Constant-roll inflation with a complex scalar field

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We consider inflation with a constant rate of rolling in which a complex scalar field plays the role of inflaton during the inflationary epoch. We implement the inflationary analysis for an accredited angular speed $\dot{\theta}$ which satisfies our dynamical equations. Scalar and tensorial perturbations generated in the framework of constant roll inflation with a complex field are studied. In this respect, we find analytically solutions to the gauge invariant fluctuations, with which an expression for the scalar power spectrum together with its scalar index spectral in this scenario were found. By comparing the obtained results with the observations coming from the cosmic microwave background anisotropies, the constraints on the parameters space of the model and also its predictions are analyzed and discussed.

I. INTRODUCTION

Inflation theory as an unavoidable part of modern cosmology attempts to describe the phenomena at the early time through the primordial perturbations coming from the quantum fluctuations. The scalar perturbations as the seed of the universe form the large-scale structures of the universe. Moreover, they are the main ones responsible for the temperature anisotropies of the cosmic microwave background (CMB) at the last scattering surface. On the other hand, monitoring the B-mode polarized CMB photons discloses the existence of primordial gravitational waves generated by the tensor perturbations of inflation [1–4]. In the standard cosmology, a single scalar field, so-that called inflaton, is the dominant matter of the universe during cosmic inflation so that it decays to the particles at the last step of inflation through the reheating process [5, 6]. Non-minimal coupling (NMC) model is another broadly-used inflationary model considering a coupling between the Ricci scalar and the inflaton field coming from the quantum corrections of scalar field [7–16]. Besides the mentioned models, modified theories of gravity introduce different mechanisms in order to explain the early-time accelerating phase of the universe. For instance, in $f(R)$ gravity, modifying the Ricci scalar can describe the inflationary era instead of considering a scalar field [17–24]. Also, in the extended versions of teleparallel gravity, i.e. modified teleparallel gravity $f(T)$ and modified symmetric teleparallel gravity $f(Q)$, two other geometrical objects, tension and non-metricity, are responsible to demonstrate the inflationary epoch [25–27]. Apart from the inflationary criteria, these models could be also tested in the context of weak gravity conjecture (WGC), in particular, in the swampland region where the low-energy effective field theories are incompatible with string theory [28–34].

A wide range of inflationary literature has been dedicated to compere the predictions of single field models with the CMB observations. Consequently, some of them are excluded while some of them are still compatible with the Planck datasets [17, 35, 36]. As a shortcoming of these models, they don't predict any non-Gaussianity in their spectrum [37] while the future CMB observations propose the existence of the non-Gaussianity in the detected spectrum of the inflationary perturbations. In such a case, single field models will be put into question. To escape from this, constant-roll inflation has been suggested in which inflaton rolls down with a constant rate from the maximum point of the potential to the minimum point at the end of inflation as

$$\ddot{\varphi} = -(3+\alpha)H\dot{\varphi}, \tag{1}$$

where α is a non-zero parameter [38, 39]. Deviation from the slow-roll approximation is also traced in the ultra slow-roll regime where we assume a non-negligible $\ddot{\varphi} = -3H\dot{\varphi}$. This class of inflationary models shows a finite value of the non-decaying mode in the spectrum of curvature perturbations [40]. Although the solutions of ultra models are situated in the non-attractor phase of inflation, sometimes they present an attractor-like behaviour [41]. Moreover,

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the large η predicted by the ultra models can't guarantee to solve the η problem introduced in supergravity [42]. Assuming a fast rate of rolling at the beginning of inflation is comprehended as another kind of deviation from the slow-roll inflation [43–45].

Recently, the idea of constant-roll has been engaged broadly for different inflationary models [14, 15, 32, 46–60]. In this work, we focus on a type of inflationary models in which a complex scalar field is assumed as inflaton to drive inflation. Complex scalar fields in quantum and classical cosmology were first introduced in [61, 62] and then were developed in [63–65]. Complex fields are comprehended as an alternative to the quintessence field in order to explain the late-time accelerating phase of the universe, so-that called dark energy (DE) [66–68]. Besides this, a complex scalar field can play the role inflaton in the early universe [69]. Consequently, complex inflation naturally presents a graceful exit from the inflationary era with a very small number of e-folds [70]. Hence, a complex scalar field can be considered as an assistant field of a real inflaton in the context of hybrid inflation in order to terminate inflation [71].

The main aim of the present manuscript is to study a constant-roll inflationary scenario by considering a complex scalar field as the scalar field driven inflation. To achieve this, we arrange the paper as follows. In Section II, we review the foundation of complex scalar field theory and its dynamical equations, briefly. We investigate the complex inflation in the context of the constant-roll approach by using the accredited form of the angular speed $\dot{\theta}$ in Section III. In Section IV, we present the cosmological perturbations of the complex constant-roll inflation. Also, we attempt to find the constraints on the parameter space of the model using the observational values of the spectral parameters. Conclusions and remarks are drawn in Section V. In the following, we chose units so that $c = \hbar = \kappa^2 = 8\pi G = 1$.

II. COSMOLOGY WITH A COMPLEX SCALAR FIELD

Let us start with the action of complex scalar theory [61, 62]

$$S = \int d^4x\sqrt{-g}\bigg(\frac{R}{2} - \frac{1}{2}g^{\mu\nu}\partial_\mu\Psi^*\partial_\nu\Psi - V(|\Psi|) + \mathcal{L}_m\bigg), \tag{2}$$

that considers a complex scalar field Ψ minimally coupled to the Ricci scalar $R = g^{\mu\nu}R_{\mu\nu}$ in the presence of ordinary matter described by a Lagrangian density $\mathcal{L}_m$. Here, we assume that the self-interacting potential V just depends on the absolute value or amplitude of the complex scalar field [68] and the quantity g corresponds to the determinant of the metric $g_{\mu\nu}$. We define the complex scalar field in terms of its amplitude $\varphi(x)$ and phase $\theta(x)$ by

$$\Psi(x) = \varphi(x)e^{i\theta(x)}. \tag{3}$$

This configuration allows us to describe both early and late-time accelerating phases of the universe only by using a complex field without any cosmological constant [66, 68–70]. Moreover, the usefulness of using the $\varphi(x)$ and $\theta(x)$ will advantage the obtaining of the motion equations (as well as the solutions) that relate the background variables, in particular the effective potential V and the Hubble parameter H. In this form, we can rewrite the action (2) as

$$S = \int d^4x\sqrt{-g}\bigg(\frac{R}{2} - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}\varphi^2g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - V(\varphi) + \mathcal{L}_m\bigg). \tag{4}$$

