likelihood (i.e., small sample variability), compared to centered parameterisation (Betancourt and Girolami, 2015). Therefore, we conducted a case study (Le, 2024) into a replication generated using a 3-part composition, 1200 clusters and cluster size of 14 , with large varying intercept variation $\left(\sigma_{u}^{2}=1.5\right)$ and residual variation $\left(\sigma_{\varepsilon}^{2}=0.5\right)$, wherein model produced low bulk ESS values for 4 out of 7 parameters. Posterior predictive distributions were checked and two methods to improve the MCMC sampling were tested: centered-parameterisation and increased iterations. We found no evidence of non-convergence (e.g., poor mixing of chains or funnel degeneracy in the posterior). Both reparameterisation or increasing iterations and warm-ups improved ESS, with centered parameterisation showing substantial gain of ESS for the same number of iterations. A sensitivity analysis comparing the model performance with and without the replications with low ESS revealed that ESS did not have an influence on the quality of parameter estimates and inference. Replications with low ESS were, therefore, kept in the subsequent evaluation of model performance.

## Quality of Parameter Estimates and Inference

Across the simulated conditions, both Bayesian compositional MLMs and Bayesian compositional substitution MLMs yielded negligible biases in the estimation of all parameters, with a mean of 0.00 and a range from -0.09 to 0.05 and mean of 0.00 and range from -0.03 to 0.04 , respectively. Both models had coverage and bias-eliminated coverage close to the nominal value, with means of 0.95 and ranges from 0.93 to 0.97 .

As there was no impact of simulation conditions on the model performance, for brevity, we presented the results for individual parameters estimated using composition with 4 parts $(D=4)$ and a medium level of modelled variance $\left(\sigma_{u}^{2}=1\right.$ and $\left.\sigma_{\varepsilon}^{2}=1\right)$ under different conditions of the number of
clusters $(J)$ and cluster size $(I)$. Full results can be accessed via the shiny app by locally running in R using package multilevelcoda. Both models performed well consistently across the number of clusters and cluster size, as demonstrated in Figure 4, 5, 6, and 7

## Discussion

EMA and wearable devices to advance clinical and health science have blossomed in the last decade. These methodologies, especially employed in intensive, longitudinal research, have enabled the full day of behaviours and experiences to be captured. In the wake of such data abundance, innovative statistical methods that appropriately address the data properties can enhance psychological studies and lead to new health insights. This paper presented a Bayesian approach to modelling multilevel compositional data, with a focus on both within-person and between-person processes. We described the theories underlying the data and models and illustrated how to perform this method in a real data application. A simulation study confirmed the overall good performance of both Bayesian compositional MLMs and the associated substitution models under different simulation scenarios.

Our empirical results demonstrated the usefulness of the proposed method in examining how day-to-day movement behaviours are associated with other daily experiences using EMA data. We showed that the reallocation of time between movement behaviours was associated with next-day sleepiness, and that this association differed by behaviours involved in the reallocation (e.g., sleep at the expense of MVPA or SB ), and whether the effect occurred at the between-person or within-person level. These findings highlight the importance of addressing multilevel and compositional nature of movement behaviours, and any other data with such properties.

Results of the simulation study showed that the quality of estimation procedure was related to sample size and variability. Divergences were observed in a small number of models fitted with small sample sizes and large sample variability, whereas inefficiency of MCMC sampling, indicated by the low ESS, was observed in models fitted with large data sets and small sample variability. The estimation procedure in the simulation study followed a common framework for MCMC sampling (Betancourt and Girolami, 2015; Betancourt, 2017; Bürkner, 2017), and diagnosing and dealing with sampling inefficiency depends on the model of interest and specific applications. Nevertheless, we suggest the following. To eliminate divergences, we recommend using data sets with more than 30 clusters with a cluster size of 3 ( $N$ $=90)$. Studies have already collected data or have sampling limitations may consider adjusting the initial
step size and target acceptance rate to assist the sampling departure and trajectories in model estimation (Stan Development Team, 2023), such as setting the "adapt_delta" control parameter to a higher value than the default of 0.80 when fitting model (Schad et al., 2021). Scenarios with convergence issues or sampling inefficiency, indicated by low ESS and high $\hat{R}$, may be improved by reparameterisation or increasing the number of warm-up iterations and/or the number of posterior draws. We found that reparameterisation, in particular, yielded the most robust ESS for the same number of iterations.

Bayesian compositional MLMs and Bayesian compositional substitution MLMs both successfully recovered all tested summary statistics, including constant and varying parameters, and residual error. Unbiased estimates and excellent coverage were consistently observed across all conditions of sample sizes, compositional parts, and the magnitude of sample variability. This performance was further not influenced by the efficiency of MCMC sampling. These findings support the advantage of Bayesian over the frequentist approach for MLMs. For frequentist MLMs, a minimum data with 30 clusters with a cluster size of 50 is recommended for models using likelihood estimation methods (either full maximum likelihood or restricted maximum likelihood) to achieve unbiased estimates (McNeish and Stapleton, 2016). MLMs with smaller sample sizes may require Kenward-Roger adjustment (Kenward and Roger, 1997). In contrast, we showed that MLMs estimated using Bayesian MCMC sampling can achieve unbiased estimates for data with 30 clusters with a cluster size of 3 , and other studies have provided evidence for data with fewer than ten clusters (Stegmueller, 2013; Browne and Draper, 2006). Another important advantage of our method lies in leveraging Bayesian approach to estimate the substitution model. Using the posterior predictive distributions, the model can directly describe the uncertainty of the estimated quantities (i.e., the predicted changes in outcomes), eliminating the computational burden of relying on resampling techniques, such as bootstrapping, which is typical with frequentist methods.

As with other Bayesian methods, the estimation time required for the models presented in this study is considerable. With more complex models, larger data sets, or when investigating model sensitivity, transforming parameterisation, the amount of time and computational resources can become increasingly substantial. However, we believe that the advantages associated with this method, including accurate and unbiased parameter estimates, straightforward estimation procedure, and minimal convergence issues, outweigh the time and computational cost. Parallelising model fits to multiple cores on a computing cluster can help speed up model estimation process. Our recommended softwares for working with multilevel
compositional data, including multilevelcoda, brms, and Stan, all provide several options for fast parallelising Bayesian models.

It is important to note that these models requires complete and non-zero data. Zeros and missing data hamper the analysis of compositional data, as the ilr transformation is essentially based on log ratios. Although dealing with zeros and missing data is outside the scope of this study, previous studies have discussed the zero composition problem (Smithson and Broomell, 2022; Martín-Fernández et al., 2003), provided a comparison of different strategies in dealing with zeros in compositional data (Rasmussen et al., 2020), and multilevel missing data (Lüdtke et al., 2017). Log-ratio Expectation-Maximization (Palarea-Albaladejo and Martín-Fernández, 2015) has been recommended for zero imputation as it preserves the relative structure (i.e., ratios) of composition (Rasmussen et al., 2020). Imputation strategy based on multivariate MLMs (Schafer and Yucel, 2002; Zhao and Schafer, 2023) has been shown to produce valid inferences for varying-intercept MLMs with missing data at the lowest level of the multilevel structure (Lüdtke et al., 2017), such as observations of movement behaviours.

