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Equivalence Principle and Machian origin of extended gravity

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Abstract

Chae's analyses on GAIA observations of wide binary stars have fortified the paradigm of extended gravity with particular attention to MOND-like theories. We recall that, starting from the origin of Einstein's general relativity, the request of Mach on the structure of the theory has been the core of the foundational debate. This issue is strictly connected with the nature of the mass-energy equivalence. This was exactly the key point that Einstein used to derive the same general relativity. On the other hand, the current requirements of particle physics and the open questions within extended gravity theories, which have recently been further strengthened by analyses of GAIA observations, request a better understanding of the Equivalence Principle. By considering a direct coupling between the Ricci curvature scalar and the matter Lagrangian a non geodesic ratio between the inertial and the gravitational mass can be fixed and MOND-like theories are retrieved at low energies.

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The Science of Mechanics by Ernst Mach [1] had a strong influence on Einstein and was very important in the development of general relativity. In Newtonian theory, acceleration is absolute. Newton deduced the existence of an absolute rotation in the famous gedankenexperiment of the rotating bucket filled with water, by observing the curved surfaces on the water. In that way, the inertia was explained via a sort of resistance to motion in the absolute space which, in turn, comes to be an agent and not a mere physical theater of coordinates, although unspecified. The philosopher George Berkeley, in his De Motu (1721), was the first who questioned the reasoning of Newton. He can be considered the precursor of Mach and Einstein [2]. In fact, after more than 150 years, Mach strongly criticized Newton's absolute space by concluding that the inertia should be an interaction which requires other bodies to manifest itself. Thus, it would make no sense in a Universe consisting of just a single mass. Mach's approach proposes a total relational symmetry and every motion, uniform or accelerated, makes sense only in reference to other bodies. Hence, the so called *Mach Principle* implies that the inertia of a body is not an intrinsic property, but depends on the mass distribution in the rest of the Universe instead. Although Einstein was very fascinated by Mach reasoning, Mach Principle is not fully incorporated into general relativity's field equations [3]. The challenge of a *Machian gravitational physics* was accepted several times (though less than expected) in the context of both classical and quantum theories. An example is Narlikar's theory with variable mass, which was derived from Wheeler-Feynman-like action at a distance theory [4, 5]. Sciama's theory [6] sees the inertia as "gravitational closeness" (and the perfect equivalence) under the precise cosmological condition $G\rho\frac{r^2}{c^2} = 1$, where r is the radius of the universe, ρ the density, c is the speed of light and G the Newtonian gravitational constant. In a quantum framework and in Higgs times, the problem results more complex [7–11].

> Einstein often stressed that some Machian effects should be present in general relativity. In particular, in the famous Lectures of 1921 [12] he argued that in general relativity there are the following effects:

1. *The inertia of a body must increase when ponderable masses are piled up in its neighbourhood.*

2. *A body must experience an accelerating force when neighboring masses are accelerated and the force must be in the same direction as that acceleration.*

3. *A rotating hollow body must generate inside of itself a Coriolis field which deflects moving bodies in the sense of the rotation and a radial centrifugal field as well.*

Following Einstein's reasoning one considers the geodesic equation

$$\frac{d^2 x_\mu}{ds^2} + \Gamma_\mu^{\alpha\beta} \frac{dx_\alpha}{ds}\frac{dx_\beta}{ds} = 0. \tag{1}$$

In the weak-field approximation, Einstein found a metric, representing the gravitational field due to a distribution of small masses corresponding to a density σ and having small velocities $\frac{dx^i}{ds}$, which is

$$g_{00} = 1 - \tfrac{2G}{c^2}\int \tfrac{\sigma dV}{r}$$

$$g_{0i} = \tfrac{4G}{c^2}\int \tfrac{dx^i}{ds}\tfrac{\sigma dV}{r} \tag{2}$$

$$g_{ij} = -\delta_{ij}\left(1 + \tfrac{2G}{c^2}\int \tfrac{\sigma dV}{r}\right).$$