By variation of the action with respect to the metric and then considering a spatially flat universe described by the Friedmann-Robertson-Walker (FRW) metric, the dynamical equations of the model are given as

$$H^2 \equiv \Big(\frac{\dot{a}}{a}\Big)^2 = \frac{1}{3}\rho = \frac{1}{3}\Big(\rho_m + \frac{1}{2}(\dot{\varphi}^2 + \varphi^2\dot{\theta}^2) + V(\varphi)\Big), \tag{5}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p) = -\frac{1}{3}\Big(\frac{1}{2}(\rho_m + 3p_m) + (\dot{\varphi}^2 + \varphi^2\dot{\theta}^2) - V(\varphi)\Big), \tag{6}$$

where the fluid filling the universe satisfies the continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{7}$$

where H is the Hubble parameter, $\rho = \rho_m + \rho_\Phi$ corresponds to the total energy density, $p = p_m + p_\Phi$ denotes the total pressure and the dot depicts the derivative with respect to cosmic time t. Here we have considered that a spatially homogeneous complex scalar field $\Phi(t)$, such that $\Phi(t) = \varphi(t)e^{i\theta(t)}$ and the tensor energy momentum $T^{\text{o.m.}}_{\mu\nu}$ associated to the ordinary matter Lagrangian can be described by a perfect fluid of the form $T^{\text{o.m}}_{\mu\nu}$ =diag$(-\rho_m, p_m, p_m, p_m)$

in which ρ_m and p_m denote the effective energy density and pressure associated to ordinary matter, respectively. Analogously, we have assumed that the energy density and pressure of the complex scalar field are given by [68]

$$\rho_\Phi = \frac{1}{2}\big(\dot{\varphi}^2 + \varphi^2\dot{\theta}^2\big) + V(\varphi), \qquad p_\Phi = \frac{1}{2}\big(\dot{\varphi}^2 + \varphi^2\dot{\theta}^2\big) - V(\varphi). \tag{8}$$

By varying the action (4) with respect to the scalar field φ, the equation of motion (EoM) for the scalar field φ becomes

$$\ddot{\varphi} + 3H\dot{\varphi} - \dot{\theta}^2\varphi + V'(\varphi) = 0, \tag{9}$$

and for phase we have the motion equation and its solution (for angular speed $\dot{\theta}$) given by

$$\ddot{\theta} + (2\dot{\varphi}/\varphi + 3H)\dot{\theta} = 0, \quad \Rightarrow \quad \dot{\theta} \propto \frac{1}{\varphi^2\, a^3}. \tag{10}$$

In the following we will consider that the prime denotes the derivative with respect to the scalar field φ.

In the context of the scenario dominated by the dark sector (DE) and in order to explain late time accelerating phase of the universe, the authors in Ref.[68] considered a non-relativistic matter with pressure $p_m = 0$ together with a complex quintessence theory in which the angular velocity $\dot{\theta}$ is proportional to $a^{-3}\varphi^{-2}$, see Eq.(10). For a review of the complex field in the framework of DE, see Refs.[66–68].

On the other hand, during the early universe the classical and quantum description of the a inflationary epoch from the complex scalar inflaton field was studied in different articles [63, 65, 72, 73] and during the reheating and the primordial black hole production in [74], see also Ref.[75]. The importance of considering a complex scalar field during this epoch lies in the fact that such fields, the non-Abelian multiplets of scalar fields and other fields arise naturally in the new theories of particle physics, string etc. In particular in Ref.[70] was utilized the Barrow's method to solve the inflationary dynamics associated to the complex field as a natural exist of inflation but with a small quantity of expansion under an angular velocity given by the expression $\dot{\theta} = \frac{M}{\varphi^2}$, where the quantity M corresponds to a constant. In particular, in the case in which the constant $M = 0$ the inflationary model reduces to, inflation with a real scalar field. Despite the mentioned success, the complex inflaton field suffers from a problem that is the number of e-folds related to the expansion of the universe is very small. Hence, using a single complex field as inflaton field will be put into doubt [69, 70].

In the following we will assume that the ordinary matter Lagrangian $\mathcal{L}_m = 0$ ($\rho = \rho_\Phi$ and $p = p_\Phi$), in order to study the constant roll inflation in the framework of the complex field. Thus, we find that the dynamical equations are given by

$$3H^2 = \frac{1}{2}\big(\dot{\varphi}^2 + \varphi^2\dot{\theta}^2\big) + V(\varphi), \qquad 2\dot{H} = -\dot{\varphi}^2 - \varphi^2\dot{\theta}^2, \tag{11}$$

and

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = \varphi\dot{\theta}^2. \tag{12}$$

From Eq.(11), we find that the effective potential can be written as

$$V(\varphi) = 3H^2 + \dot{H}. \tag{13}$$

In this form, knowing the Hubble parameter in terms of the scalar field together with the speed of the scalar field $\dot{\varphi}$, we can reconstruct the effective potential $V(\varphi)$ using Eq.(13).

III. COMPLEX CONSTANT-ROLL INFLATION

To study the complex constant-roll inflation, we start with the constant-roll condition (1) for the absolute value of the complex scalar field as

$$|\ddot{\Psi}| = -(3+\alpha)H|\dot{\Psi}|, \tag{14}$$

where as before the parameter $\alpha \neq 0$. By using the definition of the complex scalar field (3), the above constant-roll condition is reduced to the familiar form (1). From Eq.(11) and considering the definition $\dot{H} = \dot{\varphi}H'$, we find that the speed of the scalar field $\dot{\varphi}$ becomes

$$\dot{\varphi} = -H' \pm \sqrt{H'^2 - \varphi^2\dot{\theta}^2}. \tag{15}$$

In order to find Eq.(15), we have considered that the Hubble parameter is only function the scalar field φ and not of θ (or $\dot{\theta}$), since the solution given by Eq.(10) allows eliminating the dependency of the Hubble parameter of this variable (see Eq.(11)). Thus, from Eq.(10), we can write $\dot{H} = \varphi H'$ and then the relation given by Eq.(15) becomes useful to determine $\dot{\varphi}$.