The model of interest in this study was a varying-intercept MLM with a continuous, normally distributed outcome. Other outcome distributions are frequently observed in psychological research, including Bernoulli (binary data, such as depression status), Poisson (count data, such as number of cigarettes smoked per day). Frequentist MLMs with binary outcomes have been shown to be subject to more biased estimation (McNeish and Stapleton, 2016). Three-level data structure (e.g., movement behaviours nested within patients, which in turn are nested within hospitals) are relatively common, and sample sizes can significantly decrease towards the higher level of the data hierarchy. Different prior distributions and their impacts on the expected data were not investigated in our study, due to complexity of the models, the current limited knowledge about behavioural composition and its association with other outcomes. Future work may consider the application of this method and evaluate its performance with other outcome families, more complex random-effect structure (i.e., both varying intercept and varying slopes), more-than-two-level data hierarchy, and steps to build priors and prior consequences on the predictive distribution. Further, recent epidemiological research is increasingly interested in understanding the within-person variability (e.g., changes of behavioural composition at follow-up relative to baseline predicting changes in health outcomes), yet methods are not well established. Our method may be explored in such data sets to extend its impacts to other fields beyond psychology. Lastly, more tutorials detailing
step-by-step analyses of example data sets in different areas could help promote wider applications of this innovative method.

## Conclusion

We introduced an elegant methodology that integrates three statistical frameworks: compositional data analysis, multilevel modelling, and Bayesian inference. The implementation of this method in an open-source R package, multilevelcoda, with a user-friendly setup that only requires the data, model formula and minimal specification of the analysis, speaks to the feasibility of modelling multilevel compositional data in a novel way. As the availability of data with a multilevel compositional structure will grow, we believe this method will be an increasingly important tool to advance psychological research. We hope that our tutorial, evaluations, and recommendations, will motivate researchers to employ this method in their work and discipline to obtain robust answers to scientific questions that otherwise would be inaccessible.

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Figure 4
Bias of Bayesian Compositional Multilevel Models with 4-part Composition and Medium Level of Variance

| Parameter | J: 30, l: 3 | Bias | J: 30, I: 5 | Bias | J: 30, l: 7 | Bias | J: 30, I: 14 | Bias |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Y}_{0}$ | $\rightarrow-$ | 0.00 [-0.06, 0.05] | $\rightarrow$ - | 0.02 [-0.03, 0.07] | $\rightarrow$ - | -0.03 [-0.08, 0.02] | $\rightarrow-$ | -0.02 [-0.07, 0.03] |
| $\beta z_{1}{ }^{\text {b }}$ | $\rightarrow-1$ | -0.04 [-0.09, 0.00] | $\rightarrow$ | 0.01 [-0.03, 0.05] | $\cdots$ | 0.02 [-0.02, 0.07] | - | -0.01 [-0.06, 0.04] |
| $\beta \mathrm{z}_{2}{ }^{\text {b }}$ | $+$ | -0.02 [-0.05, 0.02] | + | 0.01 [-0.02, 0.05] | + | $0.00[-0.04,0.03]$ | -1 | -0.02 [-0.06, 0.02] |
| $\beta z_{3}{ }^{\text {b }}$ | - | -0.01 [-0.06, 0.03] | ! | 0.01 [-0.03, 0.06] | $!$ | 0.00 [-0.05, 0.04] | $!$ | -0.01 [-0.05, 0.04] |
| $\beta_{\mathrm{z}_{1}}{ }^{(w)}$ | - | $-0.01[-0.04,0.02]$ | - | -0.01 [-0.03, 0.01] | - | 0.00 [-0.02, 0.01] | - | 0.00 [-0.01, 0.01] |
| $\beta_{2}{ }^{(w)}$ | ${ }^{\prime}$ | -0.03 [-0.05, 0.00] | P | 0.01 [-0.01, 0.03] | ¢ | 0.00 [-0.01, 0.02] | ¢ | 0.00 [-0.01, 0.01] |
| $\beta_{z_{3}}{ }^{(w)}$ | $+$ | 0.00 [-0.03, 0.03] | 1 | -0.02 [-0.04, 0.00] | \$ | 0.00 [-0.02, 0.02] | \$ | 0.00 [-0.01, 0.01] |
| $\sigma_{u}$ | - | 0.01 [ 0.01, 0.02] | - | 0.03 [ 0.03, 0.04] | - | 0.04 [ 0.03, 0.04] | $1 \cdot$ | 0.03 [ 0.03, 0.04] |
| $\sigma_{\varepsilon}$ | $\square$ | 0.02 [ 0.01, 0.02] | $!$ | 0.01 [ 0.01, 0.01] | $\stackrel{1}{\bullet}$ | 0.01 [ 0.00, 0.01] | $\stackrel{\square}{\bullet}$ | 0.00 [ 0.00, 0.00] |
|  | -0.10.0 0.1 |  | -0.10.0 0.1 |  | -0.10.0 0.1 |  | -0.10.0 0.1 |  |
| Parameter | J: 50, l: 3 | Bias | J: 50, I: 5 | Bias | J: 50, l: 7 | Bias | J: 50, I: 14 | Bias |
| $\mathrm{Y}_{0}$ | $\rightarrow$ | -0.03 [-0.07, 0.01] | $\rightarrow$ | 0.00 [-0.04, 0.04] | $\rightarrow-$ | 0.00 [-0.03, 0.04] | - | 0.00 [-0.04, 0.04] |
| $\beta z_{1}{ }^{\text {b }}$ | - | -0.01 [-0.05, 0.02] | $\rightarrow$ | 0.00 [-0.03, 0.03] | $-$ | -0.01 [-0.04, 0.02] | - | -0.01 [-0.04, 0.02] |
| $\beta z_{2}{ }^{\text {b }}$ | - | 0.01 [-0.02, 0.03] | $\cdots$ | -0.02 [-0.04, 0.01] | 4 | -0.02 [-0.04, 0.01] | + | 0.00 [-0.02, 0.03] |
| $\beta z_{3}{ }^{\text {b }}$ | , | -0.03 [-0.06, 0.00] | $!$ | 0.01 [-0.03, 0.04] | ! | 0.01 [-0.02, 0.05] | $!$ | -0.01 [-0.04, 0.02] |
| $\beta_{\mathrm{z}_{1}}{ }^{(w)}$ | \% | 0.02 [ 0.00, 0.04] | - | -0.01 [-0.02, 0.01] | $\stackrel{\square}{6}$ | 0.00 [-0.01, 0.01] | - | 0.00 [-0.01, 0.01] |
| $\beta_{2}{ }^{(w)}$ | ¢ | 0.00 [-0.02, 0.01] | ¢ | 0.00 [-0.01, 0.01] | ¢ | 0.00 [-0.01, 0.01] | + | 0.00 [-0.01, 0.01] |
| $\beta_{z_{3}}{ }^{(w)}$ | + | 0.00 [-0.02, 0.02] | \$ | -0.01 [-0.02, 0.01] | $p$ | 0.01 [ 0.00, 0.02] | \$ | 0.00 [-0.01, 0.01] |
| $\sigma_{u}$ | - | 0.00 [ 0.00, 0.01] | 1 。 | 0.02 [ 0.01, 0.02] | 1. | 0.02 [ 0.02, 0.03] | - | 0.02 [ 0.01, 0.02] |
| $\sigma_{\varepsilon}$ | $\bullet$ | 0.01 [ 0.01, 0.01] | $\bullet$ | 0.00 [ 0.00, 0.01] | $\bullet$ | 0.00 [ 0.00, 0.00] | $\bigcirc$ | 0.00 [ $0.00,0.00]$ |
|  | -0.10.0 0.1 |  | -0.10.0 0.1 |  | -0.10.0 0.1 |  | -0.10.0 0.1 |  |
| Parameter | J: 360, I: 3 | Bias | J: 360, l: 5 | Bias | J: 30, I: 3 | Bias | J: 360, I: 14 | Bias |
| $\mathrm{Y}_{0}$ | - | 0.00 [-0.01, 0.01] | $\bullet$ | 0.00 [-0.02, 0.01] | - | 0.00 [-0.06, 0.05] | - | 0.01 [-0.01, 0.02] |
| $\beta z_{1}^{\text {(b) }}$ | - | 0.00 [-0.01, 0.01] | - | 0.00 [-0.01, 0.01] | $\rightarrow-$ | -0.04 [-0.09, 0.00] | - | -0.01 [-0.02, 0.01] |
| $\beta z_{2}{ }^{\text {b }}$ | \$ | 0.00 [-0.01, 0.01] | \$ | 0.00 [-0.01, 0.01] | $\cdots$ | -0.02 [-0.05, 0.02] | \$ | 0.00 [-0.01, 0.01] |
| $\beta z_{3}{ }^{\text {b }}$ | $!$ | 0.00 [-0.01, 0.01] | $!$ | 0.00 [-0.01, 0.01] | , | -0.01[-0.06, 0.03] | $!$ | 0.00 [-0.01, 0.01] |
| $\beta_{\mathrm{z}_{1}}{ }^{(w)}$ | - | 0.00 [-0.01, 0.01] | $\stackrel{1}{6}$ | 0.00 [-0.01, 0.00] | $\cdots$ | -0.01 [-0.04, 0.02] | - | 0.00 [ 0.00, 0.00] |
| $\beta_{z_{2}}{ }^{(w)}$ | ; | 0.00 [-0.01, 0.01] | + | $0.00[-0.01,0.00]$ | 1 | -0.03 [-0.05, 0.00] | 1 | 0.00 [ $0.00,0.00]$ |
| $\beta_{z_{3}}{ }^{(w)}$ | \$ | 0.00 [-0.01, 0.01] | \$ | 0.00 [-0.01, 0.01] | + | 0.00 [-0.03, 0.03] | \$ | $0.00[-0.01,0.00]$ |
| $\sigma_{u}$ | $!$ | 0.00 [ 0.00, 0.01] | - | 0.00 [ $0.00,0.00]$ | ' | 0.01 [ 0.01, 0.02] | $\bullet$ | 0.00 [ 0.00, 0.00] |
| $\sigma_{\varepsilon}$ | $\bigcirc$ | 0.00 [ 0.00, 0.00] | - | 0.00 [ 0.00, 0.00] | $\bullet$ | 0.02 [ 0.01, 0.02] | - | 0.00 [ 0.00, 0.00] |
|  | -0.10.0 0.1 |  | -0.10.0 0.1 |  | -0.10.0 0.1 |  | -0.10.0 0.1 |  |
| Parameter | J: 1200, l: 3 | Bias | J: 1200, I: 5 | Bias | $\mathrm{J}: 1200, \mathrm{l}: 7$ | Bias | J: 1200, l: 14 | Bias |
| $\mathrm{Y}_{0}$ | - | 0.00 [-0.01, 0.01] | - | 0.00 [-0.01, 0.01] | - | 0.00 [-0.01, 0.01] | $\bullet$ | -0.01 [-0.01, 0.00] |
| $\beta z_{1}{ }^{\text {b }}$ | - | 0.00 [-0.01, 0.01] | - | 0.00 [-0.01, 0.01] | - | 0.00 [-0.01, 0.01] | - | 0.00 [ 0.00, 0.01] |
| $\beta_{2}{ }^{\text {(b) }}$ | \$ | 0.00 [-0.01, 0.00] | \$ | $0.00[-0.01,0.00]$ | \$ | 0.00 [-0.01, 0.01] | \$ | $0.00[-0.01,0.00]$ |
| $\beta z_{3}{ }^{\text {b }}$ | $!$ | 0.00 [-0.01, 0.01] | $!$ | 0.00 [-0.01, 0.01] | $!$ | 0.00 [ 0.00, 0.01] | $!$ | 0.00 [-0.01, 0.01] |
| $\beta_{\mathrm{z}_{1}{ }^{(\mathrm{w})}}$ | ! | 0.00 [-0.01, 0.00] | ! | 0.00 [ 0.00, 0.00] | ! | 0.00 [ 0.00, 0.00] | ! | 0.00 [ $0.00,0.00$ ] |
| $\beta_{2}{ }^{(w)}$ | \$ | 0.00 [-0.01, 0.00] | \$ | 0.00 [ 0.00, 0.00] | ¢ | 0.00 [ 0.00, 0.00] | ¢ | 0.00 [ 0.00, 0.00] |
| $\beta_{z_{3}}{ }^{(\mathrm{w})}$ | \$ | 0.00 [ 0.00, 0.01] | \$ | 0.00 [-0.01, 0.00] | \$ | 0.00 [ 0.00, 0.00] | \$ | 0.00 [ $0.00,0.00$ ] |
| $\sigma_{u}$ | $!$ | 0.00 [ 0.00, 0.00] | ! | 0.00 [ 0.00, 0.00] | $!$ | 0.00 [ $0.00,0.00]$ | $!$ | 0.00 [ $0.00,0.00]$ |
| $\sigma_{\varepsilon}$ | $!$ | 0.00 [ 0.00, 0.00] | $!$ | 0.00 [ 0.00, 0.00] | ! | 0.00 [ 0.00, 0.00] | $\bigcirc$ | 0.00 [ 0.00, 0.00] |
|  | -0.10.0 0.1 |  | -0.10.0 0.1 |  | -0.10.0 0.1 |  | -0.10.0 0.1 |  |