Then, defining

$$\overline{\sigma} \equiv \tfrac{G}{c^2}\int \tfrac{\sigma dV}{r}$$

$$A \equiv \tfrac{4G}{c^2}\int \tfrac{\sigma v dV}{r} \tag{3}$$

one finds the equation of motion as

$$\frac{d}{dx^0}\left[(1+\overline{\sigma})\, v\right] = \nabla\overline{\sigma} + \frac{dA}{dx^0} + (\nabla \wedge A)\wedge v. \tag{4}$$

Einstein's interpretation was that the inertial mass m_i is proportional to $(1+\overline{\sigma})$ and, consequently, it should increase when ponderable masses approach the test body

$$m_i = m_g\left(1 + \frac{G}{c^2}\int \frac{\overline{\sigma} dV}{r}\right), \tag{5}$$

where m_g is the gravitational mass. Brans' interpretation [13], accepted by several physicists, was that only the second and third effect are contained in general relativity. At first glance it would seem that, if Einstein's interpretation were correct, there would be a violation of the Equivalence Principle. However, it should be emphasized that all bodies with different inertial masses are still falling with the same acceleration in a gravitational field. Darabi [14] analyzed what he called *Modified Mach Principle* in the context of an expanding universe. He suggested the following definitions for the inertial mass within and beyond the bulge of galaxies as

$$m_i = C \qquad\qquad r \leq R_0$$

$$m_i = \tfrac{C'}{r} = m_g \tfrac{R_0}{r} \qquad r > R_0, \tag{6}$$

where R_0 is the size of the bulge and C and C' are constants: the first one is inertial mass versus gravitational interaction within the bulge, and the second one is inertial mass versus cosmological expansion beyond the bulge. Then, the introduction of a genuine Mach's principle seems to have a need for re-introduction of the distinction between inertial mass and gravitational mass, hidden under the metric of general relativity and the strong form of the Equivalence Principle, which locally turns out to be always valid in support of the structure of general relativity, both from the classical [15] and quantum [16] point of view. On the other hand, the equivalence between inertial and gravitational mass is the axiomatic and constructive keystone not only of general relativity, but of all the metric theories of gravity. One is then faced with the foundational problem of the interpretation of the formalism able to establish the equivalence principle on the physical meaning of the relationship between inertial and gravitational mass. This, could be connected with another foundational problem in cosmology and gravitation, the one concerning the nature of Dark Matter, which is one of the unsolved mysteries in Science since C. Zwicky measured the velocity dispersion of the Coma cluster of galaxies [17]. Let us consider the equation

$$m_i \frac{v^2}{r} = \frac{G M_g m_g}{r^2}, \tag{7}$$

where m_i is a body that rotates around a gravitational mass M_g over a constant radius r. It is well known that the famous Milgrom's relation, which is founded on MOND [18, 19],

$$v = \sqrt[4]{G M_g a_0}, \tag{8}$$

with $a_0 \approx 10^{-10} \, \frac{m}{s^2}$, is in agreement with various observational evidences, although not with all, and has been recently endorsed by Chae's analyses on GAIA observations of wide binary stars [20, 21]. Hence, by combining Eqs. (7) and (8) one writes

$$v^2 = \frac{G M_g}{r} \frac{m_g}{m_i} = \sqrt{G M_g a_0}, \tag{9}$$

which imples that Milgrom's acceleration a_0 depends on the ratio between gravitational and inertial mass as

$$a_0 = \left(\frac{m_g}{m_i} \right)^2 \frac{G M_g}{r^2}. \tag{10}$$

In other words, MOND dynamics could depend on violations of the Equivalence Principle at large distances. This is not in contrast to today's strong empirical evidence of the Equivalence Principle [22], as observations and experiments on the equivalence between inertial mass and gravitational mass are conducted on Earth, or at least within the Solar System. Let us see the situation in another way. From Eq. (9) one also gets

$$\frac{m_g}{m_i} = \sqrt{\frac{a_0 r^2}{GM_g}}. \tag{11}$$

Rather than interpreting a_0 from the kinematic point of view one can interpret it in terms of a gravitational field by writing