As we see, in order to find a real solution for $\dot{\varphi}$, we have that during the constant roll scenario it is necessary that $H'^2 > \varphi^2\dot{\theta}^2$. Also, from Eq.(13), the potential and the speed of the scalar field can be driven as

$$V(\varphi) = 3H^2 - H'^2 \pm H'\sqrt{H'^2 - \varphi^2\dot{\theta}^2}. \tag{16}$$

Now, taking the derivative of Eq.(15) with respect to cosmic time t and using the constant-roll condition given by (1), we obtain a differential equation for the Hubble parameter in terms of the scalar field given by

$$-(3+\alpha)H = -H'' \pm \left[\frac{H'H'' - (\varphi^2\dot{\theta}^2/2)'}{\sqrt{H'^2 - \varphi^2\dot{\theta}^2}}\right]. \tag{17}$$

From Eq.(10), the phase speed $\dot{\theta}$ can be written as $\dot{\theta} = \sqrt{w}/\varphi^2 a^3$, with $w = constant > 0$. Then, the speed of the real scalar field (15) and the corresponding potential (16) are rewritten as

$$\dot{\varphi} = -H' \pm \sqrt{H'^2 - w/\varphi^2 a^6}, \qquad V(\varphi) = 3H^2 - H'^2 \pm H'\sqrt{H'^2 - w/\varphi^2 a^6}. \tag{18}$$

Moreover, we find that the differential equation for the Hubble parameter H (17) becomes

$$-(3+\alpha)H = -H'' \pm \left[\frac{H'H'' - (w/2\varphi^2 a^6)'}{\sqrt{H'^2 - w/\varphi^2 a^6}}\right]. \tag{19}$$

To obtain the analytical solutions for the background variables, we can consider that during inflation $1 \gg w/\varphi^2 a^6 H'^2$ with which

$$-(3+\alpha)HH' \simeq -H'H'' \pm [H'H'' - (w/2\varphi^2 a^6)'][1 + \frac{w}{2\varphi^2 a^6 H'^2} + ...], \tag{20}$$

and for the speed of scalar field in this approximation results

$$\dot{\varphi} \simeq -H' \pm H'\left[1 - \frac{w}{2\varphi^2 a^6 H'^2} + ...\right]. \tag{21}$$

By considering the positive sign and keeping the first term into the expansion given by Eq.(20), we have the reduced differential equation

$$(3+\alpha)HH' \simeq (\frac{w}{2\varphi^2 a^6})', \tag{22}$$

with the solution

$$H^2 + C \simeq \frac{w_1}{\varphi^2 a^6}, \tag{23}$$

where we have defined $w_1 = w/(3+\alpha)$ and C corresponds to an integration constant. Also, in this case the quantity $\dot{\varphi}$ (21) yields

$$\dot{\varphi} \simeq -\frac{w}{2\varphi^2 a^6 H'} = -\frac{w}{2w_1}\frac{(H^2+C)}{H'} = -\frac{(3+\alpha)}{2}\frac{(H^2+C)}{H'}. \tag{24}$$

In the particular case in which the integration constant $C = 0$ and in order to find the Hubble parameter in terms of the scalar field, we take the derivative of Eq.(23) with respect to scalar field results

$$H' = -H\left[\frac{1}{\varphi} + 3\frac{H}{\dot{\varphi}}\right], \tag{25}$$

where we have considered that $H = \tilde{H}\dot{\varphi}$ where $\tilde{H} = a'/a$. Then, using Eq.(24), the differential equation (25) reduces to

$$H' \simeq -H\left[\frac{1}{\varphi} - \frac{6}{(3+\alpha)}\,\frac{H'}{H}\right], \tag{26}$$

and its solution is given by

$$H(\varphi) = C_1\,[(3-\alpha)\varphi]^{-\beta}, \quad \text{with} \quad \beta = \left(\frac{3+\alpha}{\alpha-3}\right), \tag{27}$$

where C_1 corresponds to another integration constant. Note that Eqs.(22) and (25) are equivalent, since we have only utilized the solution given by Eq.(23) (with $C = 0$).

For the case in which the integration constant $C \neq 0$, we can introduce the variable change $\mathcal{H}^2 = H^2 + C$ into Eq.(23) and using Eq.(24), we find the same differential equation (26) for the variable $\mathcal{H}$. Thus, we obtain that the solution for the Hubble parameter as a function of the scalar field in the case in which the integration constant $C \neq 0$ yields

$$H(\varphi) = \sqrt{C_1^2[(3-\alpha)\varphi]^{-2\beta} - C}, \tag{28}$$

where as before C_1 denotes an integration constant and β is defined by Eq.(27). Clearly, when the integration constant $C = 0$, the solution given by Eq.(28) is reduced to the Eq.(27).

For simplicity, in the following we will focus our analysis on the stage in which the integration constant C is set to zero. In this form, plugging Eq.(27) into Eq.(24), the scalar field as a function of the cosmological time becomes

$$\varphi(t) = \frac{1}{(3-\alpha)}\left[\frac{C_1(3+\alpha)}{2}\,t + C_2\right]^{1/\beta}, \quad \text{with} \quad \alpha \neq 3. \tag{29}$$

The range of the validity for the scalar field during inflation can be determined from the condition $1 \gg w/(\varphi^2 a^6 H'^2)$ results

$$\varphi^{-2} \gg \frac{(3+\alpha)}{\beta^2}, \quad \text{with which} \quad 0 > \varphi > -\frac{|\beta|}{\sqrt{(3+\alpha)}}, \quad \text{or} \quad 0 < \varphi < \frac{|\beta|}{\sqrt{(3+\alpha)}}, \tag{30}$$

where we have used Eqs.(23) and (27), respectively.