Figure 5
Coverage of Bayesian Compositional Multilevel Models with 4-part Composition and Medium Level of Variance


| Parameter | J: 50, I: 3 | Coverage | J: 50, I: 5 | Coverage | J: 50, I: 7 | Coverage | J: 50, I: 14 | Coverage |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Y}_{0}$ | $\rightarrow$ | 0.94 [ 0.93, 0.95] | - | 0.96 [ 0.95, 0.96] | $\rightarrow$ | 0.95 [ 0.94, 0.96] | - | 0.96 [ 0.95, 0.96] |
| $\beta \mathrm{z}_{1}^{(\mathrm{b})}$ | - | 0.94 [ 0.93, 0.95] | $\cdots$ | 0.95 [ 0.94, 0.96] | $\cdots$ | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] |
| $\beta z_{2}^{(b)}$ | $+$ | 0.95 [ 0.94, 0.96] | $+$ | 0.95 [ 0.94, 0.96] | $+$ | 0.95 [ 0.94, 0.96] | $\frac{1}{6}$ | 0.96 [ 0.95, 0.96] |
| $\beta_{z_{3}}{ }^{\text {b }}$ | 1 | 0.94 [ 0.93, 0.95] | $!$ | 0.95 [ 0.94, 0.96] | $!$ | 0.95 [0.94, 0.96] | $!$ | 0.95 [0.94, 0.96] |
| $\beta \mathrm{z}_{1}{ }^{(w)}$ | $\cdots$ | 0.95 [0.94, 0.96] | $\cdots$ | 0.95 [ 0.94, 0.96] | - | 0.96 [ 0.95, 0.96] | - | 0.95 [ 0.94, 0.96] |
| $\beta_{z_{2}}{ }^{(w)}$ | - | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | $-$ | 0.94 [0.93, 0.95] | - | 0.95 [0.94, 0.96] |
| $\beta_{z_{3}}{ }^{(w)}$ | $+$ | 0.95 [0.94, 0.96] | + | 0.95 [ 0.94, 0.96] | $\frac{1}{1}$ | 0.95 [ 0.94, 0.96] | + | 0.95 [ 0.94, 0.96] |
| $\sigma_{u}$ | $\cdots$ | 0.95 [ 0.94, 0.96] | $\stackrel{-}{-}$ | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | $\stackrel{-}{+}$ | 0.95 [ 0.94, 0.96] |
| $\sigma_{\varepsilon}$ | $\cdots$ | 0.95 [ 0.94, 0.96] | $\rightarrow$ | 0.94 [ 0.93, 0.95] | $\rightarrow$ | 0.94 [ 0.93, 0.95] | - | 0.94 [ 0.93, 0.95] |
|  | 0.951 |  | 00.951 |  | 00.951 .0 |  | ( 90.951 |  |