$$\frac{m_g}{m_i} = \sqrt{\frac{g_0}{g}}, \tag{12}$$

where $g = \frac{GM_g}{r^2}$ is the standard Newtonian acceleration. According to Mach's interpretation, the inertial mass of a body arises as a consequence of its interactions with the Universe. Thus, one assumes that

$$\frac{m_g}{m_i} \equiv \mu, \tag{13}$$

where $\mu = 1$ for $\left|\frac{g_0}{g}\right| \ll 1$ (relatively small distances) and $\mu = \sqrt{\frac{g_0}{g}}$ for $\left|\frac{g_0}{g}\right| \gg 1$ (large distances). A possible form of μ could be

$$\mu \equiv \sqrt{\frac{g_0 + g}{g}}, \tag{14}$$

where in this case g_0 is the Machian gravitational field generated by all the masses of the Universe different from M_g. It can easily be verified that, when $g \gg g_0$ the circular velocity decreases with increasing distance from M_g, according to the Newtonian law, but, when $g \ll g_0$ one obtains

$$v^2 = \frac{GM_g}{r}\sqrt{\frac{g_0}{g}} = GM_g\sqrt{\frac{g_0}{GM_g}} = \sqrt{GM_g g_0}, \tag{15}$$

which leads immediately to

$$v = \sqrt[4]{GM_g g_0}. \tag{16}$$

Obviously, the value of g_0 which fits the majority of the data of galaxies rotation curves is about $10^{-10}\,\frac{m}{s^2}$. If, on the one hand, the relations (8) and (16) coincide from the mathematical point of view, on the other hand from the physical point of view the situation is different. At every point in the Universe Newton second law continues to be valid even in the presence of small accelerations. This is due to the fact that the Machian gravitational field generated by all the masses of the Universe different from M_g, which still has Newtonian origin, dominates over the Newtonian gravitational field generated by M_g. It is important to ask what could be the geometric-relativistic counterpart of the weak field approach developed so far. An intriguing interpretation in a geometric-relativistic sense is the following. In 2007 Bertolami and others [23] proposed an explicit coupling between an arbitrary function of the scalar curvature, R, and the Lagrangian density of matter in the framework of $f(R)$ gravity via the action

$$S = \int \left\{ \frac{1}{2\kappa} f_1(R) + \left[1 + \lambda f_2(R) \right] L_m \right\} \sqrt{-g} dx^4, \tag{17}$$

where $\kappa \equiv 8\pi G c^{-4}$ is the Einstein gravitational constant and L_m is the Lagrangian density corresponding to matter. By setting $f_1(R) = f_2(R) = R$, $\lambda \ll \frac{1}{2\kappa}$, then the theory arising from the corresponding action

$$S = \int \left(\frac{1}{2\kappa} R + \lambda R L_m + L_m \right) \sqrt{-g} dx^4, \tag{18}$$

which only includes a weak coupling between the Ricci scalar and the matter Lagrangian, represents a weak deviation from standard general relativity and can, in principle, pass the solar system terms. Adapting the analysys in [23] to the theory arising from the action of Eq. (18), one introduces the standard energy-momentum tensor of a perfect fluid $T_{\mu\nu}^{(m)} \equiv (\epsilon + p) u_\mu u_\nu$, where ϵ and p are the overall energy density and the pressure, respectively. u_μ is the four-velocity satisfying $u_\mu u^\mu = 1$ and $u^\mu u_{\mu;\nu} = 0$. Then, one finds that the coupling between the Ricci scalar and the matter Lagrangian generates a non-geodesic equation compatible with a violation of the Equivalence Principle at large distances [23]

$$\frac{d^2 x_\mu}{ds^2} + \Gamma_\mu^{\alpha\beta} \frac{dx_\alpha}{ds} \frac{dx_\beta}{ds} = F^\alpha, \tag{19}$$

due to the presence of extra force orthogonal to the four-velocity of the particle [23]

$$F^\alpha = \frac{1}{\epsilon + p} \left[\frac{\lambda}{1 + \lambda R} (L_m + p) \nabla_\beta R + \nabla_\beta p \right] h^{\alpha\beta}, \tag{20}$$

where the projection operator $h_{\mu\nu} \equiv g_{\mu\nu} - u_\mu u_\nu$ has been introduced, which satisfies $h_{\mu\nu} u^\mu = 0$. The weak field limit in three dimensions of Eq. (19) is [23]

$$\overrightarrow{a}_{tot} = \overrightarrow{g} + \overrightarrow{a}_{ex}. \tag{21}$$