Besides, we find that the phase speed $\dot{\theta}$ in terms of the scalar field becomes

$$\dot{\theta} = \frac{\sqrt{w}}{\varphi^2\,a^3} = \dot{\theta}_0\,\varphi^{-(\beta+1)}, \quad \text{where} \quad \dot{\theta}_0 = C_1(3+\alpha)^{1/2}\,(3-\alpha)^{-\beta}. \tag{31}$$

Thus, combining Eqs.(29) and (31) we obtain that the phase as a function of the time is given by

$$\theta(t) = \theta_1\left[\frac{C_1(3+\alpha)}{2}\,t + C_2\right]^{-1/\beta} + C_3, \tag{32}$$

where the quantity θ_1 is a constant and it is defined as

$$\theta_1 = \frac{-2\beta\,\dot{\theta}_0}{C_1\,(3+\alpha)\,(3-\alpha)^{-(\beta+1)}}, \tag{33}$$

and C_3 corresponds to another integration constant. Also, we find that the effective potential (13) as a function of the scalar field becomes

$$V(\varphi) = \left(\frac{3-\alpha}{2}\right)C_1^2[(3-\alpha)]^{-2\beta}\,\varphi^{-2\beta} = V_0\,\varphi^{-2\beta}, \tag{34}$$

where we have defined $V_0 = \frac{(3-\alpha)}{2}C_1^2[(3-\alpha)]^{-2\beta}$. Here we observe that from the approximation $w(\varphi\,a^3\,H')^{-2} \ll 1$, as much as the solution for the Hubble parameter $H(\varphi)$ and the reconstructed potential $V(\varphi)$ do not depend on the parameter w associated to the angular speed $\dot{\theta}$. This is due to the fact that we have considered the first term into the expansion (20) and then Eq.(24) does not depend on w. In the situation in which we keep the first and second term of the expansion given by Eq.(20), the differential equation (22) is modified to

$$\left[\frac{2(3+\alpha)}{w}\right] HH' \simeq \left(\frac{1}{\varphi^2 a^6}\right)' \left[1+\frac{w}{2\varphi^2 a^6 H'^2}\right] - \frac{H''}{\varphi^2 a^6 H'}, \tag{35}$$

and together with the equation $\dot{\varphi} \simeq -w/(2\varphi^2 a^6 H')$ obtained of (21) are part of the system to be solved (recall that $H = (a'/a)\dot{\varphi}$). Thus, in order to solve (numerically) this coupled system of differential equations we would need to specify the value of the parameter w.

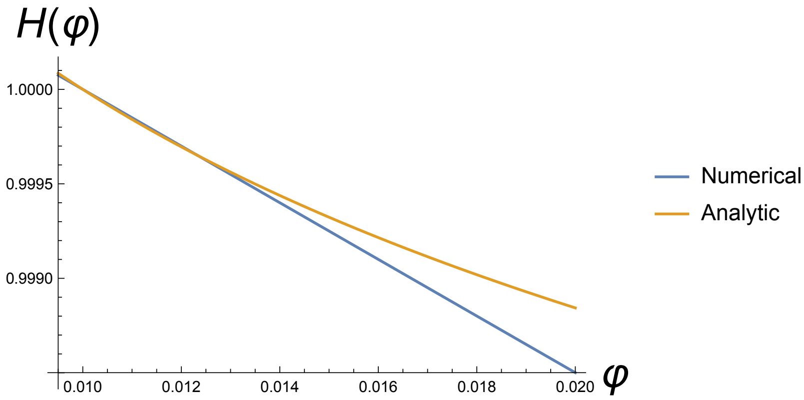

Figure 1: The plot shows the comparison between the analytic expression given by Eq.(27) and the numerical solution for the Hubble parameter $H(\varphi)$ in terms of the scalar field φ. Here we have used the positive signs of Eqs.(18) and (19) together with the values of $\alpha = -2.99$, $w = 0.01$ and $C_1 = 0.99$, respectively.

The Fig.1 shows the comparison between the analytic solution given by Eq.(27) where the approximation $1 \gg w/\varphi^2 a^6 H'^2$ is used and the numerical solution for the Hubble parameter as a function of the scalar field. Here we have utilized the values $\alpha = -2.99$, $w = 0.01$ and $C_1 = 0.99$. We note that the comparison is suitable between the numerical solution and the analytical solution inside of the validity range defined by Eq.(30) for the scalar field.

Besides, the number of $e-$folds in this case results

$$N = -\int_{t_*}^{t_f} H dt = -\int_{\varphi_*}^{\varphi_f} \frac{H}{\dot{\varphi}} d\varphi = \frac{2w_1}{w}\int_{\varphi_*}^{\varphi_f} \frac{H'}{H} d\varphi = -\frac{2}{(\alpha-3)} \ln\left[\frac{\varphi_f}{\varphi_*}\right]. \tag{36}$$

In the following, the subscripts f and $*$ are used to describe to the epoch the end of inflation and when the cosmological scales exit the horizon, respectively.

By assuming that the end of inflation occurs when the slow roll parameter $\epsilon_1 \simeq \frac{1}{2}(V'/V)^2 = 1$, then we find that the value of the scalar field at the end of inflation is given by

$$\epsilon_1(\varphi = \varphi_f) \simeq \frac{2\beta^2}{\varphi_f^2} = 1, \quad \Rightarrow \quad \varphi_f = \pm\sqrt{2}\beta. \tag{37}$$

In this way, we obtain that the value of the scalar field when the cosmological scales exit the horizon yields

$$\varphi_* = -\sqrt{2}\,\beta\, e^{(\alpha-3)N/2}, \tag{38}$$

where we have considered the negative sign of φ_f (see Eq.(37)), since as we will see later the parameter β is a negative quantity.