| Parameter | $\mathrm{J}: 360, \mathrm{l}: 3$ | Coverage | J: 360, I: 5 | Coverage | J: 30, I: 3 | Coverage | J: 360, I: 14 | Coverage |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Y}_{0}$ | - | 0.96 [ 0.95, 0.97] | - | 0.96 [ 0.95, 0.97] | - | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] |
| $\beta_{z_{1}}{ }^{\text {b }}$ | $\cdots$ | 0.95 [ 0.94, 0.96] | $\cdots$ | 0.95 [0.94, 0.96] | - | 0.96 [ 0.95, 0.97] | $\cdots$ | 0.95 [ 0.94, 0.95] |
| $\beta z_{2}^{(b)}$ | \% | 0.96 [ 0.95, 0.97] | - | 0.96 [ 0.95, 0.97] | - | 0.95 [0.94, 0.96] | - | 0.95 [0.94, 0.96] |
| $\beta z_{3}^{(b)}$ | 1 | 0.96 [ 0.96, 0.97] | ! | 0.95 [ 0.94, 0.96] | 1 | 0.96 [ 0.95, 0.96] | $\cdots$ | 0.94 [ 0.93, 0.95] |
| $\beta_{z_{1}}{ }^{(w)}$ | $\cdots$ | 0.95 [0.94, 0.96] | \% | 0.96 [ 0.95, 0.96] | - | 0.94 [ 0.93, 0.95] | - | 0.96 [0.95, 0.96] |
| $\beta \mathrm{z}_{2}{ }^{(\mathrm{w})}$ | - | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | b | 0.95 [0.94, 0.96] | - | 0.96 [0.95, 0.97] |
| $\beta_{Z_{3}}{ }^{(w)}$ | $+$ | 0.95 [0.94, 0.96] | $\frac{1}{1}$ | 0.96 [ 0.95, 0.96] | $+$ | 0.95 [0.94, 0.96] | $\frac{1}{1}$ | 0.95 [0.94, 0.95] |
| $\sigma_{u}$ | $\cdots$ | 0.94 [ 0.93, 0.95] | - | 0.96 [ 0.95, 0.97] | $\rightarrow$ | 0.95 [0.94, 0.96] | - | 0.96 [0.95, 0.96] |
| $\sigma_{\varepsilon}$ |  | 0.96 [ 0.95, 0.96] | - - | 0.95 [ 0.94, 0.96] | $\cdots$ | 0.94 [ 0.93, 0.95] | $\stackrel{-}{-}$ | 0.95 [ 0.94, 0.96] |
|  | 00.951 .00 |  | 00.951 .00 |  | 00.951 .00 |  | $90 \quad 0.95 \quad 1.00$ |  |


| Parameter | J: 1200, I: 3 | Coverage | J: 1200, I: 5 | Coverage | J: 1200, l: 7 | Coverage | J: 1200, l: 14 | Coverage |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $Y_{0}$ | - | 0.96 [ 0.95, 0.96] | - | 0.95 [ 0.94, 0.96] | - | 0.94 [ 0.93, 0.95] | - | 0.95 [ 0.94, 0.96] |
| $\beta z_{1}{ }^{\text {(b) }}$ | $\bullet$ | 0.95 [ 0.94, 0.96] | $\bullet$ | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] |
| $\beta_{2}{ }^{\text {(b) }}$ | + | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | ¢ | 0.95 [ 0.94, 0.96] | + | 0.95 [ 0.94, 0.96] |
| $\beta z_{3}{ }^{\text {(b) }}$ | $!$ | 0.95 [ 0.94, 0.96] | - | 0.96 [ 0.95, 0.96] | - | 0.95 [ 0.94, 0.96] | $!$ | 0.95 [ 0.94, 0.96] |
| $\beta_{\mathrm{z}}{ }^{(w)}$ | - | 0.95 [ 0.94, 0.96] | 0 | 0.96 [ 0.95, 0.97] | $\cdots$ | 0.95 [ 0.94, 0.96] | $\cdots$ | 0.95 [ 0.94, 0.96] |
| $\beta \mathrm{z}_{2}{ }^{(\mathrm{w})}$ | - | 0.95 [ 0.94, 0.96] | + | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | + | 0.95 [ 0.94, 0.96] |
| $\beta_{z_{3}}{ }^{(w)}$ | $+$ | 0.95 [ 0.94, 0.96] | -1 | 0.94 [ 0.93, 0.95] | -1 | 0.94 [ 0.93, 0.95] | 1 | 0.95 [ 0.94, 0.96] |
| $\sigma_{u}$ | $+$ | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | $-$ | 0.95 [ 0.94, 0.96] | $\rightarrow$ | 0.95 [ 0.94, 0.96] |
| $\sigma_{\varepsilon}$ | $\cdots$ | 0.95 [ 0.94, 0.96] | $\cdots$ | 0.94 [ 0.93, 0.95] | - | 0.96 [ 0.95, 0.96] | - | 0.96 [ 0.95, 0.97] |
|  | 0.900 .951 .00 |  | 900.951 .00 |  | 900.951 .00 |  | 900.951 .00 |  |

Figure 6
Bias of Bayesian Compositional Substitution Multilevel Models with 4－part Composition and Medium Level of Variance