Hence, the total acceleration $\overrightarrow{a}_{tot}$ turns out to be the sum of the standard Newtonian one, $\overrightarrow{g}$, plus that (per unit mass) due to the presence of the extra force, $\overrightarrow{a}_{ex}$. From Eq. (21), a bit of three-dimensional geometry [23] permits one to write the Newtonian acceleration as

$$\overrightarrow{g} = \frac{1}{2}\left(a_{tot}^2 - g^2 - a_{ex}^2\right)\frac{\overrightarrow{a}_{tot}}{a_{tot}a_{ex}}. \tag{22}$$

In the limit in which $\overrightarrow{a}_{ex}$ dominates, that is $g \ll a_{tot}$, one obtains [23]

$$\overrightarrow{g} \simeq \frac{a_{tot}\overrightarrow{a}_{tot}}{2a_{ex}}\left(1 - \frac{a_{ex}^2}{a_{tot}^2}\right) = \frac{a_{tot}}{g_0}\overrightarrow{a}_{tot}, \tag{23}$$

where [23]

$$a_0 = g_0 \equiv 2a_{ex}\left(1 - \frac{a_{ex}^2}{a_{tot}^2}\right)^{-1}. \tag{24}$$

Eq. (23) implies $a_{tot} \simeq \sqrt{g_0 g}$, which is completely consistent with Eq. (15). This consistence enables one to combine Eq. (24) with Eqs. (11) and (12) obtaining

$$\frac{m_g}{m_i} = \sqrt{\frac{g_0}{g}} = \sqrt{\frac{2a_{ex}}{g\left(1 - \frac{a_{ex}^2}{a_{tot}^2}\right)}}. \tag{25}$$

Thus, in the current approach the ratio between gravitational and inertial mass is explained in an elegant, geometric way, via a direct coupling between the Ricci curvature scalar and the matter Lagrangian which generates a non geodesic motion of test particles. It should be emphasized that, in the current approach, slightly different from a pure MOND approach, the Machian gravitational field g_0 is not strictly constant as it depends both on local characteristics of the curvature and the direct coupling between curvature and matter. This seems consistent with the fact that, although MOND appears to be able to explain many astrophysical observations, for example the GAIA data analyzed by Chae [20, 21], it cannot explain all of them. For example MOND does not seem completely consistent with recent data on the rotation curve of the Milky Way because the decreasing behavior of the rotation curve beyond 20 kpc [24].

In summary, in this Essay it has been shown that an approach to gravitation conforming to the Mach Principle allows one to rediscover MOND-like theories through a violation of the Equivalence Principle at large distances. The geometric-relativistic counterpart of this approach is based on a weak modification to the standard Einstein-Hilbert action which admits a weak direct coupling between the Ricci scalar and the Lagrangian of matter. If on the one hand the weakness of this modification to standard general relativity allows the theory to pass, in principle, the solar system tests, on the other hand it is precisely this direct coupling between the Ricci scalar and the Lagrangian of matter that generates the violation of the Equivalence Principle at large distances, which allows one to find the MOND-like behavior in the weak field approximation. Thus, the Machian approach in this Essay obtains strong observational consistency with the GAIA data analyzed by Chae [20, 21], while the non-strict constance of the Machian gravitational field g_0 can, in principle justify variations from the pure MOND regime in some astrophysical observations, like the decreasing behavior of the rotation curve of the Milky Way beyond 20 kpc [24].

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The Authors dedicate this Essay to the memory of their beloved mothers.

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