IV. COSMOLOGICAL PERTURBATIONS

In this section, we will analyze the scalar and tensor perturbations, in which the former will be characterized by considering the longitudinal gauge on the metric. Thus, by considering the longitudinal gauge in the perturbed FRW metric we write

$$ds^2 = (1+2\Phi)dt^2 - a^2(t)(1-2\Phi)\delta_{ij}dx^i dx^j, \tag{39}$$

where the function $\Phi = \Phi(t, \mathbf{x})$ corresponds to the Bardeen's gauge invariant variable [76]. Assuming the spatial dependence $e^{i\mathbf{kx}}$, where the quantity k is the wavenumber, each Fourier mode satisfies the following perturbed equations of motion given by

$$\dot{\Phi} + H\Phi = \frac{1}{2}\left[\dot{\varphi}\delta\varphi + \varphi^2\dot{\theta}\delta\theta\right], \tag{40}$$

$$\delta\ddot{\varphi} + 3H\delta\dot{\varphi} + \left[\frac{k^2}{a^2} + V'' - \dot{\theta}^2\right]\delta\varphi - 2\varphi\dot{\theta}\delta\dot{\theta} = 4\dot{\varphi}\dot{\Phi} - 2V'\Phi, \tag{41}$$

and

$$\delta\ddot{\theta} + \left[3H + \frac{2}{\varphi}\dot{\varphi}\right]\delta\dot{\theta} + \frac{k^2}{a^2}\delta\theta - \frac{2\dot{\varphi}}{\varphi}\delta\dot{\varphi} = 4\dot{\theta}\dot{\Phi}. \tag{42}$$

Here, the quantities $\delta\varphi$ and $\delta\theta$ correspond to the gauge invariant fluctuation variables associated to the respective fields φ and θ, respectively. In order to establish that the quantities $\delta\phi$ and $\delta\theta$ are gauge invariant fluctuation variables, we can consider an infinitesimal gauge transformation on the coordinates $\tilde{x^\nu} = x^\nu + \delta x^\nu$ together with the most generic perturbed metric; $ds^2 = a^2[(1+2\Phi)d\tau^2 - 2\partial_i B d\tau dx^i - [(1-2\psi)\delta_{ij} + (\partial_i\partial_j - (1/3)\delta_{ij}\nabla^2)E]dx^i dx^j]$, with τ the conformal time and a detailed explication see Refs. [77, 78]. Since φ and θ are scalar fields (amplitude and phase), these satisfy the transformation laws for gauge invariant (GI); $\delta\varphi_{GI} = \delta\varphi - \varphi'(E'/2 - B)$ and $\delta\theta_{GI} = \delta\theta - \theta'(E'/2 - B)$, where a prime now indicates differentiation wrt to the conformal time [77, 78]. Thus, from the longitudinal gauge on the metric given by Eq.(39), we have $B = E = 0$ and $\Phi = \psi$ and then the quantities $\delta\varphi$ and $\delta\theta$ are gauge invariant fluctuation variables.

By considering large scale perturbations in which $k \ll aH$ and neglecting the term $\dot{\Phi}$ and those terms which include second order time derivative, then the above equations of motion reduce to

$$H\Phi \simeq \frac{1}{2}\left[\dot{\varphi}\delta\varphi + \varphi^2\dot{\theta}\delta\theta\right], \tag{43}$$

$$3H\delta\dot{\varphi} + V''\delta\varphi \simeq -2V'\Phi, \tag{44}$$

and

$$3H\delta\dot{\theta} \simeq 0, \quad \Rightarrow \quad \delta\theta = \mathcal{C}. \tag{45}$$

Here we have considered that as we need the non-decreasing modes on large scale in our model, which are determined to be weakly time-dependent [79, 80], then we can consistently ignore the term associated to $\dot{\Phi}$, including those involving two time derivatives. In relation to the existence of growing and decaying modes is attributed to the the observation outlined in Ref.[81], with a more detailed explication provided in Ref.[79].

Thus, we find that the Eq.(44) can be written as

$$\delta\varphi' + \frac{1}{3H\dot{\varphi}}\left[V'' + \frac{V'}{H}\dot{\varphi}\right]\delta\varphi + \left(\frac{V'\varphi^2\dot{\theta}}{3H\dot{\varphi}}\right)\mathcal{C} = 0, \tag{46}$$

where we have used $\delta\dot{\varphi} = \dot{\varphi}\delta\varphi'$.

We note that the relation between the real variables $\delta\varphi$ and $\delta\theta$ with the variation of the complex field $\delta\Psi$ from Eq.(3) becomes $\delta\Psi = (\Psi/\varphi)[\delta\varphi + i\,\delta\theta]$. Under the large scale approximation, we find that the relation between the Bardeen's variable Φ and the variation of the complex field $\delta\Psi$ can be written as $2H\Phi \simeq \delta\Psi[(\varphi/\Psi)\dot{\Psi} - i\,\dot{\theta}](\varphi/\Psi) - \mathcal{C}[\dot{\theta}(1-\varphi^2) + i\,(\varphi/\Psi)\dot{\Psi}]$, where we have utilized Eqs.(43) and (45), respectively. Thus, this relation indicates that the metric perturbation is determined when the evolution of the fields are known.

In order to find an analytical solution to the gauge invariant variable $\delta\varphi$, we consider for the integration constant as $\mathcal{C} = 0$. In this way, we find that the solution of Eq.(46) can be written as

$$\delta\varphi = \mathcal{C}_0\,\exp\left[-\int_{\varphi}^{\varphi_f}\frac{1}{3H\dot{\varphi}}\left(V'' + \frac{V'}{H}\dot{\varphi}\right)d\varphi\right], \tag{47}$$

where $\mathcal{C}_0$ corresponds to an integration constant. Therefore, the variable Φ becomes

$$\Phi \simeq \frac{1}{2}\frac{\dot{\varphi}}{H}\mathcal{C}_0\,\exp\left[-\int_{\varphi}^{\varphi_f}\frac{1}{3H\dot{\varphi}}\left(V'' + \frac{V'}{H}\dot{\varphi}\right)d\varphi\right]. \tag{48}$$