| Parameter | $\mathrm{J}: 30, \mathrm{l}: 3$ | Blas | J：30，1： 5 | Blas | $\mathrm{J}: 30, \mathrm{l}: 7$ | Blas | J：30，I： 14 | Blas |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Delta \hat{y}_{(\text {Slepe－SB）}}^{(\mathrm{w})}$ | $\bullet$ | 0.00 ［－0．01，0．01］ | $\bullet$ | 0.00 ［－0．01，0．01］ | $\bullet$ | －0．01［－0．02，0．01］ | $\bullet$ | 0.00 ［－0．02，0．01］ |
| $\Delta \widehat{y}_{(\text {Sleap－MVPA）}}^{(\mathrm{w})}$ | ！ | 0.01 ［－0．01，0．02］ | － | 0.00 ［－0．01，0．01］ | ！ | 0.00 ［－0．01，0．01］ | － | 0.00 ［－0．01，0．01］ |
| $\left.\Delta \widehat{y}_{(\text {Sleap }- \text { LPA }}^{(\mathrm{w}}\right)$ | ； | 0.00 ［ $0.00,0.01]$ | ； | 0.00 ［ $0.00,0.00]$ | ； | $0.00[-0.01,0.00]$ | ； | 0.00 ［0．00，0．00］ |
| $\Delta \widehat{y}_{(\text {MVPA }}^{(\mathrm{w})}{ }_{\text {d }}{ }^{\text {a }}$ | ＋ | $0.00[-0.03,0.03]$ | ＋ | －0．01［－0．04，0．02］ | ＋ | 0.01 ［－0．02，0．03］ | ＋ | 0.01 ［－0．02，0．04］ |
| $\Delta \widehat{y}_{(\text {MVPA LLPA }}^{(\mathrm{w})}$ | － | $0.00[-0.02,0.02]$ | － | －0．01［－0．03，0．01］ | － | 0.01 ［－0．01，0．03］ | － | 0.01 ［－0．01，0．03］ |
| $\Delta \hat{y}_{(\text {LPA－SB）}}^{(\mathrm{w})}$ | ¢ | $0.00[-0.02,0.01]$ | i | 0.00 ［－0．01，0．01］ | ； | $0.00[-0.01,0.02]$ | ＋ | 0.00 ［－0．02，0．01］ |
| $\Delta \widehat{y}_{\text {（Slep－SB）}}^{(\mathrm{b})}$ | \＄ | $-0.01[-0.01,0.00]$ | 中 | 0.00 ［ $0.00,0.01]$ | \＄ | 0.00 ［ 0．00，0．01］ | \＄ | 0.00 ［ 0．00，0．00］ |
| $\Delta \widehat{y}_{\text {（Slepp－MVPA）}}^{(\mathrm{b})}$ | ； | 0.00 ［ $0.00,0.01]$ | ！ | $0.00[-0.01,0.00]$ | ！ | 0.00 ［ $0.00,0.00]$ | ！ | 0.00 ［ 0．00，0．00］ |
| $\left.\Delta \widehat{y}_{(\text {Sleap }- \text { LPA }}^{(\mathrm{b}}\right)$ | ！ | 0.00 ［ 0．00，0．00］ | ！ | 0.00 ［ $0.00,0.00]$ | ！ | 0.00 ［ $0.00,0.00]$ | ！ | 0.00 ［ 0．00，0．00］ |
| $\Delta \hat{y}_{(\text {MVPA }- \text { B }}(\mathrm{b})$ | ？ | 0.02 ［ 0．00，0．04］ | d | －0．01［－0．02，0．00］ | ＋ | 0.00 ［－0．01，0．01］ | ＋ | 0.00 ［－0．01，0．01］ |
| $\Delta \widehat{\mathrm{y}}_{(\text {MVPA－LPA）}}^{(\mathrm{b})}$ | p | 0.01 ［ 0．00，0．03］ | $!$ | －0．01［－0．01，0．00］ | $!$ | 0.00 ［－0．01，0．01］ | － | 0.00 ［ 0．00，0．01］ |
| $\Delta \widehat{y}_{(\text {LPA－SB }}^{(\mathrm{b})}$ | － | $0.00[-0.01,0.00]$ | ： | 0.01 ［ 0．00，0．01］ | $\bullet$ | 0.00 ［－0．01，0．01］ | － | 0.00 ［ 0．00，0．00］ |
|  | －0．10．0 0.1 |  | $-0.10 .00 .1$ |  | －0．10．0 0.1 |  | －0．10．0 0.1 |  |
| Parameter | J：50，I： 3 | Blas | J：50，l： 5 | Blas | J：50，l： 7 | Blas | J：50，I： 14 | Blas |
| $\left.\Delta \hat{y}_{(\text {Slepp }}^{(\mathrm{w}} \mathrm{FB}\right)$ | － | 0.01 ［ 0．00，0．01］ | － | －0．01［－0．01，0．00］ | $\bullet$ | 0.00 ［－0．01，0．01］ | $\bullet$ | 0.00 ［－0．01，0．01］ |
| $\Delta \hat{\mathrm{y}}_{\text {（Slepp－MVPA }}^{(\mathrm{w})}$ | － | $0.00[-0.01,0.00]$ | － | 0.00 ［－0．01，0．01］ | i | 0.01 ［ 0．00，0．01］ | i | 0.00 ［－0．01，0．01］ |
| $\Delta \widehat{y}_{(\text {Slepp－LPA }}^{(\mathrm{w})}$ | 1 | 0.00 ［ 0．00，0．00］ | i | 0.00 ［ 0．00，0．00］ | 1 | 0.00 ［ 0．00，0．00］ | i | 0.00 ［ 0．00，0．00］ |
|  | 1 | －0．01［－0．04，0．01］ | p | 0.01 ［－0．01，0．03］ | $p$ | $0.01[-0.01,0.03]$ | ＋ | －0．01［－0．03，0．01］ |
| $\Delta \widehat{y}_{(\text {MVPA－LPA }}^{(\mathrm{w})}$ | ＇ | －0．01［－0．02，0．01］ | $\stackrel{\square}{\bullet}$ | $0.01[-0.01,0.02]$ | $!$ | 0.01 ［－0．01，0．02］ | ！ | 0.00 ［－0．02，0．01］ |
| $\Delta \hat{y}_{(\text {LPA－SB }}^{(\mathrm{w})}$ | i | 0.01 ［ 0．00，0．02］ | ¢ | 0.00 ［－0．02，0．01］ | ¢ | $-0.01[-0.02,0.00]$ | ¢ | 0.00 ［－0．01，0．01］ |
| $\Delta \hat{y}_{(\text {Slepp }- \text { SB }}(\mathrm{b})$ | 1 | $0.00[-0.01,0.00]$ | 中 | 0.00 ［ $0.00,0.01]$ | \＄ | 0.00 ［ 0．00，0．00］ | 中 | 0.00 ［ 0．00，0．00］ |
| $\Delta \widehat{y}_{\text {（Slepp－MVPA）}}^{(\mathrm{b})}$ | $\stackrel{1}{1}$ | $0.00[-0.01,0.00]$ | $!$ | 0.00 ［ 0．00，0．00］ | $!$ | 0.00 ［ 0．00，0．00］ | $!$ | 0.00 ［ 0．00，0．00］ |
| $\Delta \widehat{y}_{(\text {（Slep－LPA }}(\mathrm{b})$ |  | 0.00 ［ $0.00,0.00]$ | 1 | 0.00 ［ $0.00,0.00]$ | 1 | 0.00 ［ $0.00,0.00]$ | ＇ | 0.00 ［ 0．00，0．00］ |
| $\Delta \hat{y}_{(\text {MVPA }- \text { SB }}{ }^{\text {b }}$ ） | \＄ | $0.01[-0.01,0.02]$ | \＄ | $0.00[-0.01,0.01]$ | ＋ | 0.00 ［－0．01，0．01］ | \＄ | 0.00 ［－0．01，0．01］ |
| $\Delta \widehat{y}_{(\text {MVPA－LPA）}}^{(\mathrm{b})}$ | ！ | 0.01 ［ $0.00,0.02]$ | ！ | $0.00[-0.01,0.01]$ | ¢ | 0.00 ［ 0．00，0．01］ | ！ | 0.00 ［ 0．00，0．00］ |
| $\Delta \widehat{y}_{(\text {LPA－SB）}}^{(\mathrm{b})}$ | － | $0.00[-0.01,0.01]$ | ： | 0.00 ［ 0．00，0．01］ | ： | 0.00 ［－0．01，0．00］ | － | 0.00 ［ 0．00，0．00］ |
|  | －0．10．0 0.1 |  | －0．10．0 0.1 |  | －0．10．0 0.1 |  | －0．10．0 0.1 |  |
| Parameter | $\mathrm{J}: 360, \mathrm{l}: 3$ | Blas | $\mathrm{J}: 360, \mathrm{l}: 5$ | Blas | $\mathrm{J}: 30, \mathrm{l}: 3$ | Blas | $\mathrm{J}: 360, \mathrm{l}: 14$ | Blas |
| $\Delta \hat{y}_{(\text {Slepe SB）}}^{(\mathrm{w})}$ | － | 0.00 ［ 0．00，0．00］ | $\bullet$ | 0.00 ［ 0．00，0．00］ | $\bullet$ | 0.00 ［－0．01，0．01］ | $\bullet$ | 0.00 ［ 0．00，0．00］ |
| $\Delta \hat{\mathrm{y}}_{\text {（Slepp－MVPA）}}^{(\mathrm{w})}$ | ！ | 0.00 ［ $0.00,0.00]$ | ， | 0.00 ［ $0.00,0.00]$ | ； | 0.01 ［－0．01，0．02］ | i | 0.00 ［ 0．00，0．00］ |
| $\Delta \widehat{y}_{(\text {Sllep }}^{(\text {（ })}$－LPA $)$ | ＋ | 0.00 ［ $0.00,0.00]$ | ¢ | 0.00 ［ $0.00,0.00]$ | ¢ | 0.00 ［ $0.00,0.01]$ | 1 | 0.00 ［ 0．00，0．00］ |
|  | ¢ | $0.00[-0.01,0.00]$ | ¢ | 0.00 ［－0．01，0．01］ | 中 | $0.00[-0.03,0.03]$ | ¢ | 0.00 ［－0．01，0．01］ |
| $\Delta \hat{y}_{(\text {MVPA－LPA }}^{(\mathrm{w})}$ | ！ | $0.00[-0.01,0.00]$ | － | 0.00 ［－0．01，0．00］ | － | 0.00 ［－0．02，0．02］ | － | 0.00 ［－0．01，0．00］ |
| $\Delta \widehat{y}_{(\text {LPA }}^{(\mathrm{w})}(\mathrm{SB})$ | ¢ | 0.00 ［ 0．00，0．00］ | i | 0.00 ［ 0．00，0．00］ | ＋ | 0.00 ［－0．02，0．01］ | ¢ | 0.00 ［－0．01，0．00］ |
| $\Delta \hat{\mathrm{y}}_{\text {（Sleep－SB）}}^{(\mathrm{b})}$ | ＋ | 0.00 ［ $0.00,0.00]$ | ＋ | 0.00 ［ $0.00,0.00]$ | 1 | $-0.01[-0.01,0.00]$ | \＄ | 0.00 ［ 0．00，0．00］ |
| $\Delta \widehat{y}_{(\text {Sleep－MVPA）}}^{(\mathrm{b})}$ | 1 | 0.00 ［ $0.00,0.00]$ | ＋ | 0.00 ［ 0．00，0．00］ | ， | 0.00 ［ 0．00，0．01］ | 1 | 0.00 ［ 0．00，0．00］ |
| $\Delta \hat{y}_{(\text {Sleep－LPA }}^{\text {（ }}$ | ！ | 0.00 ［ $0.00,0.00]$ | ！ | 0.00 ［ 0．00，0．00］ | ！ | 0.00 ［ 0．00，0．00］ | 1 | 0.00 ［ 0．00，0．00］ |
| $\Delta \hat{y}_{(\text {MVPA }- \text { SB }}(\mathrm{b})$ | \＄ | $0.00[-0.01,0.00]$ | 中 | 0.00 ［ $0.00,0.01]$ | ？ | 0.02 ［ 0．00，0．04］ | \＄ | 0.00 ［ 0．00，0．00］ |
| $\Delta \widehat{y}_{(\text {MVPA－LPA）}}^{(\mathrm{b})}$ | ！ | 0.00 ［ $0.00,0.00]$ | － | 0.00 ［ $0.00,0.00]$ | ！ | 0.01 ［0．00，0．03］ | ！ | 0.00 ［ 0．00，0．00］ |
| $\Delta \widehat{y}_{(\text {LPA－SB）}}^{(\mathrm{b})}$ | $\stackrel{1}{+}$ | 0.00 ［ 0．00，0．00］ | － | 0.00 ［ 0．00，0．00］ | $\bigcirc$ | $0.00[-0.01,0.00]$ | － | 0.00 ［ 0．00，0．00］ |
|  | －0．10．0 0.1 |  | －0．10．0 0.1 |  | $-0.10 .00 .1$ |  | －0．10．0 0.1 |  |
| Parameter | J：1200，l： 3 | Blas | J：1200，I： 5 | Blas | J：1200，l： 7 | Blas | J：1200，l： 14 | Blas |
|  | － | 0.00 ［ 0．00，0．00］ | $\bullet$ | 0.00 ［ 0．00，0．00］ | $\bullet$ | 0.00 ［ 0．00，0．00］ | Q | 0.00 ［ 0．00，0．00］ |
| $\Delta \hat{\mathrm{y}}_{(\text {Slepp }-\mathrm{MVPA})}^{(\mathrm{w})}$ | ！ | 0.00 ［ $0.00,0.00]$ | i | 0.00 ［ $0.00,0.00]$ | － | 0.00 ［ $0.00,0.00]$ | ； | 0.00 ［ 0．00，0．00］ |
| $\Delta \widehat{y}_{(\text {Slepp－LPA }}^{(\mathrm{w})}$ | ¢ | 0.00 ［ $0.00,0.00]$ | \＄ | 0.00 ［ $0.00,0.00]$ | ＋ | 0.00 ［ $0.00,0.00]$ | ¢ | 0.00 ［ 0．00，0．00］ |
|  | ¢ | $0.00[-0.01,0.00]$ | ¢ | 0.00 ［ $0.00,0.00]$ | ¢ | 0.00 ［ $0.00,0.00]$ | $\phi$ | 0.00 ［ 0．00，0．01］ |
| $\Delta \widehat{y}_{(\text {（MVPA－LPA }}^{(\mathrm{w})}$ | ！ | 0.00 ［ $0.00,0.00]$ | － | 0.00 ［ $0.00,0.00]$ | － | 0.00 ［ $0.00,0.00]$ | － | 0.00 ［ 0．00，0．01］ |
| $\Delta \widehat{y}_{(\text {（PPA－SB）}}^{(\mathrm{w})}$ | － | 0.00 ［ $0.00,0.00]$ | ¢ | 0.00 ［ $0.00,0.00]$ | ¢ | 0.00 ［ $0.00,0.00]$ | i | 0.00 ［ 0．00，0．00］ |
| $\Delta \widehat{y}_{\text {（liep }- \text { SB }}^{(\mathrm{b})}$ | \＄ | 0.00 ［ 0．00，0．00］ | \＄ | 0.00 ［ 0．00，0．00］ | \＄ | 0.00 ［ 0．00，0．00］ | \＄ | 0.00 ［ 0．00，0．00］ |
| $\left.\Delta \hat{y}_{(\text {Slepp }}^{\mathrm{b}} \mathrm{MVPA}\right)$ | ！ | 0.00 ［ $0.00,0.00]$ | $!$ | 0.00 ［ 0．00，0．00］ | $!$ | 0.00 ［ 0．00，0．00］ | $!$ | 0.00 ［ 0．00，0．00］ |
| $\Delta \widehat{y}_{(\text {（lapeLPPA）}}^{(\mathrm{b})}$ | 1 | 0.00 ［ $0.00,0.00$ ］ | 1 | 0.00 ［ $0.00,0.00$ ］ | 1 | 0.00 ［ $0.00,0.00$ ］ | 1 | 0.00 ［ 0．00，0．00］ |
|  | ＋ | 0.00 ［ $0.00,0.00]$ | \＄ | 0.00 ［ $0.00,0.00]$ | \＄ | 0.00 ［ $0.00,0.00]$ | \＄ | 0.00 ［ 0．00，0．00］ |
| $\Delta \widehat{y}_{(\text {MVPA-LPA }}^{(b)}$ | ！ | 0.00 ［ 0．00，0．00］ | i | 0.00 ［ 0．00，0．00］ | i | 0.00 ［ 0．00，0．00］ | ？ | 0.00 ［ 0．00，0．00］ |
| $\Delta \widehat{y}_{(\text {LPA－SB）}}^{(\mathrm{b})}$ | $\stackrel{1}{+}$ | 0.00 ［ $0.00,0.00$ ］ | ！ | 0.00 ［ $0.00,0.00$ ］ | $!$ | 0.00 ［ $0.00,0.00$ ］ | $\bigcirc$ | 0.00 ［ 0．00，0．00］ |
|  | －0．10．0 0.1 |  | －0．10．0 0.1 |  | －0．10．0 0.1 |  | －0．10．0 0.1 |  |