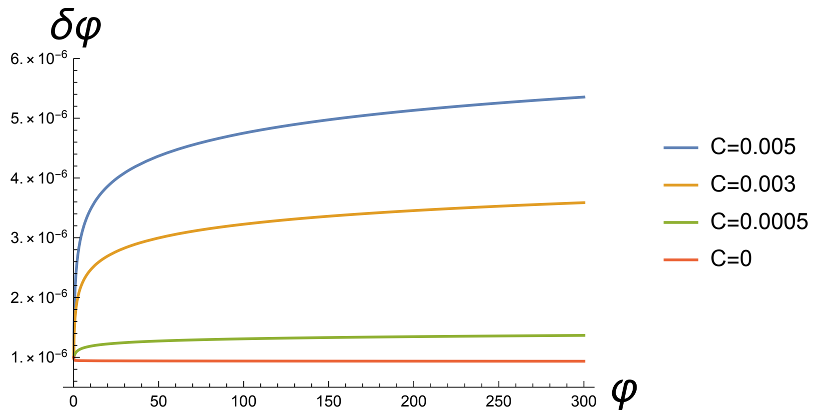

Figure 2: The plot shows the numerical solution for the gauge invariant fluctuation variable $\delta\varphi$ versus the scalar field φ given by Eq.(46), for different values of the constant $\mathcal{C}$. In this plot we have utilized the values $C_1 = 0$, $C_2 = 0$ and $\alpha = -2.99$.

Thus, from Eq.(24), we find that Eq.(47) can be rewritten as

$$\delta\varphi \simeq \mathcal{C}_0 \exp\left[\int_{\varphi}^{\varphi_f} \frac{2w_1 H'}{3wH(H^2+C)}\left(V'' - \frac{w(H^2+C)V'}{2w_1 H\, H'}\right) d\varphi\right], \tag{49}$$

and analogously for Φ, we get

$$\Phi \simeq -\mathcal{C}_0 \left(\frac{w(H^2+C)}{4w_1 H\, H'}\right) \exp\left[\int_{\varphi}^{\varphi_f} \frac{2w_1 H'}{3wH(H^2+C)}\left(V'' - \frac{w(H^2+C)V'}{2w_1 H\, H'}\right) d\varphi\right]. \tag{50}$$

The Fig.2 shows the numerical solution given by Eq.(46) for the fluctuation variable $\delta\varphi$ in terms of the scalar field φ, for different values of the constant $\mathcal{C}$ defined by Eq.(45). Here we have used the values $C_1 = 1$, $C_2 = 0$ and $\alpha = -2.99$. In order to write down values for the numerical solution of Eq.(46), we have utilized Eqs.(27), (29), (31) and (34) for the background variables. From this plot, we observe that for values of integration constant $\mathcal{C} \ll 1$, the numerical solutions of $\delta\varphi(\varphi)$ exhibit behaviors closely resembling those observed when $\mathcal{C} = 0$. Hence, given that the gauge-invariant fluctuation variable $\delta\varphi$ remains significantly smaller than unity ($\delta\varphi \ll 1$), it is pertinent to consider the scenario where $\mathcal{C} = 0$ as a suitable approximation.

Following Refs.[82–84], we assume that gauge invariant density fluctuation $\delta\rho^c/\rho = \delta_H$ is defined as $\delta_H = -(2/3)(k/aH)^2\Phi$ and considering that the curvature perturbation due to primordially adiabatic fluctuation $\Phi = [1 + 2/(3(1+p/\rho))]^{-1}\mathcal{C}_0$, then from Eq.(49) we find that the density perturbation at the horizon crossing in which $k = aH$, results

$$\delta_H = f\left(\delta\varphi\ \ \exp\left[-\int_{\varphi}^{\varphi_f} \frac{2w_1 H'}{3wH(H^2+C)}\left(V'' - \frac{w(H^2+C)V'}{2w_1 H\, H'}\right) d\varphi\right]\right)\Big|_{\varphi=\varphi_*} = f\left(\delta\varphi\, e^{-\mathcal{F}(\varphi)}\right)\Big|_{\varphi=\varphi_*}, \tag{51}$$

where the function $\mathcal{F}(\varphi)$ is defined as

$$\mathcal{F}(\varphi) = \int_{\varphi}^{\varphi_f} \frac{2w_1 H'}{3wH(H^2+C)}\left(V'' - \frac{w(H^2+C)V'}{2w_1 H\, H'}\right) d\varphi. \tag{52}$$

Further, the quantity f is a constant and in the specific case associated to the radiation domination f takes value $f = 4/9$ and for matter domination we have $f = 2/5$. Here the constant $f = (2/3)[1+2/(3(1+p/\rho))]^{-1}$ is introduced by normalization at the second crossing after inflation during the radiation or matter domination Refs.[82–84]. Also the minus sign can be omitted, since this sign may be absorbed into the stochastic variable $\delta\varphi$. Thus, the large-angular-scale anisotropy of background due to the Sachs-Wolfe effect is well defined as $\delta T/T = \Phi/3$ [85].

Besides, the fluctuations $\delta\varphi$ are generated by small scale perturbations and then they can be assumed as free massless scalar field in which are described by independent random variables such that at the horizon crossing we have $\delta\varphi = H/2\pi$, see e.g. Refs.[4, 86]. By considering that the scalar power spectrum $\mathcal{P}_{\mathcal{S}} = (25/4)\delta_H^2$ [87], then we find that the spectrum at the horizon crossing can be written as

$$\mathcal{P}_{\mathcal{S}}(\varphi = \varphi_*) = \left(\frac{25}{4}\right)\left(\frac{f}{2\pi}\right)^2 \left[H^2\, e^{-2\mathcal{F}(\varphi)}\right]\Big|_{\varphi=\varphi_*}. \tag{53}$$

Further the scalar spectral index n_s is defined as

$$n_s - 1 = \frac{d\ln \mathcal{P_S}}{d\ln k}, \tag{54}$$

in which the wavenumber k is related to the number of e-folds N by the relationship $d\ln k \simeq dN$. Thus, from Eq.(53) we find that the index n_s can be written as

$$n_s \simeq 1 + 2\frac{\dot{\varphi}}{H}\left[\frac{H'}{H} - \mathcal{F}'(\varphi)\right] \simeq 1 - \left(\frac{w}{w_1}\right)\frac{(H^2+C)}{H\,H'}\left[\frac{H'}{H} - \mathcal{F}'(\varphi)\right]. \tag{55}$$