Figure 7
Coverage of Bayesian Compositional Substitution Multilevel Models with 4-part Composition and Medium Level of Variance

| Parameter | J: 30, I: 3 | Coverage | J: $30, \mathrm{l}: 5$ | Coverage | J: 30, 1: 7 | Coverage | J: 30, I: 14 | Coverage |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Delta \hat{y}_{\text {(Slepp SB) }}^{(\mathrm{w})}$ | - | 0.95 [ 0.94, 0.96] | - | 0.96 [ 0.95, 0.96] | - | 0.96 [ 0.95, 0.97] | $\cdots$ | 0.96 [ 0.95, 0.96] |
| $\Delta \widehat{y}_{(\text {Slepe }- \text { MVPA }}^{(\mathrm{w})}$ | $\cdots$ | 0.95 [ 0.94, 0.96] | $\cdots$ | 0.95 [ 0.94, 0.96] | - | 0.96 [ 0.95, 0.97] | - | 0.96 [ 0.95, 0.97] |
| $\Delta \widehat{y}_{(\text {Slepe }- \text { LPA }}^{(\mathrm{w})}$ | - | 0.96 [ 0.95, 0.97] | - | 0.96 [ 0.95, 0.97] | - | 0.96 [ 0.95, 0.97] | 1 | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{y}_{(\text {MVPA }}^{(\mathrm{w})}{ }^{\text {d }}$ ) | $+$ | 0.95 [0.94, 0.96] | + | 0.95 [ 0.94, 0.96] | - | 0.96 [0.95, 0.97] | - | 0.96 [ 0.95, 0.97] |
| $\Delta \widehat{y}_{(\text {MVPA LLPA }}^{(\mathrm{w})}$ | $\cdots$ | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | 1 | 0.96 [ 0.95, 0.97] |
| $\Delta \widehat{\mathrm{y}}_{(\text {LPA }}^{(\mathrm{w})}(\mathrm{SB})$ | - | 0.95 [ 0.95, 0.96] | - | 0.96 [ 0.95, 0.97] | ; | 0.96 [ 0.95, 0.97] | - | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{y}_{\text {(Slepp-SB) }}^{(\mathrm{b})}$ | $p$ | 0.95 [ 0.94, 0.96] | p- | 0.96 [ 0.95, 0.96] | + | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{y}_{\text {(Sleep-MVPA) }}^{(\mathrm{b})}$ | - | 0.95 [ 0.94, 0.96] | - | 0.94 [ 0.93, 0.95] | ; | 0.95 [ 0.94, 0.96] | ! | 0.94 [ 0.93, 0.95] |
| $\left.\Delta \widehat{y}_{(\text {Sleep }- \text { LPA }}^{(\mathrm{b}}\right)$ | ' | 0.94 [ 0.93, 0.95] | ! | 0.95 [ 0.94, 0.96] | 1 | 0.95 [ 0.94, 0.96] | 1 | 0.96 [ 0.95, 0.97] |
| $\Delta \widehat{\mathrm{y}}_{(\text {MVPA }}^{(\mathrm{b})}$ | - | 0.96 [ 0.95, 0.97] | $+$ | 0.95 [ 0.94, 0.96] | $+$ | 0.95 [ 0.94, 0.96] | $\bigcirc$ | 0.94 [ 0.93, 0.95] |
| $\Delta \widehat{y}_{(\text {MVPA-LPA })}^{(\mathrm{b})}$ | - | 0.96 [ 0.95, 0.97] | - | 0.95 [0.94, 0.96] | - | 0.95 [0.94, 0.96] | -1 | 0.94 [ 0.93, 0.95] |
| $\Delta \widehat{y}_{(\mathrm{LPA}-\mathrm{SB})}^{(\mathrm{b})}$ | - | 0.96 [ 0.95, 0.96] | $\cdots$ | 0.95 [ 0.94, 0.95] | $\cdots$ | 0.95 [ 0.94, 0.96] | $\cdots$ | 0.94 [ 0.93, 0.95] |
|  | 00.951 |  | 00.951 |  | 0.95 |  | 0.90 .951. |  |


| Parameter | J: 50, I: 3 | Coverage | J: 50, I: 5 | Coverage | J: 50, I: 7 | Coverage | J: 50, I: 14 | Coverage |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Delta \widehat{\mathrm{y}}_{\text {(Sleep-SB) }}^{(\mathrm{w})}$ | - | 0.96 [ 0.95, 0.97] | $\cdots$ | 0.96 [ 0.95, 0.97] | $\cdots$ | 0.96 [ $0.95,0.97]$ | $\cdots$ | 0.95 [ $0.95,0.96$ ] |
| $\left.\Delta \hat{\mathrm{y}}_{(\text {Slepp }}^{\mathrm{w})} \mathrm{MVPA}\right)$ | $\cdots$ | 0.95 [ 0.94, 0.96] | $-$ | 0.95 [ 0.94, 0.96] | io | 0.96 [ 0.95, 0.97] | 1 | 0.95 [ 0.95, 0.96] |
| $\Delta \widehat{y}_{(\text {Slep }- \text { LPA }}^{(\mathrm{w})}$ | $\rightarrow$ | 0.94 [ 0.93, 0.95] | - | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] |
| $\left.\Delta \widehat{y}_{(\text {(wVPA }}^{(\mathrm{w})} \mathrm{SB}\right)$ | + | 0.96 [ 0.95, 0.96] | + | 0.95 [ 0.94, 0.96] | - | 0.96 [ 0.95, 0.97] | - | 0.96 [ 0.95, 0.97] |
| $\Delta \widehat{y}_{(\text {MVPA }- \text { LPA }}(\mathrm{w})$ | - | 0.95 [ 0.94, 0.96] | - | 0.96 [ 0.95, 0.97] | - | 0.96 [ $0.95,0.96$ ] | - | 0.96 [ 0.95, 0.96] |
| $\Delta \widehat{y}_{(\text {LPA }}^{(\mathrm{w})}{ }_{\text {dB }}$ | $\square$ | 0.95 [ 0.94, 0.96] | $\cdots$ | 0.94 [ 0.93, 0.95] | - | 0.96 [ $0.95,0.96$ ] | - | 0.95 [ 0.95, 0.96] |
| $\Delta \widehat{\mathrm{y}}_{\text {(Sleep-SB) }}^{(\mathrm{b})}$ | - | 0.96 [ 0.95, 0.96] | $+$ | 0.95 [ 0.94, 0.96] | $\cdots$ | 0.94 [0.93, 0.95] | $+$ | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{y}_{(\text {Sleep-MVPA) }}^{(\mathrm{b})}$ | i | 0.95 [ 0.94, 0.96] | - | 0.94 [ 0.93, 0.95] | i | 0.95 [ 0.94, 0.96] | $i$ | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{y}_{\text {(liep-LPA) }}^{(\mathrm{b})}$ | 1 | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.95, 0.96] | + | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{y}_{(\text {MVPA }- \text { SB }}(\mathrm{b})$ | - | 0.96 [ 0.95, 0.97] | $\frac{1}{1}$ | 0.95 [ 0.94, 0.95] | $+$ | 0.95 [0.94, 0.96] | $+$ | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{y}_{(\text {( }}^{(\mathrm{bVPA}}$-LPA) | + | 0.95 [ 0.94, 0.96] | $-$ | 0.95 [ 0.94, 0.96] | $\rightarrow$ | 0.94 [ 0.93, 0.95] | - | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{\mathrm{y}}_{(\mathrm{LPA}-\mathrm{SB})}^{(\mathrm{b})}$ | $\cdots$ | 0.95 [ 0.94, 0.96] | $\cdots$ | 0.94 [ 0.93, 0.95] | $\cdots$ | 0.95 [ 0.94, 0.95] | $\cdots$ | 0.95 [ 0.94, 0.96] |
|  | 00.95 |  | 00.95 |  | 0.95 |  | 00.951. |  |