Besides, it is well known that the production of tensor perturbations during the inflationary epoch would produce gravitational wave. In this context, the spectrum of the tensor perturbations $\mathcal{P_T}$ after Hubble exist can be approximated to [86, 87]

$$\mathcal{P_T} \simeq 4\left(\frac{H}{\pi}\right)^2\Big|_{\varphi=\varphi_*}. \tag{56}$$

Also, an important observational parameter corresponds to the tensor to scalar ratio which is defined as

$$r = \frac{\mathcal{P_T}}{\mathcal{P_S}}, \tag{57}$$

and in our case this ratio can be written as

$$r \simeq \frac{64}{25}\frac{1}{f^2}e^{2\mathcal{F}(\varphi)}. \tag{58}$$

For our model, from Eq.(23), we find that the function $\mathcal{F}(\varphi)$ defined by Eq.(52) yields

$$\mathcal{F}(\varphi) = \frac{\beta(\alpha-3)}{3}\left\{\ln(\varphi) + \frac{\beta(1+2\beta)}{\varphi^2}\frac{w_1}{w}\left(1 + {}_2F_1\left[1, -\frac{1}{\beta}, \frac{-1+\beta}{\beta}, -\frac{C(\varphi(\alpha-3))^{2\beta}}{C_1^2}\right]\right)\right\}\Bigg|_{\varphi_f}^{\varphi},$$

where the quantity ${}_2F_1$ corresponds to the hypergeometric function. In the particular case in which the integration constant $C = 0$, the above function reduces to

$$\mathcal{F}(\varphi) = \frac{\beta(3-\alpha)}{3}\left\{\ln(\varphi) + \frac{\beta(1+2\beta)}{\varphi^2}\frac{w_1}{w}\right\}\Bigg|_{\varphi_f}^{\varphi}. \tag{59}$$

By considering Eqs.(55) and (59) we obtain the that scalar spectral index n_s can be written as

$$n_s(\varphi=\varphi_*) = n_{s_*} \simeq -(2+\alpha) + \frac{(\alpha^2-9)}{3}\left[1 - \frac{2\beta(1+2\beta)}{\varphi^2}\frac{w_1}{w}\right]\Bigg|_{\varphi_*}. \tag{60}$$

Also, the tensor-to-scalar ratio of the model is given by

$$r_* \simeq \frac{64}{25}\frac{1}{f^2}\exp\left[\frac{2\beta(3-\alpha)}{3}\left(\ln(\frac{\varphi}{2\sqrt{\beta}}) + \frac{1+2\beta}{3+\alpha}(\frac{\beta}{\varphi^2} - \frac{1}{4})\right)\right]\Bigg|_{\varphi_*}. \tag{61}$$

Thus, considering that the scalar spectral index at the horizon crossing takes the value $n_{s*} = 0.967$ and using Eqs.(37) and (38), we find numerically that the parameter α has a negative value given by

$$\alpha \simeq -2.989. \tag{62}$$

Here we have considered that at the crossing the number of $e-$ folds $N = 60$.

Besides, from the scalar power spectrum defined by Eq.(53) and considering that at the crossing this quantity is $\mathcal{P_S}(\varphi=\varphi_*) \simeq 2.2\times10^{-9}$ together with $\alpha = -2.989$, we find that the integration constant $C_1 = \pm 1.02\times10^{-4}$. However, as the Hubble parameter is a positive quantity we only have to consider $C_1 = 1.04\times10^{-4}$. This value of C_1 suggests that the quantity $V_0 = \frac{(3-\alpha)}{2}C_1^2(3-\alpha)^{-2\beta}$ (amplitude of the potential) associated to the reconstructed potential $V(\varphi) = V_0\varphi^{-2\beta}$ becomes $V_0 \simeq 3.1 \times 10^{-8}$, when $\alpha = -2.989$ and $C_1 \simeq 10^{-4}$. In this form, we find that the value of the effective potential at the end of inflation results $V(\varphi = \varphi_f) \simeq 3.03 \times 10^{-8}$ (in units of M_p^4, where M_p the Planck mass) and at the crossing in which the number of $e-$folds $N = 60$ the potential $V(\varphi = \varphi_*) \simeq 5.88 \times 10^{-8}$ (in units of M_p^4). In this context, we obtain that the energy density of the universe during inflation in our model becomes $\rho \sim V \sim \mathcal{O}(10^{-7}) \sim \mathcal{O}(10^{-8})\ M_p^4$ and this energy scale is similar to that obtained in different inflationary models during the early universe, see e.g., [1–3, 87]. As before we have used that the number of $e-$folds $N = 60$ and also we have considered the value $f = 4/9$.

Also, from Eq.(61) we find that the tensor to scalar ratio $r_* \sim 10^{-78}$, when we consider the values of $N = 60$, $f = 4/9$, together with the value given by Eq.(62) for the parameter α. In this sense, we find that our model predicts that the tensor to scalar ratio ratio $r \simeq 0$, see right panel of Fig.3.

In Fig.3 we show the evolution of the observational parameters n_s and r versus the parameter α associated to Eq.(14). The left panel shows that the observational value for the scalar spectral index $n_s = 0.967$ takes place for the value $\alpha \simeq -2.989$. In order to write down the scalar spectral index n_s as a function of the parameter α, we consider Eq.(60) together with the Eq.(38) in which the number of $e-$folds $N = 60$. The right panel shows the evolution of the tensor to scalar ratio r (logarithmic scale) in terms of the parameter α. Also, in order to write down the observational parameter r as a function of α, we consider Eqs.(37), (38) and (61), where we have assumed $N = 60$ and the parameter $f = 4/9$, respectively. From this panel, we note that the tensor to scalar ratio $r \simeq 0$, when the observational parameter $n_s \sim 0.97$. On the other hand, in order to give an appropriate comparison and distinction