| Parameter | J: 360, I: 3 | Coverage | J: 360, l: 5 | Coverage | $\mathrm{J}: 30,1: 3$ | Coverage | J: 360, I: 14 | Coverage |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Delta \widehat{y}_{\text {(Slepp-SB) }}^{(\mathrm{w})}$ | - | 0.96 [ 0.95, 0.97] | $\cdots$ | 0.96 [ 0.95, 0.96] | - | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{y}_{(\text {Slep }- \text { MVPA }}^{(\mathrm{w})}$ | - | 0.96 [ 0.95, 0.97] | - | 0.96 [ 0.95, 0.97] | $\div$ | 0.95 [ 0.94, 0.96] | $\cdots$ | 0.94 [ 0.93, 0.95] |
| $\Delta \widehat{y}_{(\text {Slep }}^{(\mathrm{w})}(\mathrm{LPPA})$ | b | 0.95 [ 0.94, 0.96] | + | 0.95 [ 0.94, 0.96] | 1- | 0.96 [ 0.95, 0.97] | -1 | 0.95 [ 0.94, 0.95] |
|  | io. | 0.96 [ 0.95, 0.97] | + | 0.95 [ 0.94, 0.96] | $+$ | 0.95 [ 0.94, 0.96] | - + | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{y}_{\left(\begin{array}{l}\text { (wVPA-LPA }\end{array}\right)}$ | ${ }^{\text {co}}$ | 0.96 [ 0.95, 0.97] | - | 0.96 [ 0.95, 0.96] | - | 0.95 [ 0.94, 0.96] | $\square$ | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{\mathrm{y}}_{(\text {LPA-SB }}^{(\mathrm{w})}$ | - | 0.96 [ 0.95, 0.97] | - | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.95, 0.96] | $\rightarrow$ | 0.94 [ 0.93, 0.95] |
| $\Delta \widehat{y}_{\text {(Slepe }- \text { SB }}^{(\mathrm{b})}$ | + | 0.95 [ 0.94, 0.96] | + | 0.95 [ 0.94, 0.96] | $t$ | 0.95 [ 0.94, 0.96] | $+$ | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{y}_{(\text {Sliep }- \text { MVPA }}(\mathrm{b})$ | - | 0.96 [ 0.95, 0.96] | \% | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | , | 0.94 [ 0.93, 0.95] |
| $\Delta \widehat{y}_{(\text {Slep }}^{\text {b }}$ (LPA) | I | 0.95 [ 0.94, 0.96] | , | 0.96 [ 0.95, 0.97] | , | 0.94 [ 0.93, 0.95] |  | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{y}_{(\text {MVPA }}(\mathrm{s}$ S $)$ | t | 0.96 [ 0.95, 0.96] | 4 | 0.95 [ 0.94, 0.96] | - | 0.96 [ 0.95, 0.97] | 4 | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{y}_{(\text {MVPA }- \text { LPA }}(\mathrm{b})$ | - | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | - | 0.96 [ 0.95, 0.97] | - | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{y}_{(\text {LPA-SB })}^{(\mathrm{b})}$ | - | 0.96 [ 0.95, 0.97] | - | 0.95 [ 0.94, 0.96] |  | 0.96 [ 0.95, 0.96] | , | 0.94 [ 0.93, 0.95] |
|  | $0.90 \quad 0.951 .00$ |  | $0.90 \quad 0.951 .00$ |  | 0.900 .951 .00 |  | $0.90 \quad 0.951 .00$ |  |
| Parameter | J: 1200, I: 3 | Coverage | J: 1200, l: 5 | Coverage | J: 1200, I: 7 | Coverage | J: 1200, I: 14 | Coverage |
| $\Delta \widehat{y}_{(\text {Slep-SB) }}^{(\mathrm{w})}$ | - | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | $\bigcirc$ | 0.94 [ 0.93, 0.95] |
| $\left.\Delta \widehat{y}_{(\text {Slep }}^{(\mathrm{w})} \mathrm{MVPA}\right)$ | $\cdots$ | 0.95 [ 0.94, 0.96] | $\cdots$ | 0.95 [ 0.94, 0.96] | $\stackrel{-}{-}$ | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{y}_{(\text {(slep-LPA }}(\mathrm{w})$ | - | 0.95 [ 0.94, 0.96] | $\frac{1}{6}$ | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | , | 0.95 [ 0.94, 0.96] |
| $\left.\Delta \widehat{y}_{(\text {MVPA }}^{(\mathrm{w})} \mathrm{SB}\right)$ | + | 0.95 [ 0.94, 0.96] | + | 0.95 [ 0.94, 0.96] | $p$ | 0.95 [ 0.95, 0.96] | + | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{y}_{(\text {(mVPA-LPA }}(\mathrm{w})$ | $\cdots$ | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | - | 0.96 [ 0.95, 0.96] | $\square$ | 0.94 [ 0.93, 0.95] |
|  | - | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | , | 0.95 [ 0.94, 0.96] |
|  | - | 0.95 [ 0.94, 0.96] | P | 0.96 [ 0.95, 0.96] | + | 0.95 [ 0.94, 0.96] | 4 | 0.95 [ 0.94, 0.96] |
|  | - | 0.95 [ 0.95, 0.96] | $\cdots$ | 0.95 [ 0.94, 0.96] | $\cdots$ | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.95, 0.96] |
| $\Delta \hat{\mathrm{y}}_{\text {(Slep-LPA }}(\mathrm{b})$ | , | 0.95 [ 0.94, 0.96] | - | 0.96 [ 0.95, 0.96] | $!$ | 0.95 [ 0.94, 0.96] | 1 | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{y}_{(\text {SVPA }}$ (b) ${ }^{\text {b/ }}$ ) | ' | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | + | 0.95 [ 0.94, 0.96] | + | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{\mathrm{y}}_{(\text {MVPA-LPA }}(\mathrm{b})$ | - | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] | - | 0.95 [ 0.94, 0.96] |
| $\Delta \widehat{\mathrm{y}}_{(\mathrm{LPA}-\mathrm{SB})}^{(\mathrm{b})}$ | $\cdots$ | 0.95 [ 0.94, 0.96] | $\cdots$ | 0.94 [ 0.93, 0.95] | - | 0.94 [ 0.93, 0.95] | $\cdots$ | 0.95 [ 0.95, 0.96] |
|  | 0.900 .951 .00 |  | $0.90 \quad 0.951 .00$ |  | $0.90 \quad 0.951 .00$ |  | $0.90 \quad 0.951 .00$ |  |