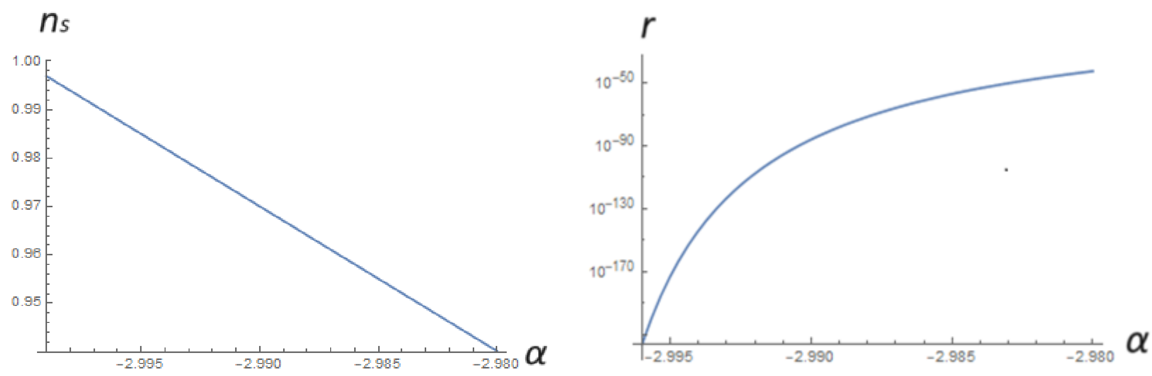

Figure 3: Observational parameters: The left panel shows the behavior of the scalar spectral index versus the parameter α and the right panel shows the tensor to scalar ratio r (logarithmic scale) versus the parameter α. From the left panel we observe that the value $n_s = 0.967$ for the scalar spectral index corresponds to $\alpha = -2.989$. Additionally, from the right panel we note that the tensor to scalar ratio $r \sim 0$ for values of α in which $n_s \simeq 0.97$. Here we have used that at the crossing the number of $e-$folds $N = 60$ and $f = 4/9$.

between the real inflaton field and the complex field, we have to consider the situation in which the parameter $\alpha > -3$. In this context, when we analyze the case in which $\alpha > -3$ for a real field, see Ref.[38], the reconstructed potentials in terms of the scalar field are an exponential potential associated to power-law inflation [88] and a hyperbolic cosine potential which is similar to that found in Ref.[89] (inflationary solutions). For the complex field we have found that the reconstructed potential corresponds to a power-law potential $V(\varphi) \propto \varphi^{-2\beta}$ becoming different to the real field. Here the evolution of the scalar field on the reconstructed potential in the framework of complex field becomes similar to the hyperbolic cosine potential, since both potentials present a minimum at $\varphi = 0$, and the evolution of the field takes place from large-φ towards the minimum. In relation to the observational parameters and in particular from the scalar spectral index n_s, it is found that the parameter α takes the value $\alpha = -3.02$ when the index $n_s = 0.96$ [38]. In our case using a complex field we have obtained the value $\alpha = -2.989$ and it suggests that both constrains on the parameter α are similar from observational parameter n_s. However, the analysis realized assuming a real inflaton field predicts $\alpha < -3$ and from a complex inflaton field suggests $\alpha > -3$ from the scalar spectral index.

V. CONCLUSIONS AND REMARKS

In the present manuscript, we have studied the complex inflation in which a complex field as inflaton is the main responsible to drive inflation. We have worked with an expression for the angular velocity $\dot{\theta} \propto \varphi^{-2}\, a^{-3}$ determined from the motion equation for the phase θ that leads to some interesting inflationary solution.

By applying the constant-roll condition on the absolute value of the complex scalar field, we have found a specific form for the Hubble parameter H and of the effective potential as a function of the scalar field, by assuming that during inflationary era the term $w(\varphi\, a^3\, H')^{-2} \ll 1$. Here the solutions for the Hubble parameter and the effective potential have a dependency power-law type with the scalar field φ. Also, we have found that the evolution of the phase as a function of the cosmological time scale as $\theta(t) \propto t^{-1/\beta}$. In this context, we have noted that in the approximation $w(\varphi\, a^3\, H')^{-2} \ll 1$ the parameter related to the angular speed w is not present in the solutions of $H(\varphi)$ and $V(\varphi)$, then this parameter can not be constrained. Besides, from the condition $w(\varphi\, a^3\, H')^{-2} \ll 1$ we have found different ranges of validity for the scalar field, in which an upper bound is obtained for positive values of φ or a lower bound for negative values of φ, see Eq.(30). In this form, we have utilized these ranges for the scalar field to constraint the different results found on parameter-space from the observational parameters.

Additionally, we investigated the corresponding scalar perturbations where we have obtained the perturbed equations of motion using the longitudinal gauge. By considering large scale perturbations and neglecting some terms we have reduced the equations system and then we have found solutions for the Bardeen's variable and the perturbation associated to the scalar field. A general relation for the scalar power spectrum and the scalar spectral index n_s are given by Eqs.(53) and (55), respectively. In particular from the background solutions for the Hubble parameter and the effective potential in terms of the scalar field we have found analytical quantities for the observational parameters such as; scalar spectral index and the tensor to scalar ratio.

By comparing the obtained results with the datasets coming from CMB anisotropies, we have attained the observational constraints on the parameters space of the model, in particular, the constant-roll parameter α see Eq.(62) and the integration constant associated to the Hubble parameter $C_1 \sim \mathcal{O}(10^{-4})$.

Also, from Eq.(61) we have obtained that the tensor to scalar ratio $r_* \sim \mathcal{O}(10^{-78})$, when we consider the values of $N = 60$ and $\alpha = -2.989$. This value for the tensor to scalar ratio suggests that our analysis done in the framework of constant roll with a complex inflaton, predicts a ratio $r \sim 0$, see Fig.3. Additionally, from Fig.3 (left panel) we have ratified that the observational parameter $n_s \sim \mathcal{O}(1)$ when the parameter $\alpha \sim -3$.

In relation to the condition give by Eq.(1), we see the difficult to understand how the scalar field could begin its movement on the reconstructed potential given by (34) with the appropriate initial condition $\ddot{\varphi} = -(3+\alpha)H\varphi$, in which the potential satisfies the relation $V'(\varphi) = \alpha H\dot{\varphi} + \varphi\dot{\theta}^2$. In this sense, the study of this issue concerning the consequences of the constant roll initial condition from the dynamics of the inflaton and how would be the special features that would be a consequence this initial conditions in our scenario would be a very interesting subject for future research concerning our model.

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